International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 15 Issue 6, June-2026

Free Vibration and Buckling Analysis of Tapered Beam with open Transverse Crack

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ABSTRACT

Tapering beams are used in diversities for their economic, aesthetic and other considerations in architecture, aeronautics, robotics and other innovative engineering applications. More recently they have been the subject of numerous studies. Present study deals with the vibration and buckling behavior of linearly tapered beams with single open transverse cracks. The variety of natural frequency and buckling load with distinctive parameters including relative crack depth, position of cracks and the slope of the beam are analyzed using finite element methods (FEM). A Matlab code is developed for the computation of natural frequencies and buckling loads of the cracked beam. Crack in the beam is represented by a rotating spring in line with T. D. Chaudhari, S. K. Maiti (1999). Stiffness matrix of the cracked tapered beam element is obtained from the flexibility matrix of the intact beam and the additional flexibility matrix due to crack. Free vibration frequencies and buckling load of a cracked cantilever beam reduce with an increase in crack depth. And also frequencies reduce more with the crack located nearer to the fixed end than the free end. For an intact tapered beam the frequencies vary more for the depth ratio compared to breadth ratio. The free vibration frequencies of a single cracked beam also vary more for depth ratio compared to breadth ratio. The vibration results can also be utilized as a tool for structural health monitoring, testing of structural integrity, execution and safety.

INTRODUCTION

It is known that beams are the basic structural components and can be classified according to their geometric configuration. They are usually uniform or non-uniform, and slender or thick. Non-prismatic members are increasingly being used in diversities as for their economic, aesthetic, and other considerations. If we analyze the non-uniform beams more practically provide a better or more suitable distribution of mass and strength than uniform beams and therefore can meet special functional requirements in architecture, aeronautics, robotics, and other innovative engineering applications and they have been the subject of numerous studies. Tapered beam have functions in turbine machinery. For long spans, tapered beams are the alternatives for uniform beams, which give economically good results.

The number of tapered beams used in construction has increased in recent years because of the economic benefits gained from optimizing different cross-sections within the lengths of those beams. Compared with prismatic beams, the effect of a variable cross-section will affect the shear stress distributions in tapered beams.

In the 19th century, Russian engineer D.I. Zhuravskii first deduced a formula to calculate shear stresses in prismatic beams with rectangular cross-sections based on the static equilibrium in a small element of a prismatic beam and the theorem of conjugate shearing stress. The resultant formula is the well-known Zhuravskii shear stress formula found in most mechanics of materials textbooks. However, the Zhuravskii shear stress formula is limited by the fact that it is only applicable to rectangular prismatic beams within the elastic range.

LITERATURE REVIEW

Many engineering structures may have structural defects such as cracks due to mechanical vibrations, environmental attack, corrosion, long term service and cyclic load etc. A crack on a beam element introduces local flexibility due to strain energy concentrations in the vicinity of the crack tip under the load. This flexibility changes the dynamic behavior of the beam. The dynamic characteristics of cracked beams are of considerable importance in many designs.

Gupta (1985) derived the stiffness and consistent mass matrices for tapered beam with linearly variable cross-sectional elements in explicit form. He derived expression for any type of cross sectional area. He obtained Numerical results of free vibration for some tapered beams using the derived matrices. These obtained results are compared with the analytic solution of uniform beam elements. Convergence characteristics and solution accuracy are examined. Rosa and Auciello (1996) examined the dynamic behavior of tapered beams with variable cross-section. He considered the ends of the beams to be flexible both rotationally and axially. For this, they used Bessel functions for solving equation of motion. When they applied boundary conditions, the equation obtained is a function of four flexibility functions. The cross-sectional parameters such as height and depth are linearly varied. Chaudhari and Maiti (1999) proposed the transverse vibration of the tapered beam with constant thickness and linearly variable depth with an `open' edge crack present normal to its axis. They introduced the concept of rotational spring to represent the crack section. The Frobenius method was used to detect the possible location of the crack. A number of numerical examples are discussed to show the effectiveness of the inverse problem.

