

# Comprehensive study on black-hole thermodynamics and its related theorems

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## ABSTRACT

In this paper, the author has studied a generalisation of thermodynamic geometry to degenerate quantum ground states at zero temperatures exemplified by charged extremal black holes in type II string theories. Several examples of extremal charged black holes with non degenerate thermodynamic geometries and finite but non zero state space scalar curvatures are established. These incorporate dark openings portrayed by D1-D5-P and D2-D6-NS5-P brane frameworks and furthermore two charged little black holes in Type II string speculations. We additionally investigate the adjustments to the state space geometry and the scalar bend because of the higher subsidiary commitments and string circle amendments just as a precise entropy articulation from quantum data hypothesis. Our development portrays state space geometries emerging out of a potential constraining thermodynamic characterisation of ruffian quantum ground states at zero temperatures.

**Keywords:** black-hole, thermodynamics, theorem, energy.

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## INTRODUCTION

In physics, black hole thermodynamics is the area of study that seeks to reconcile the laws of thermodynamics with the existence of black-hole event horizons. As the investigation of the factual mechanics of black holes radiation prompted the coming of the hypothesis of quantum mechanics, the push to comprehend the measurable mechanics of black holes has had a profound effect upon the comprehension of quantum gravity, prompting the definition of the holographic guideline.

Over the previous decade, black holes thermodynamics has risen as a significant hypothetical research facility to test issues of quantum gravity with regards to string hypotheses. The zone has seen significant advances particularly toward a goals of the minuscule factual premise hidden the plainly visible entropy of extremal and approach extremal black holes in string hypothesis [1]. Perceptibly dark openings are known to be thermodynamic frameworks with a trademark Hawking temperature and an entropy, which, upto driving request, is relative to the zone of the occasion skyline in Planck units. The entropy is an element of the mass (interior vitality)  $M$ , charge  $Q$  and the precise energy  $J$  for the most broad charged turning dark openings. These fill in as broad thermodynamic factors gave we believe the dark gap to be a subsystem of a bigger thermodynamic framework with which it is in balance. An enormous class of extremal BPS black holes happen in the low vitality supergravity hypotheses emerging from string hypothesis. Specifically the entropy of extremal dark gaps in supergravity hypotheses with  $N \geq 2$  are known to forces a hidden minuscule measurable portrayal as far as D-brane frameworks or essential string states.

Albeit such extremal dark openings have zero Hawking temperature, they have a non zero thermodynamic entropy and are depicted by degenerate quantum ground states. A tiny state including in the related conformal field hypothesis at that point repeats the thermodynamic entropy as an asymptotic extension in the huge charge limit. The low vitality successful activity of  $N \geq 2$  supergravity following from type II string compactifications include higher subsidiary terms in a  $\alpha'$  development. These terms change the Bekenstein Hawking region law and acquaints subleading revisions with the entropy. These remedies might be processed from the Wald detailing of commonly covariant higher subordinate hypotheses of gravity [2]. It has been conceivable in the

ongoing past to represent these subleading adjustments from a minute point of view following from the hidden string hypothesis. Precise coordinating between the perceptible and minuscule entropy upto different subleading orders in an asymptotic development have been acquired for various extremal dark openings.

One of the broad parameters, normally the volume, was held fixed to give a physical scale and forestall the improvement of negative eigenvectors of the measurement. Albeit fascinating, the physical pertinence of this structure appeared to be tricky. Ruppeiner reformulated the Weinhold internal item in the entropy portrayal as far as the (negative of the) Hessian lattice of the entropy as for the broad thermodynamic factors. This likewise prompted a positive unequivocal Riemannian geometric structure in the thermodynamic state space which was conformally identified with the Weinhold geometry, with the temperature as the conformal factor. Ruppeiner indicated that thought of thermodynamic variance hypothesis [3] notwithstanding the thermodynamic laws permitted a momentous physical translation of this geometric structure as far as the likelihood circulation of the vacillations and a connection of the scalar ebb and flow with basic wonders.

