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Economic Design of X Control Chart using Particle Swarn Optimization

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ABSTRACT

Control chart is the most widely used tools for statistical process control. For detecting shift in process mean, X chart is the simplest and most commonly used. Control chart should be designed economically in order to achieve minimum quality control costs. The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action can be taken before a large quantity of nonconforming product is manufactured. The X control chart dominates the use of any other control chart technique if quality is measured on a continuous scale. The design of a control chart refers to the selection of three parameters i.e., sample size, width of control limit, and interval between samples. Economic design of control chart has gained considerable importance in providing better quality of end products to customer at less cost. In the present work, a computer program in C language based on a non-traditional optimization technique namely particle swarm optimization has been developed for the economic design of the X control chart giving the optimum values of the sample size, sampling interval and width of control limits such that the expected total cost per hour is minimized. The results obtained are found to be better compared to that reported in the literature.

INTRODUCTION

Quality Control

Quality is one of the most important consumer decision factors in the selection among competing products and services. The phenomenon is widespread, irrespective of the fact that the consumer is an individual, a retail store, an industrial organization or a defense program. Consequently, understanding and improving quality is a key factor leading to business success, an enhanced competitive position, and growth in the market. In the quality control a substantial return on investment from successfully employing quality and from improved quality is an integral part of business strategy. Thus, quality can be defined as the fitness for use, in other words quality is inversely proportional to variability or quality improvement is the reduction of variability in processes and products. Quality engineering is the set of managerial, operational, and engineering activities in a company uses to confirm that the quality characteristics of a product are at the required levels.

Statistical Process Control

Statistical process control (SPC) is a powerful collection of statistical methods to the monitoring and control of a process to ensure that it operates as its full potential to produce conforming product. Under SPC, a process desired to produce as much conforming product as possible with the least possible waste. Control chart is a key tool of SPC. Most of the power of SPC lies in the ability to examine a process and the sources of variation in that process using tools that give weight to objective analysis over subjective opinions and that allow the strength of each source is to be determined numerically. Variations in the process may affect the quality of product. With its emphasis on early detection and prevention of problems, SPC has a distinct advantage over other quality methods, such as inspection, that apply resources to detecting and correcting problems after they have occurred. In addition to that, SPC can lead to a reduction in the time required to produce the product. Basically SPC has seven following major tools: -

1. Scatter diagram
2. Check sheet
3. Histogram

4. Cause-and-effect diagram
5. Control chart
6. Pareto chart
7. Defect concentration diagram

Control Chart

Control chart is one of the widely used statistical process control (SPC) tools. It is used to statistically monitor the process through sampling inspection instead of 100% inspection. It only indicates whether the process is in-control or out-of-control but it cannot on its own rectify the process.

Assuming that quality characteristics with process mean μ0 and process standard deviation σ, is normally distributed with sample size n and width of control limit k, the upper and lower control limits can be given by following equations:

$$UCL=\mu_0+k\sigma/\sqrt{n} \quad (1.1)$$
$$LCL=\mu_0-k\sigma/\sqrt{n} \quad (1.2)$$

The upper and lower control limits on a control chart are usually set as ±3 standard deviations from the mean.If it is assumed that the data exhibit a normal distribution, these control limits will capture 99.74 percent of all the items produced as shown in Fig.1.2. Similarly, if control limits are set at ±2 standard deviations from the mean, the two control limits would capture 95.44 percent of the values.

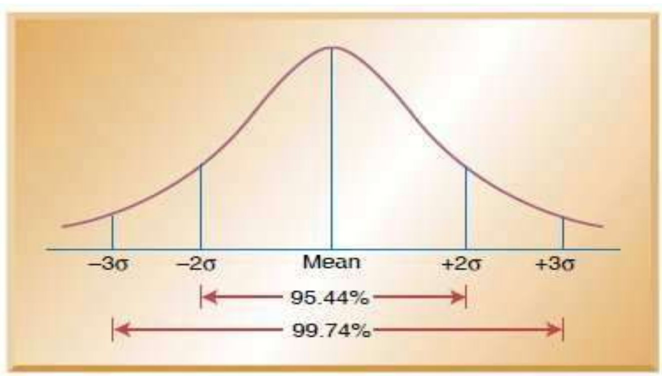

Fig. 1.2. Area Under Normal Curve

LITERATURE REVIEW

Introduction:

In this chapter few selected research paper related to economic design of $\bar{X}$ control chart has considered. On economic design lot of work has been done already, based on this some research work some papers has classified into four categories i.e. based on economic design, statistical design, types of distribution and particle swarm optimization for economic design.