Mazanoglu *et. al.* (2008) paper presented the energy-based method for the vibration identification of non- uniform Euler-Bernoulli beams having different open cracks. The dissemination of energy expended is dictated by considering the both strain change at the cracked beam surface and extensive impact of the stress field created by the angular displacement of the beam because of bending. The Rayleigh-Ritz approximation strategy is utilized as a part of the analysis.

Rezaee and Hassannejad (2010) proposed energy balance method for free vibration analysis of cracked cantilever by considering both the damping due to the crack and structural damping. The stiffness changes in the crack location are thought to be a nonlinear amplitude dependent function which causes the frequencies and mode shapes of the beam to change constantly with time.

Achawakorn and Jearsiripongkul (2012) introduced an approximate method to analyze uniform and non-uniform beam. Euler-Bernoulli thin beam equation is the base of the differential equation formation. This analytical method gives approximate results at the highest degree of calculations. Galerkin"s method is used in this analysis.

THEORITICAL MODELLING OF A TAPERED BEAM

The beam element is assumed to be associated with two degrees of freedom, one rotation and one translation at each node. The location and positive directions of these displacements in a linearly varying tapered beam element are demonstrated in fig below. Some commonly utilized cross-sectional shapes of beams are demonstrated in Table. The depths of the cross sections at the ends are represented by h1 (at fixed end) and h2 (at free end), similarly the widths at the ends are represented by b1 (at fixed end) and b2 (at free end) respectively. Length of the beam is taken as „l". The axis about which bending is assumed to occur is demonstrated by a line in the center coinciding with the neutral axis.

FINITE ELEMENT FORMULATION

Finite element method is the most suitable technique for digitalized computers. It includes a body to be discretized into smaller bodies having equivalent system. Then the whole body is represented by assembling such small bodies. Every subsystem is comprehended separately and the outcomes so acquired are then joined to get solution for the entire body. The finite element method is relevant to the extensive variety of problems, including nonlinear stress- strain relations, non-homogeneous materials and confounded boundary conditions. Such problems are typically handled by one of the three methodologies, namely,

(1). Displacement Method or Stiffness Method (2). Equilibrium Method or Force Method
(3). Mixed Method

Our concern is particularly on Displacement method, is widely used method because of the simplicity and can handled easily with a computer.
A polynomial function should possess the following requirements,

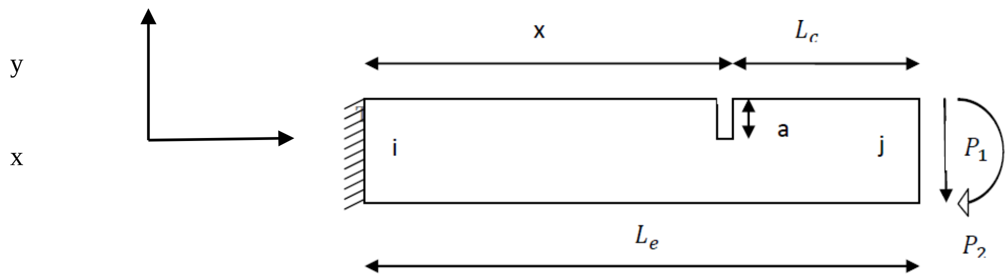
1.It is to be continuous within the elements and should be compatible between the adjacent elements.
2.It should include the rotations and rigid body displacements of the element.

3.Should possess a strained state that is consistent.

Mathematical Formulation for Crack Stiffness

A regular cracked uniform cantilever beam element of the rectangular cross sectional areawith „b‟ as the breadth, ‟h‟ as the depth of the beam and the depth of the crack are indicated by
„a‟ as shown in fig.4.2.

The left side end, which is fixed, is denoted with node „I‟ and right side node is denoted with „j‟. A shearing force „P_1' and bending moment „P_2' is subjected by the beam element. The overseeing equations of the vibration analysis of the uniform beam with open transverse crackare processed on the premise of the FEM model proposed by Zheng(2004).