Thermodynamic geometry of the harmony state space depicted above may likewise be applied to examine dark openings considered as thermodynamic frameworks. Late investigations of the thermodynamics of differing dark openings right now have explained intriguing parts of stage changes and relations to moduli spaces of  $N \geq 2$  supergravity compactifications with regards to extremal dark gap arrangements in these hypotheses. It might be contended, notwithstanding, that the association of this detailing to variance hypothesis for application to dark gaps requires a few changes. The geometric plan in the state space was first applied to  $N \geq 2$  supergravity extremal dark gaps in  $D = 4$  which emerge as low vitality viable field speculations from compactification of Type II string hypotheses on Calabi-Yau manifolds. From that point forward, a few creators have endeavored to comprehend this association [4], both for supersymmetric just as non-supersymmetric extremal and non extremal dark gaps and five dimensional pivoting dark rings. Right now, had investigated the state space geometry of both non extremal turning BTZ dark gaps and BTZ-Chern Simons (BTZ-CS) black holes in the Ruppeiner detailing.

It is a characteristic inquiry to present whether the formalism of thermodynamic geometry of the balance state space can be summed up to frameworks at zero temperature. Dark openings at extremality give the most regular research facility to contemplate this issue. Unmistakably, ideas of traditional thermodynamics are not expected to be substantial right now, one would expect adjustments to the equivalent. The primary inquiry that one can address, in any case, is whether a geometric portrayal of the state space is at all conceivable at extremality. It is one of the issues that we will address right now, we will see this is in reality the case.

## LITERATURE REVIEW

In this section, the author presented a brief review of the essential features of thermodynamic geometries and their application to the thermodynamics of black holes, in particular extremal black holes. This will serve to to set the notations and conventions used in the rest of this paper. An intrinsically geometric structure in equilibrium thermodynamics was introduced by Weinhold [5] through an inner product in the space of equilibrium thermodynamic macrostates defined by the minima of the internal energy function  $U = U(S/T, V/T, \mu_i/T)$  as the Hessian [5].

$$h_{ij} = \partial_i \partial_j U \quad (1)$$

As mentioned in the introduction, the quantities  $\mu_i$ ,  $T$ ,  $V$ ,  $S$  are the chemical potentials, temperature, volume and entropy respectively and the volume or any other parameter is held fixed to provide a physical scale and to restrict negative eigenvectors of the metric. Although such a Riemannian geometric structure was interesting, no physical significance could be ascribed to it. The inner product on the state space was later reformulated by Ruppeiner [6] in the entropy representation as the negative of the Hessian matrix of the entropy with respect to the extensive variables.

The thermodynamic macrostates underlying the equilibrium state space being now described by the maxima of the entropy function  $S = S(U, V, N)$ . Explicitly the Ruppeiner metric in the state space was given as and was conformal to the Weinhold metric with the inverse temperature as the conformal factor. The negative sign was necessary to ensure positive definiteness of the metric, as the entropy is a maximum in the equilibrium state. It could be shown that the Riemannian structure defined by the Ruppeiner metric was closely related to classical thermodynamic fluctuation theory [8] and critical phenomena.

$$g_{ij} = -\partial_i \partial_j S(U, V, N) \quad (2)$$

Having provided thus a brief account of thermodynamic geometries, in the following sections, the state space geometry in the Ruppeiner framework for several charged extremal black holes in Type II string theories are systematically explored. Our developments would give a geometric acknowledgment of a constraining harmony thermodynamics at zero temperatures. This would be the initial step to address the issue of the thermodynamics of extremal dark openings from the attractor fixed point and a geometric portrayal of the attractor system.