Economic Design

*Weiler[1]*represented that the optimum sample size should minimize the total amount of

Inspection required to detect a specified shift for $\bar{X}$ control chart. If the shift is from an in- control state μ_0 to out-of-control state $\mu_1=\mu_0+\delta\sigma$.

Duncan [2] proposed the optimum economic design of the$\bar{X}$control chart. This paper was the first to deal with a fully economic model of a Shewhart-type control chart and to incorporate formal optimization methodology into determining the control chart parameters.

Economic Statistical Design

*Taylor[10]*hasreportedthatcontrol proceduresbased ontaking a sample of constant size at fixed intervals of time is non-optimal. He suggests that sample size and sampling frequency should be determined at each point in time based on the posteriorprob ability thattheprocessis in an out-of-control state. Dynamic programming methods were utilized extensively in the development.

Types of Distribution

Girshick and Rubin [13] have investigated a process model in which a machine producing items characterizedby ameasur able quality characteristic can be in one off our states. States 1 and 2 are production states. In state 1 the output quality characteristic is described by the probability density function $f_i(x)$, i=1, 2, State 1 is the "in-control" state.
While in state 1, there is a constant probability ofa shifttostate 2.They treatboth100% inspection and periodic inspection rules. The economic criterion is to maximize the expected income from the process. Their work is of significant theoretical value. They were the first researchers to propose the expected cost per unit time criterion and rigorously show its appropriateness for the problem.

Particle Swarm Optimization

Zee-Lee[18] proposed a particleswarm optimization (PSO) method for solving the economic dispatch(ED) problem in power systems. Many nonlinearcharacteristics ofthe generator, such as ramp rate limits, prohibitedoperatingzone,and non-smooth cost functions are considered using the proposed method in practical generator operation. The feasibility of the proposed method is demonstrated for three different systems, and it is compared with the Gamethod in terms of the solution quality and computation efficiency. The experimental results show that the proposed PSO method was indeed capable of obtaining higher quality solutions efficiently in ED problems.

Objective of the Present Work

From the literaturere view, It is observed that lot of work has been done on design of

X control chart of various types following various approaches and optimization techniques.The particle swarm optimization technique has been observed to have application in wide variety of fields. However, this technique has not been tried in the economic design of control chart. With this motivation, the objective of the present work is to use the PSO technique in economic design

of*X* control chart where quality characteristic is normally distributed and compare the resultswith that reported in literature.

METHODOLOGY

For comparing the results of the economic design of control chart using particle swarm optimization the following example earlier solved by Montgomery [22] and Shiau [8] has been considered in this work.

Example: A manufacturer produces non returnable glass bottles for packaging a carbonated soft-drink beverage. The wall thickness of the bottles is an important quality characteristic. If the wall is too thin, internal pressure generated during filling will cause the bottle to burst. In an effort to reduce costs, the manufacturer wishes to design an economically

Optimum *X* chart for the process.

Based on an analysis of quality control technicians' salaries and the costs of test equipment, it is estimated that the fixed cost of taking a sample is $1. The variable cost of sampling is estimated to be $0.10 per bottle, and it takes approximately 1 min (0.0167h) to measure and record the wall thickness of a bottle.

The process is subjected to several different types of assignable causes. However, on the average, when the process goes out-of-control, the magnitude of the shift is approximately two standard deviations. Process shifts occur at random with a frequency of about one every 20 h of operation. Thus, the exponential distribution with parameter λ= 0.05 is a reasonable model of the runlengthin-control.