RESULTS AND DISCUSSION

The dynamic and static behavior of a beam can be studied with its stiffness properties. Structural defects are origin for local flexibilities results deficiency in structural resistance. The presence of cracks in a structure results, changes in its stiffness. We can observe the changes in the local flexibilities Structural deficiencies like cracks give deficiencies in the local flexibilities. In this chapter the following contents are discussed,

1. Convergence study
2. Comparison with previous data
3. Results on the effects of various parameters on the vibration and buckling of intact and single cracked tapered beam are presented.

A MATLAB code is developed to calculate the natural frequencies and mode shapes of the tapered cantilever beam by using the Finite element method. The solutions thus obtained are compared with previously established results to check the accuracy of the lower four natural frequencies for various crack depths and various crack positions.

For the fourth mode of frequency, it is observed from fig. 5-10 that for the crack location factor 0.2, the natural frequency reduced approximately 0.09%, 0.20% and 0.44% for relative crack depths 0.1, 0.3 and 0.5 respectively. And for the crack location factor 0.9 from the fixed end the non-dimensional frequencies are reduced by 0.41%, 0.97% and 3.5%. The maximum drop in the non-dimensional frequencies 1.12%, 2.59% and 7.65% located for a crack location factor of 0.81 from the fixed end.

CONCLUSION AND FUTURE WORK

Free vibration and buckling analysis of a tapered beam subjected to a transverse crack hasbeen carried out using Finite element method in Matlab environment. The following observations are concluded from the present study,

- Mathematical formulation for free vibration and buckling analysis of a tapered beam with transverse open edge crack is presented.
- Free vibration frequencies for both intact and single cracked tapered beams increase with increase in depth ratio (α) whereas the breadth ratio (β) has a detrimental effect. This is a very useful concept that can be used in structures or machine members where strength to weight ratio is important to be considered for minimal weight and highest strength, simultaneously increasing the fundamental frequency.
- The natural frequencies for a single cracked taper beam are influenced by crack depth, location of the crack and

taper ratio.

- In a cracked tapered cantilever beam, the first mode of frequency has a maximum drop at the fixed end; the second mode of frequency has a maximum drop for a location factor of 0.6 from the fixed end. Similarly, the third and fourth mode of frequencies has a maximum drop at a location factor of 0.4 and 0.81 from the fixed ends respectively.
- Buckling loads for both intact and single cracked beam s increase rapidly with increase in the depth ratio α than breadth ratio β.

REFERENCES

1. Achawakorn, K. and Jearsiripongkul, T.(2012), "Vibration Analysis of Exponential Cross- Section Beam Using Galerkin‟s Method", *International Journal of Applied Science and Technology, Vol. 2 No. 6*
2. Auciello, N.M.(2013), "Dynamic analysis of rotating tapered Rayleigh beams using two general approaches", *Electronic International Interdisciplinary Conference September, 2. – 6.*
3. Bayat, M., Pakar, I. and Bayat, M.(2011), "Analytical study on the vibration frequencies of tapered beams", *Latin American Journal of Solids and Structures 8, 149-162.*
4. Bazoune, A., Khulief Y.A., Stephen, N.G. and Mohiuddin, M.A., (2001), "Dynamic response of spinning tapered Timoshenko beams using modal reduction", *Finite Elements in Analysis and Design, 37, 199-219.*
5. Behzad, M., Meghdari, A. and Ebrahimi, A., (2005) "A new approach for vibrational analysis of a cracked beam", *international journal of engineering, Vol. 18, No. 4, 319.*
6. Chaudhari, T.D. and Maiti, S.K., (1999), "Modeling of transverse vibration of beam of linearly variable depth with edge crack", *Engineering Fracture Mechanics 63, 425-445.*
7. Cheng, Y., Zhigang Y., Xun W. and Yuan, Y.(2011), "Vibration analysis of a cracked rotating tapered beam using the p-version finite element method", *Finite Elements in Analysis and Design 47, 825–834*
8. Daniel J. Marquez-Chisolm, "Natural frequencies and mode shapes of non-linear uniform cantilever beam", *AFIT/GAE/ENY/06-S06.*
9. De Rosa, M. A. and Auciello, N. M.(1996), "Free vibration of tapered beams with flexible ends", *Computers & Structures Vol. 60, No 2, 197-202.*