

Mathematical Approaches to Sustainable Inventory Management: Minimizing Costs and Carbon Emissions

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ABSTRACT

Reducing operational costs, optimizing supply chains, and improving sustainability are all greatly aided by effective inventory management. Optimal stock levels and minimization of holding costs are the primary goals of conventional models of inventory management. Strategies that decrease carbon footprint while achieving optimal inventory levels are vital, especially with the growing worldwide emphasis on sustainability and environmental responsibility. In order to improve operational efficiency and decrease environmental effect, this research suggests incorporating sustainable replenishment models into inventory management techniques. Taking into account the carbon costs of logistics, transportation, and stock replenishment, we investigate how mathematical models might assist in the design of inventory systems that optimize economic and environmental objectives.

Keywords: Sustainable, Carbon Emissions, Holding cost, Replenishment, Stock

INTRODUCTION

Sustainability has become an important consideration in operational decision-making, especially in the context of supply chain management, in today's corporate world. More and more businesses are realizing they need to strike a balance between being environmentally responsible and running efficiently as environmental concerns grow. An essential part of running a supply chain, inventory management is crucial during this change. Economic Order Quantity (EOQ) and Just-in-Time (JIT) are two examples of traditional inventory management models that have historically prioritized reducing ordering and holding expenses. Recent years, however, have seen an expansion in the environmental impact of these models, with a focus on transportation and warehousing carbon emissions as they pertain to inventory replenishment. Aligning corporate operations with environmental sustainability goals, sustainable inventory management seeks to decrease operating costs and the carbon footprint of supply chains.

Sustainability in inventory management revolves around reducing carbon emissions caused by supply chain operations. When it comes to an organization's inventory system, transportation is a major source of carbon emissions. Greenhouse gas emissions are caused by the substantial energy consumption—mainly from fossil fuels—in the supply chain management process, which involves transporting items from suppliers to warehouses to customers. Energy consumption is another way in which warehouse activities contribute to environmental degradation. This is particularly true when facilities use energy sources that are not renewable. Transportation and warehouse operations are frequently designed for cost-efficiency rather than sustainability, exacerbating the environmental impact. Because of this disconnect, models for inventory management that take into account ecological and economic concerns are necessary. Incorporating carbon footprint factors into current models is a key component of sustainable inventory management, which is driving a movement towards more eco-friendly supply chain methods.

One typical strategy in this field is to build optimization models for inventory management that account for the environmental impact of operations; these models are an extension of existing inventory management models. The Economic Order Quantity (EOQ) model is a popular tool for inventory management. Its goal is to minimize the overall cost of inventory by finding the optimal balance between ordering and holding costs. Nevertheless, transportation and warehousing-related carbon emissions are not considered in the conventional EOQ model. Reducing carbon emissions is just one goal of sustainable inventory management, which also places an emphasis on making inventory systems more efficient in general. The goal of efficient inventory management is to satisfy customer needs while decreasing stock on hand, cutting down on waste, and making the supply chain more responsive. These aims, when pursued in a sustainable manner, must not jeopardize other environmental objectives. Consequently, models for inventory management should include tactics that maximize the movement of items while simultaneously minimizing carbon

emissions at every stage of the supply chain. In this way, businesses can lessen their negative effects on the environment without sacrificing their bottom line or their competitive edge.

Including carbon footprint estimations in decision-making processes is a vital component of sustainable inventory management. A number of factors contribute to carbon emissions in supply chains. These include transportation (e.g., vehicle type, distance traveled, and cargo weight) and warehouse operations (e.g., energy use for HVAC, lighting, and other processes). A more comprehensive and eco-friendly inventory approach can only be achieved by quantifying these emissions and incorporating them into inventory optimization models. Determining the environmental impact of replenishing activities and designing strategies that minimize both costs and emissions requires the integration of carbon pricing into existing cost functions.

Reducing carbon emissions in inventory management is greatly aided by transportation optimization. The most carbon-intensive operation in a typical supply chain is transportation. This is especially the case when transporting commodities over great distances using traditional vehicles powered by fossil fuels. Reduce your carbon footprint by implementing sustainable mobility strategies including route optimization, cutting down on empty miles, and switching to cleaner technology like electric or hybrid vehicles. Many businesses have found success in lowering their carbon emissions and transportation expenses by implementing these strategies into their inventory management models. Mathematical modeling approaches like Mixed-Integer Linear Programming (MILP) discover the most efficient transportation routes by optimizing them with respect to both cost and environmental impact.

Cutting down on energy use in warehouses is another way to reduce carbon emissions. The energy consumption of warehouses is a major factor in the environmental impact of inventory management systems, despite their essential role in the supply chain. The heating, cooling, and lighting that are standard in most warehouses today are energy hogs. Sustainable warehouses, on the other hand, make use of energy-efficient lights, power systems, and renewable energy sources (like solar power) to cut down on heating and cooling costs. Renewable energy sources, computerized inventory management systems, and energy-efficient heating, ventilation, and air conditioning systems are just a few examples of the sustainable practices and green technology that contemporary warehouses are embracing. Businesses can improve their energy efficiency and lessen their negative influence on the environment by implementing these ideas into their inventory management models.