It might likewise be an introduction to the utilization of the formalism of thermodynamic geometries to the investigation of zero temperature quantum stage advances among particular vacua in the moduli space of string hypothesis compactifications. We should specify here that the job of the scalar arch over the state space in the Ruppeiner formalism isn't clear at this phase in the situation being imagined. In any case, utilization of this geometric formalism to non extremal dark openings and the ensuing divergences of the equivalent at known basic focuses [1] demonstrates that a non zero scalar ebb and flow for state spaces of extremal dark gaps may likewise propose a hidden connecting measurable framework.

Right now, divergences of the scalar bend may imply zero temperature quantum stage advances among unmistakable vacua in the moduli space. The attractor instrument for extremal dark openings and the resulting stream of the scalar moduli to fixed qualities at the skyline as far as the charges additionally appears to recommend such an association of the state space scalar arch with the structure of the moduli space. It is this point of view that we will receive in the present investigation and regardless of the considerable number of provisos, we will investigate the scalar ebbs and flows over the state space of extremal dark openings and their affectability to higher subordinate redresses to the entropy. Following the contentions introduced before we will decipher the scalar ebb and flow as demonstrative of an associating minute factual framework fundamental extremal dark gaps at zero temperatures [3].

### BLACK HOLE UNIQUENESS AND RELATED THEOREMS

There is just a restricted group of stationary, asymptotically level, black holes answers for the Einstein conditions. Such a spacetime is one, that has an occasion skyline and a Killing vector that is timelike at limitlessness. A static spacetime is a stationary one that additionally has a period reflection balance. In this way a pivoting dark opening is stationary however not static, while a nonrotating one is static. Various dark gap uniqueness hypotheses have been demonstrated under different sensibly very much spurred suspicions. The EF metric (1.2) gives the one of a kind static vacuum arrangement with an occasion skyline. The main stationary vacuum arrangement with a skyline is the Kerr arrangement, parametrized by the all out mass  $M$  and rakish force  $J$ . Counting an electromagnetic field, the main static arrangement with a skyline with one associated segment is the Reissner-Nordstrom arrangement parametrized by mass and electric and attractive charges  $Q_e$ ,  $Q_m$ . Since the electromagnetic pressure vitality tensor is duality pivot invariant, the measurement relies just upon the blend  $Q^2_e + Q^2_m$ . At last, considering precise energy, the extraordinary stationary dark opening arrangement with electromagnetic field is the Kerr-Newman metric [4].

### Positive Energy Theorem

Vitality of a secluded (asymptotically level) framework in GR can be characterized as the floating mass as estimated at endlessness, times  $c^2$ . This vitality, which is the numerical estimation of the Hamiltonian that produces the time interpretation balance at endlessness, is a saved amount all in all relativity. The vitality can be negative for example in the event that we basically put  $r_s < 0$  in the Eddington-Finkelstein line component, yet this yields an exposed peculiarity. On the off chance that one accept (i) spacetime can be spread over by a nonsingular Cauchy surface whose lone limit is the one at limitlessness, and (ii) matter has positive energy (all the more definitely, the pressure vitality tensor fulfills the predominant vitality condition, which for diagonalizable  $T_{ab}$  implies that the vitality thickness is more prominent than the extent of any chief weight), at that point it very well may be demonstrated that the all out vitality of the spacetime is essentially positive. This was first demonstrated in a geometrical manner by Schoen and Yau, and presently demonstrated in a more straightforward path route by Witten [3].

The thought for this confirmation originated from quantum supergravity, where the Hamiltonian has the plainly positive structure  $H = Q^2$  regarding the supersymmetry generator  $Q$ . Witten's verification goes generally as follows. The vitality is composed as a motion fundamental including first subsidiary of the measurement at

interminability which takes out the coefficient of the  $1/r$  term in the measurement. This is at times called the ADM vitality. This is then reexpressed, utilizing the Einstein conditions, as a volume basic over a spacelike Cauchy surface with an integrand containing a term quadratic in the subordinate of a self-assertive spinor field and a term in the vitality thickness of issue. In the event that the spinor field is picked to fulfill a specific elliptic differential condition, at that point the quadratic spinor term turns out to be clearly positive. The main zero vitality arrangement is vacant level spacetime [5].

