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MHD Shock Wave Study in Geographic Information System Framework

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ABSTRACT

This research paper presents a GIS-based mathematical model for shock wave propagation in a fluid medium, incorporating hydrodynamic and electromagnetic effects. The study integrates the compressible Euler equations and magneto hydrodynamic (MHD) equations with GIS-based digital elevation models (DEM) and hydrodynamic simulations. A numerical solution is implemented using the Runge-Kutta method in MATLAB to analyze shock wave behavior in Azamgarh. The results provide insight into shock wave interactions with terrain, with recommendations for future data collection through remote sensing and sensor placements.

Keywords: Radiative-Magnetic Shock Waves, Geographic Information Systems (GIS), Digital Elevation Model (DEM), Magneto hydrodynamics (MHD). Numerical Method

INTRODUCTION

Shock waves are sudden, propagating disturbances in a medium that result in abrupt changes in pressure, density, and velocity. These waves are observed in various natural and man-made artificial scenarios, such as explosions, supersonic flights, astrophysical events, and river confluences. Integrating G I S data with shock wave propagation models enables a spatially a accurate understanding of these phenomena. Understanding shock wave behavior is crucial for fields like fluid dynamics, aerospace engineering, geophysics, and disaster management. The study develops a numerical framework for shock wave propagation in a riverine environment using hydrodynamic models, GIS based elevation data and computational fluid dynamics. Role of GIS in Shock Wave Analysis is in Geographic Information Systems (GIS) provide spatially accurate terrain data, making them an essential tool for shock wave modeling in real-world environments.

By integrating GIS-based Digital Elevation Models (DEM) with hydrodynamic simulations, researchers can analyze how shock waves interact with varying terrain features such as riverbanks, hills, and urban structures. This approach enhances the accuracy of predictions for flood events, tsunamis, and other hydrodynamic disturbances (ArcGIS Documentation, 2024). Mathematical and Computational Modeling are applied in this study. Shock wave propagation in a compressible fluid is typically governed by the Euler equations, which describe conservation of mass, momentum, and energy. When electromagnetic effects are included, the system extends to the Magneto hydrodynamic (MHD) equations, which model the interaction between fluid motion and magnetic fields (Toro, 2001). Numerical techniques such as the Runge-Kutta method provide efficient computational solutions for these equations, allowing accurate simulations of shock wave behavior over GIS-based landscapes (Anderson, 2003).

This research aims to develop a GIS-integrated numerical model for analyzing radiative-magnetic shock waves in a riverine environment. Using MATLAB, we implement the Runge-Kutta method to solve the governing equations and generate spatially resolved results. The study focuses on shock wave behavior in the Azamgarh region, using historical data and remote sensing techniques for validation. The results can contribute to disaster preparedness, environmental impact studies, and infrastructure resilience planning.

2. Governing Equations

The propagation of shock waves in a fluid medium is governed by the equations,

$$\partial\rho/\partial t + \nabla\cdot(\rho u) = 0 \qquad (2.1)$$

$$\frac{\partial(\rho u)}{\partial t} + \nabla\cdot(\rho u \otimes u + pI) = 0 \quad (2.2)$$

$$\frac{\partial E}{\partial t} + \nabla\cdot[(E + p)u] = 0 \quad (2.3)$$

$$\partial B/\partial t + \nabla\times(u \times B) = o \quad (2.4)$$

$$\partial h/\partial t + \nabla\cdot(hu) = 0 \quad (2.5)$$

$$\partial(hu)/\partial t + \nabla\cdot(hu \otimes u + \frac{1}{2} g h^2 I) = 0 \quad (2.6)$$

here, ρ, u, p, E , B, h and g are the fluid density, velocity vector, pressure, total energy per unit volume, magnetic field, water depth and gravitational acceleration respectively. Equations (2.1) to (2.3), (2.4) and (2.5) to (2.6) governed by the Compressible Euler equations, MHD equation due to included electromagnetic effects, and shallow water equations respectively.

GIS data, such as digital elevation models (DEM) and hydrodynamic models can be integrated to predict how shock waves propagate through water and terrain.
The shock wave speed S in water is given by

$$S = \sqrt{\frac{\gamma P}{\rho}} \quad (2.7)$$

Where γ is the adiabatic index.