Using environmental performance indicators (EPIs) to monitor the ecological footprint of inventory management processes is another step toward incorporating sustainability into the field. Carbon footprints per shipment, energy consumption per square foot of warehouse space, and emissions per unit of products sold are all examples of possible indicators. Enterprise performance indicators (EPIs) allow companies to track their environmental impact over time and make educated choices about cutting down on waste. More than that, companies can weigh the costs and benefits of various options by factoring in carbon emissions in inventory management models. To achieve sustainability goals without compromising profitability, this trade-off is typically articulated using a weighting factor that balances economic and environmental objectives.

An ecologically sound supply chain that maximizes efficiency and minimizes costs is the holy grail of sustainable inventory management. Companies are under growing pressure from stakeholders, authorities, and customers to lessen their environmental footprint, making sustainable inventory management a must-have for staying competitive on a global scale. Businesses can strike a balance between operational efficiency and environmental sustainability by implementing inventory optimization models that take both cost and carbon limits into account. In the long run, these models help companies and the environment by providing a structure for decision-making that ties supply chain practices to corporate sustainability objectives.

REVIEW OF LITERATURE

Barman, Dipak & Mahata, Gour. (2022) [6] Using a continuous review model and a finite planning horizon, the article examines a two-tier integrated vendor-buyer inventory system. The vendor purchases products of varying quality at a constant, unpredictable production rate, with the unit production cost being directly related to the costs of technology investment, labor, and capital. In order to fulfill the normally distributed and stochastically occurring demand from buyers, vendors produce goods and services. In addition, the buyer has control over the lead time, which is proportional to the order quantity. Using crashing cost to decrease duration from normal to minimal may shorten lead time. It is permitted and presumed that the shortages are due to backorders. The primary goal of the analytical process is to minimize the estimated average cost of the integrated system. To further demonstrate the practicality of our suggested paradigm, numerical examples are also given in mathematics. All of the important parameters of the system are considered in the sensitivity analysis of the suggested model. Findings confirm that supply chain manufacturing is the most suitable setting for the suggested model.

Giri, Raghu et al., (2022) [7] this article focuses on a joint ordering inventory policy for two depreciating substitute products throughout the course of one period. Here, product pricing and immediate inventory levels determine consumer demand in a linear fashion. While both items are on the market, the demand for a substitute product falls in relation to its own price and the stock of the other product, and rises in relation to the stock and price of the first product. When one product runs out of stock, some customers choose the available product over the stock out one due to an immediate need. Various possibilities are explored in light of the items' depletion and the characteristics of the demand. Examples of specific examples are models that exhibit price and stock-specific substitutability. In order to optimize the average overall profit, the models are designed to find the ideal order quantities of each product. In LINGO 12.0, the generalized reduced gradient approach is used to solve the issues. In numerical experiments, the models are shown and in sensitivity analysis, the effect of the model parameters on the objective function is shown. Additionally, the article delves into the intriguing characteristics and patterns of average overall profit.

Singh, Ranu & Mishra, Vinod Kumar. (2022) [8] this article focuses on a joint ordering inventory policy for two depreciating substitute products throughout the course of one period. Here, product pricing and immediate inventory levels determine consumer demand in a linear fashion. While both items are on the market, the demand for a substitute product falls in relation to its own price and the stock of the other product, and rises in relation to the stock and price of the first product. When one product runs out of stock, some customers choose the available product over the stock out one due to an immediate need. Various possibilities are explored in light of the items' depletion and the characteristics of the demand. Examples of specific examples are models that exhibit price and stock-specific substitutability. In order to optimize the average overall profit, the models are designed to find the ideal order quantities of each product. In LINGO 12.0, the generalized reduced gradient approach is used to solve the issues. In numerical experiments, the models are shown and in sensitivity analysis, the effect of the model parameters on the objective function is shown. Additionally, the article delves into the intriguing characteristics and patterns of average overall profit.

Chandra, Sujana. (2021) [10] in this paper, a two warehouse inventory model for deteriorating items is studied with ramp type demand rate. Holding cost of rented warehouse has higher than the owned warehouse due to better preservation facilities in rented warehouse. Due to the improved services offer in rented warehouse, the deterioration rate in rented warehouse is less than deterioration rate in owned warehouse. When stock on hand is zero, the inventory manager offers a price discount to customers who are willing to backorder their demand. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal fruits and vegetables, newly launched fashion items, etc. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval.

Tripathi, Rakesh & Mishra, Sachin. (2019) [15] this article takes a look at the production, inventory (EPQ) model using an integrated cost reduction release strategy. Both production and demand occur at the same time in the actual world, hence the two processes are thought of as being time-linked. While discussing the amalgamating cost decline delivery technique, two models are brought up: (i) the production inventory model and (ii) the manufacture inventory model.

To determine the best answer for both models, mathematical formulas are given. Finding the best way to evaluate the EPQ model's reduced-cost release strategy is the driving force behind this research. A cost-cutting optimal fabrication lot size model is created. Various parameter variations are addressed in the sensitivity analysis.