On the off chance that a dark gap is available, at that point the Cauchy surface can be picked to plunge underneath the arrangement of the occasion skyline, in this manner keeping away from the nearness of an internal limit or peculiarity superficially. On the other hand, the commitment from an internal limit situated at a clear skyline can be demonstrated to be certain. Inspiration of the all out vitality at vastness doesn't really imply that the framework can't emanate an unending vitality while falling, since both the vitality of the radiation and the vitality of the extra framework are remembered for the all out vitality. An alternate meaning of vitality, called the Bondi vitality, permits one to assess only the "extra" vitality. The Bondi vitality is the floating mass as observed by light beams proliferating out to vastness in the lightlike heading, instead of the spacelike course. Basically a similar contention as before shows that the Bondi vitality is likewise fundamentally nonnegative. Along these lines just a limited vitality can be transmitted away. A positive vitality hypothesis has additionally been demonstrated within the sight of a negative cosmological consistent, in which case the asymptotic structure of the spacetime is hostile to de-Sitter as opposed to level [6].

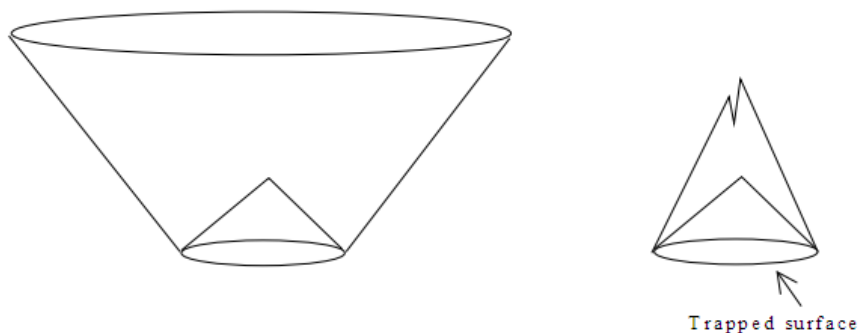
### Singularity Theorem

One may have believed that the peculiarity at  $r = 0$  is only an antique of immaculate round balance, that in an unbalanced breakdown a large portion of the mass would "miss" instead of impact and no vast thickness or ebb and flow would create. A solid proposal this isn't the situation originates from the way that the precise force boundary for circles of test particles in a dark opening spacetime offers route to a negative  $1/r^3$  - term of absolutely relativistic starting point which creates a limitless well as  $r$  goes to zero. That it is in reality false was demonstrated by Penrose. The possibility of Penrose's confirmation lays on the idea of a caught surface.

This is a shut, spacelike, 2-surface whose ingoing and active invalid ordinary congruences are both meeting (see Fig. 1). For instance, a circle at consistent  $r$  and  $v$  in Eddington-Finkelstein organizes is a caught surface in the event that it lies inside the skyline. Be that as it may, even in a to some degree lopsided breakdown it is normal that a caught surface will frame. Penrose contends that the presence of a caught surface  $T$  infers the presence of a peculiarity on the limit  $\partial F$  of its future  $F$ . (The "future" of a set is the assortment of all spacetime focuses that can be come to by future-going timelike or invalid bends from that set.) Very generally his thinking is this: the invalid normals to  $T$  begin meeting all over along these lines, since gravity is appealing, they should keep combining and will fundamentally arrive at intersection focuses (in fact, conjugate focuses) in a limited relative parameter.  $\partial F$  must "end" previously or when the intersection focuses are come to (in light of the fact that the limit  $\partial F$  must be locally digression to the light cones) so  $\partial F$  must be smaller [3].