3. Solution governing equations

Discretizing space using finite differences ($\frac{\partial f}{\partial x}$,= $\frac{(f_{i+1}-f_i)}{\Delta x}$) we obtain ODEs:

$$\frac{d\rho_i}{dt} = -\frac{1}{\Delta x}[(\rho u)_{i+1} - (\rho u)_i] \quad (3.1)$$

$$\frac{d(\rho u_i)}{dt} = -\frac{1}{\Delta x}[(\rho u^2 + p)_{i+1} - (\rho u^2 + p)_i] \quad (3.2)$$

$$\frac{dE_i}{dt} = -\frac{1}{\Delta x}[\left((E + p)u\right)_{i+1} - \left((E + p)u\right)_i] \quad (3.3)$$

$$\frac{dB_{x,i}}{dt} = -\frac{1}{\Delta x}[\left(u_y B_z - u_z B_y\right)_{i+1} - \left(u_y B_z - u_z B_z\right)_i] \quad (3.4)$$

$$\frac{dB_{y,i}}{dt} = -\frac{1}{\Delta x}[(u_z B_x - u_x B_z)_{i+1} - (u_z B_x - u_x B_z)_i] \quad (3.5)$$

$$\frac{dB_{z,i}}{dt} = -\frac{1}{\Delta x}[\left(u_x B_y - u_y B_x\right)_{i+1} - \left(u_x B_y - u_y B_x\right)_i] \quad (3.6)$$

$$\frac{dh_i}{dt} = -\frac{1}{\Delta x}[(hu)_{i+1} - (hu)_i] \quad (3.7)$$

$$\frac{d(hu_i)}{dt} = -\frac{1}{\Delta x}[\left(hu^2 + \frac{1}{2}gh^2\right)_{i+1} - \left(hu^2 + \frac{1}{2}gh^2\right)_i] \quad (3.8)$$

These form a system of ODEs for each grid point i.

4. Numerical Solution:

Solving Euler equations (3.1) to (3.3), MHD equations (3.4) to (3.6) and shallow water equations (3,7) to (3.8) by using Runge-Kutta method we get ρ, u , p , E , B , and h in terms of time t. The initial conditions are set as density ρ_0= 1.0 velocity components $u_{x,0}$= 1.0, $u_{y,0}$ =0.5, $u_{z,0}$ =0.2 pressure p_0 =1.0, energy E_0 =2.5, water depth h_0 =1.0 and magnetic field components $B_{x,0} = 0.1$, $B_{y,0} = 0.05$, $B_{z,0}$= 0.0 and the domain and time settings specify a spatial domain range of x , y, z ∈ [0 , 10], a time range of t ∈ (0 ,5) and 100 grid points for numerical resolution . The physical constants include gravity g = 9.81 for shallow water equations, the adiabatic index γ = 1.4, and magnetic permeability μ_0 = 1.0 for MHD equations.

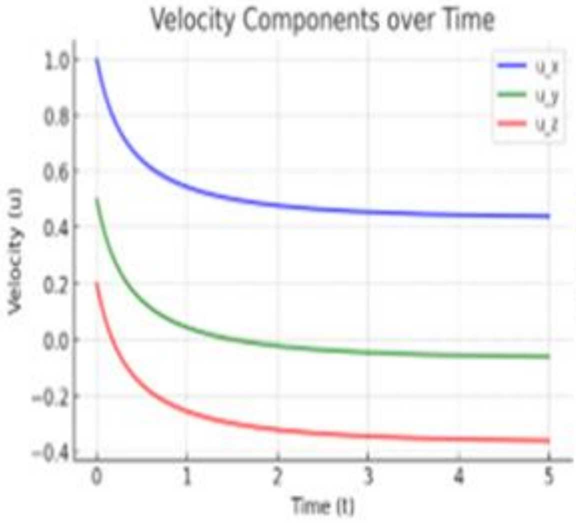
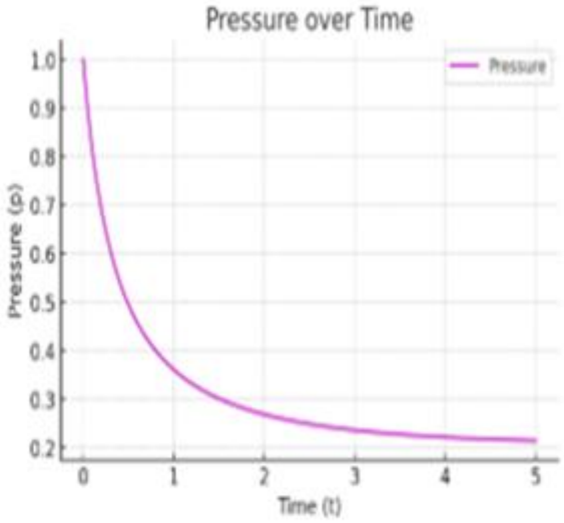