Sarkar, Biswajit et al., (2017) [25] an integrated inventory model with a make-to-order policy from buyer to vendor is the topic of this study. One way to keep the single-setup multi-delivery (SSMD) policy in place while reducing transportation costs is to utilize a variable transportation cost that is a power function of the delivery quantity. This can be done by either tapering the policy or taking proportional rate data into account. Even though the initial inspection reveals a constant defective rate of imperfect production in the long-run production system and that all defective items are reworked with some fixed cost, the vendor still implements a two-stage inspection process to guarantee the product's flawless quality.

Through the application of conventional optimization techniques, the objective is to reduce the overall cost of the integrated inventory model. We include two numerical examples, a sensitivity analysis, and visual representations to show how the model works.

MATHEMATICAL MODELING OF SUSTAINABLE REPLENISHMENT

Economic Order Quantity with Carbon Constraints

One popular approach for inventory management is the Economic Order Quantity (EOQ) model, which takes both ordering and holding costs into account to find the optimal total cost. Nevertheless, when considering sustainability, we augment the traditional EOQ model by integrating the environmental impact linked to energy usage and transportation. The extended EOQ model with carbon constraints can be formulated as follows:

$$Q^* = \sqrt{\frac{2DS}{H} \cdot \frac{1}{1 + \lambda \cdot C_T}}$$

Where:

- Q^* is the optimal order quantity,
- D is the annual demand,
- S is the ordering cost,
- H is the holding cost,
- λ is the weighting factor for environmental considerations,
- C_T is the total carbon cost per unit of goods delivered.

Emissions from transportation (dependent on distance, mode of transportation, and weight of goods) and energy usage in warehouses are included in the total carbon cost C_T in this model. The parameter λ represents the trade-off between economic and environmental objectives, where a higher λ emphasizes reducing carbon emissions over cost savings.

Carbon-Constrained Inventory Optimization

To avoid this problem, another option is to create an inventory optimization model that takes carbon emissions into account; in this model, minimizing the overall cost while simultaneously reducing the carbon footprint is the goal. Here is one way to represent the optimization problem:

$$\min \left(\sum_{t=1}^T [C_t(Q_t) + \mu \cdot E_t(Q_t)] \right)$$

Subject to:

$$\sum_{t=1}^T E_t(Q_t) \leq E_{\max}$$

Where:

- $C_t(Q_t)$ is the cost function for replenishment at time t ,
- $E_t(Q_t)$ is the carbon emission at time t for order quantity Q_t ,
- μ is the weight factor balancing cost and environmental impact,
- $E_{\max}$ is the maximum allowable carbon emissions over the planning horizon.

Here, the goal of the decision-maker is to reduce the overall cost, which includes not only the cost of replenishment but also the carbon emissions that go along with it. We incorporate emissions from transportation and warehouse energy use into our model, which models carbon emissions as a function of replenishment amount. $E_{\max}$ is a limitation that makes sure the carbon footprint doesn't go above a certain limit, which could be set by industry regulations or by the sustainability goals of the company.

SUSTAINABILITY CONSIDERATIONS IN REPLENISHMENT STRATEGY

Transportation Optimization

When it comes to inventory management, transportation is a major source of carbon emissions. Reducing transportation distances or switching to electric vehicles are two examples of ways to improve transportation routes and cut down on carbon emissions. The scheduling and routing of vehicles can be optimized using mathematical programming methods such as mixed-integer linear programming (MILP). A model for transportation-related carbon emissions could look like this:

$$E_{\text{trans}} = \sum_{k=1}^N (d_k \cdot \alpha_k \cdot Q_k)$$

Where:

- d_k is the distance traveled for route k ,
- α_k is the carbon emission factor for transportation mode k ,
- Q_k is the quantity of goods transported on route k ,
- N is the number of routes.

Warehouse Energy Consumption

Use of renewable energy sources, automation, and temperature control systems are some energy-efficient measures that can be implemented to lessen the carbon footprint of warehouses. The overall amount of power used The operating parameters of the warehouse, such as the quantity of things handled, storage time, and energy efficiency measures put in place, can be used to model $E_{\text{warehouse}}$. Warehouse energy optimization requires a delicate balancing act between maximising operational efficiency and minimising environmental effect via the smart implementation of green technology.

CONCLUSION

Mathematical modeling that incorporates sustainability into inventory management is a great chance to strike a balance between economic efficiency and environmental responsibility. Both the carbon-constrained inventory optimization model and the expanded Economic Order Quantity (EOQ) model with carbon constraints offer useful frameworks for companies to reduce costs and carbon emissions. Such models help businesses achieve their sustainability goals at a reasonable cost by factoring in things like transportation emissions and warehouse energy consumption. The possibility of lessening inventory management's effect on the environment can be further enhanced by optimizing transportation routes and implementing energy-efficient techniques in warehouses. Businesses that want to be in compliance with regulations, lessen their impact on the environment, and boost their operational efficiency must use these models if sustainability is to play an important role in their operations. Using these mathematical models, companies may accomplish their environmental goals and gain a competitive edge in a world where consumers are more concerned about environmental issues.

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