This is an odd structure for the limit of things to come of  $T$ , and in actuality is contradictory with other sensible prerequisites on the spacetime (see underneath). The main way out is if at any rate one of the invalid normals can't be stretched out far enough to arrive at its intersection point. This nonextendibility is what is implied in the hypothesis by the presence of a peculiarity. Einstein's condition comes into the confirmation just in guaranteeing that the at first uniting invalid normals to  $T$  must arrive at an intersection point in a limited relative parameter.

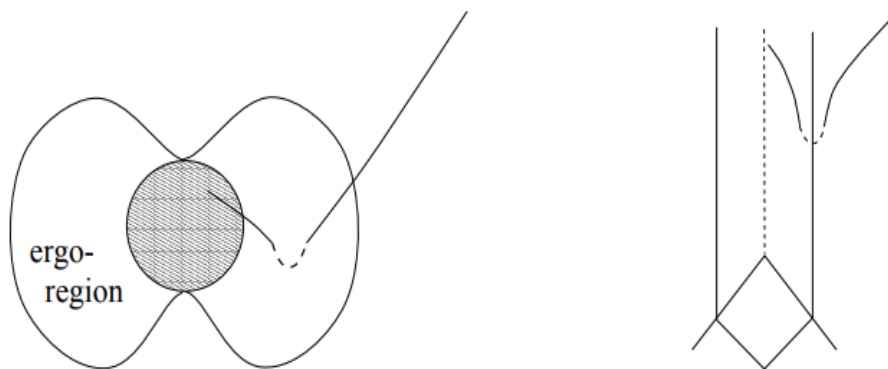
It merits clarifying this in more detail, since it includes innovation that figures in numerous improvements when all is said in done relativity and dark opening thermodynamics, specifically, the centering condition (which is frequently called the Raychaudhuri condition, or Sach's condition, or Newman-Penrose condition). This condition relates the centering of a heap of light beams (called an invalid geodesic consistency) to the Ricci tensor. Consider an invalid geodesic coinciding that radiates from one side of a spacelike 2-surface. Characterize the combination  $\rho$  of the harmoniousness as the fragmentary pace of progress of a microscopic cross-sectional territory  $\delta A$ :  $\rho := d \ln \delta A / d\lambda$ , where  $\lambda$  is a relative parameter for the invalid geodesics [7].



**Figure 1: Boundary of the future of an ordinary untrapped surface (left) and a trapped surface (right).**

### QUANTUM BLACK HOLE THERMODYNAMICS

Classical black hole physics cries out for the incorporation of  $\hbar$  effects, so the thermodynamic “analogy” can become true thermodynamics. Since general relativity is relativistic, it isn't quantum mechanics yet relativistic quantum field hypothesis that is called for. In this way, on a fundamental level, one ought to consider “quantum gravity”, whatever that might be. Albeit nobody knows without a doubt what quantum gravity really is, formal treatment of its semiclassical limit by Gibbons and Hawking in a way fundamental system uncovered one manner by which the similarity can turn into a personality. This will be talked about later. An other semiclassical approach—and generally the first—is to consider quantum fields in a fixed black hole foundation. A quantum field has vacuum vacillations that saturate all of spacetime, so there is continually something going on, even in the “unfilled space” around a dark gap. In this manner turning on the vacuum changes of quantum fields can profoundly affect the thermodynamics of dark gaps. The chief impact is the presence of Hawking radiation. The authentic course to Hawking's disclosure merits referencing. After the Penrose procedure was developed, it was just a short advance to consider a comparable procedure utilizing waves as opposed to particles, a wonder named “super-brilliance”. Quantum precisely, superradiance compares to invigorated emanation, so it was then normal to ask whether a pivoting black hole would immediately transmit. In attempting to enhance the figurings for unconstrained discharge, Hawking faltered onto the way that even a non-turning dark opening would produce particles, and it would do as such with a warm range at a temperature [8].