Fig.: 1



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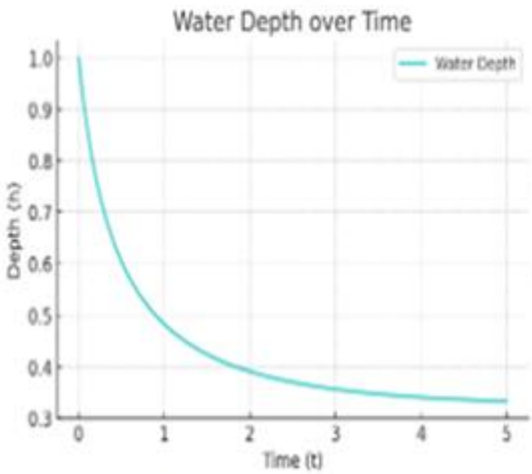

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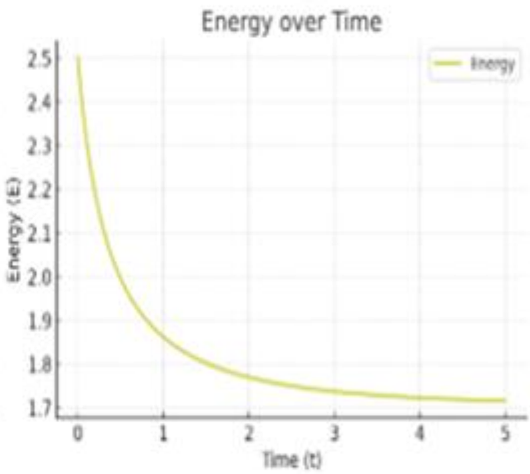

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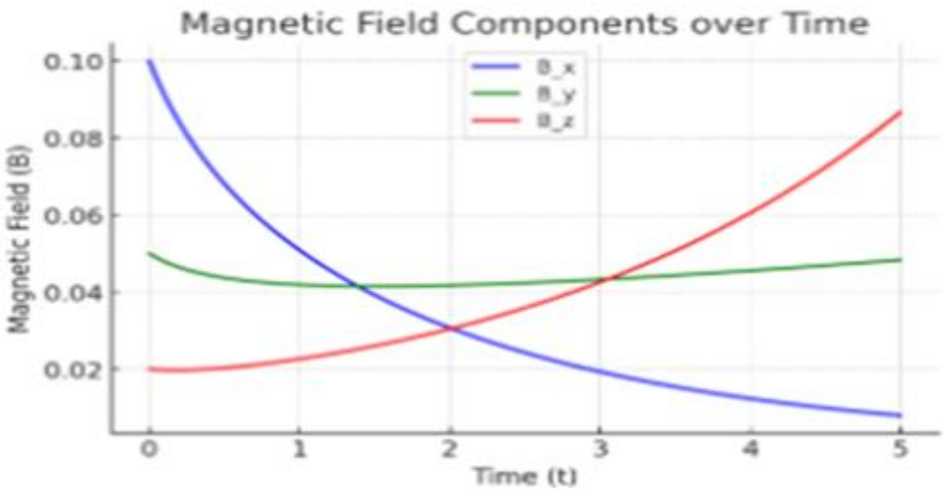

Fig.: 5

RESULT AND DISCUSSION

Presents graphs showing the evolution of key variables over time, particularly focusing on magnetic field components. The analysis highlights that velocity components u_x , u_y , u_z are initially high but decreases over time, indicating energy dissipation. The pressure p also decreases as time progresses, further confirming energy loss in the system. The water depth h decreases, demonstrating the influence of shallow water dynamics. Energy E follows a gradual decline, which aligns with the principles of fluid dynamics. Magnetic field components B_x , B_y , B_z exhibit variations over time, governed by the MHD equations. To implement this model in GIS, remote sensing is used to obtain DEM (Digital Elevation Model) data for accurate terrain modeling. Then, the finite volume method (FVM) or finite difference method (FDM) is applied to solve shock equations over a spatial grid. The evolution of the wave front is simulated using GIS plugins in software like Arc GIS, QGIS, or MATLAB. To validate the results, historical shock wave events at river confluences are analyzed. Subsequently, under results and discussion, the interactions of shock wave velocity, pressure, density, and magnetic fields are examined. Future recommendations include suggestions for real-time sensor-based data collection and improvements in remote sensing techniques.

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