The bottlesare soldtoa soft-drink bottler.If thewallsare toothin, anexcessive number of bottles will burst when they are filled. When this happens, the bottler's standard practice is to back charge the manufacturer for the costs of cleanup and lost production. Based on this practice, the manufacturer estimates that the penalty cost of operating in the out-of-control state for onehour is $100.

PSO Algorithm

PSO simulates the behaviors of bird flocking. Let a group of birds are randomly searching food in an area and there is only one piece of food in the area being searched. The group behavior learned by the birds while flying to reach their destination in minimum possible time is simulated in solving the optimization problems. In PSO, each single solution isa"bird" in the search space i.e.called a "particle". All the particles have fitness values which are evaluated by the fitness function to be optimized, and have velocities which direct the flying of the particle. The particles fly through the problem space by following the current optimum particles.

RESULT AND DISCUSSION

The output from the C program based on PSO algorithm give the result of optimum solution of the example taken from Montgomery [22] giving the economic design of *X* chart. This program calculates the optimum values of sampling frequency hand width of control limit k for different values of sample size n, and also computes the corresponding minimum values of the cost function E(L) as defined in Equation 3.13.

Table 5.1. Economic Design of *X*Chart for 3-sigma Limits (k=3)

Sample Size, N	Width of Control Limit, K	Optimum Sampling Interval, H	Optimum Cost, E(L)
1	3	0.231	16.794
2	3	0.418	12.232
3	3	0.586	10.622
4	3	0.717	9.910
5	3	0.807	9.592
6	3	0.86	9.463
7	3	0.906	9.422
8	**3**	**0.937**	**9.419**
9	3	0.963	9.431
10	3	0.988	9.446

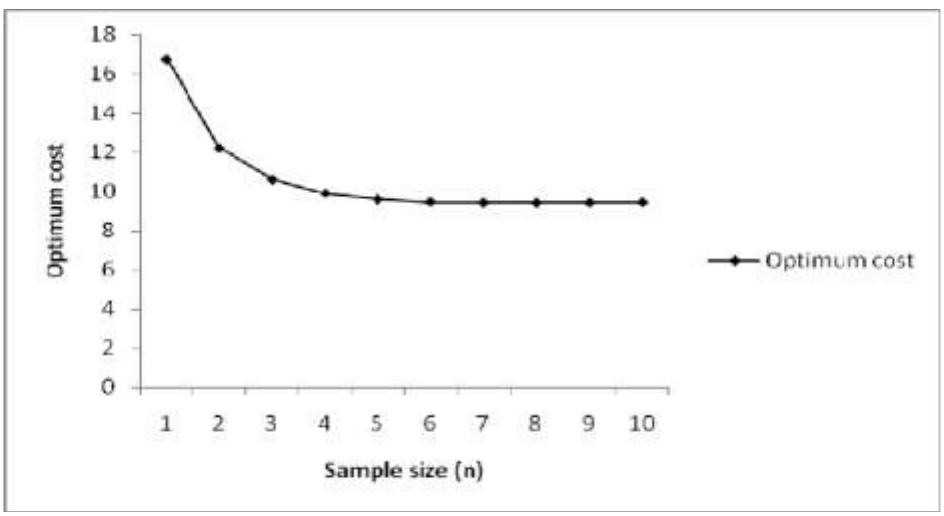

Fig. 5.1.Optimum Cost For 3-Sigma Limit (K=3)

CONCLUSIONS

In this project work, the X control chart has been designed for economic purpose by using Particle Swarm Optimization and based on the results the following conclusions are made. The economic design of X chart using PSO is found to be superior to that obtained

- Earlier in the literature. For x chart with 3-sigma limits (k = 3), minimum cost (i.e., 9.419) is obtained with
- Sample size n = 8 and sampling interval h = 0.937. With variable k, minimum cost (i.e., 9.352) is obtained with

sample size n=9, sampling

- Interval h = 0.963 and width of control limit k = 3.360. The best economic design is thus found to be at sample size 9, width of control limits
- 3.36 sigma and sampling interval 0.963 h

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