**Fig. 2: Pair production in the ergoregion of a rotating black hole (left); and Hawking effect: pair production straddling the horizon (right)**

Killing energy and angular momentum must be conserved, so the two particles must have opposite values for these. In the ergoregion there are negative vitality states for genuine particles, so such a couple can be made there, with the negative energy accomplice later falling over the occasion skyline into the dark opening. In the nonrotating case the ergoregion exists just into the great beyond, anyway the pair creation procedure can straddle the skyline (Fig. 2). This ends up having a warm plentifulness, and offers ascend to the Hawking impact.

## The Hawking impact

At the core of the Hawking impact is the Unruh impact. The key material science in both is the related structure of the vacuum at short separations. These connections show themselves as the Hawking impact when the quantum field is engendering out of sight of a stationary black hole. Instead of remaining beside the skyline everlastingly, the active quanta outside the occasion skyline step by step climb away from the skyline, leaving their connected accomplices on the opposite side to fall into the peculiarity. Right now, first depict the Hawking impact accentuating the connection to acceleration radiation, and featuring the job of the gravitational redshift. After quickly showing the ramifications for dark opening dissipation, I at that point disclose how to utilize the outcomes to determine the Hawking impact. At last, the upsetting pretended by self-assertively high recurrence field modes in the Hawking impact is examined [9].

## The transplanckian puzzle

There is something upsetting about Hawking's thinking be that as it may. As the wavepacket is proliferated in reverse in time along the skyline, it is blueshifting exponentially regarding Killing time. For the absolute first Hawking quanta that develop after a black hole structure this is maybe not all that genuine, since they have not experienced much blueshifting. However, for quanta that rise a period  $t$  after the dark gap framed, there is a blueshift of request  $\exp(\kappa t)$ . For a Schwarzschild dark gap,  $\kappa = 1/2R_s$ , so after, state,  $t = 1000R_s$ , the blueshift factor is  $\exp(500)$ . That is, the ingoing mode has recurrence  $\exp(500)$  times the recurrence of the active Hawking quantum at endlessness. For a sun oriented mass dark gap, the factor is  $\exp(10^5)$  after just 2 seconds have passed. Obviously, we can't be sure that we realize what material science resembles at such discretionarily high, "transplanckian" frequencies. Obviously on the off chance that careful neighborhood lorentz invariance is accepted, at that point any recurrence can be Doppler moved down to a low recurrence, just by a difference in reference outline. Be that as it may, the boundless extrapolation of neighborhood lorentz invariance to self-assertive lift factors (and the related endless thickness of states) must be respected with suspicion [10].

## CONCLUSIONS

In this paper, the author has discussed about the formalism of thermodynamic geometries to degenerate quantum ground states at zero temperatures, exemplified by extremal black holes in Type II string theories. Such frameworks displaying perceptible declines are notable in the material science of dense issue like turn glasses. Our inspiration has been to investigate Riemannian geometric structures hidden the harmony thermodynamic state spaces of extremal black holes in Type II supergravities which emerge as low vitality cutoff points of string hypotheses. As expressed, the entropy of extremal dark gaps recommend a constraining characterisation of customary thermodynamics to decline quantum ground states at zero temperatures. It is notable that black hole arrangements in  $N \geq 2$  supergravity includes moduli spaces with exceptional Kähler geometry. Specifically, they display an attractor marvels as an outcome of which moduli fields stream under outspread advancement to fixed qualities as far as the charges at the skyline which is a fixed purpose of the stream. The entropy is subsequently a component of the charges just, and free of the asymptotic estimations of the moduli guaranteeing the legitimacy of a basic tiny factual premise regarding major string states or D-brane frameworks. The present examination fills in as an introduction to investigate the thermodynamics of extremal black holes in Type II string speculations, away from the attractor fixed point and a subsequent geometrical comprehension of the attractor instrument and the attractor fixed point as potential limitations in the harmony state space reached out by the moduli factors.

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