On Super Mean Labeling for Line and Middle Graphof Path and Cycle

M. Lakshmi Prasanna[1], Ch Radhika[2], K Vanama Devi[3]

[1,2,3]K.B.N. College (Autonomous), Vijayawada-520001, Andhra Pradesh, India

---

ABSTRACT

Let G be a (p, q) graph. A (p, q) graph is a graph with p vertices and q edges. Let us definean injection mapping $f : V(G) \to \{1,2, \dots, p+q\}$, for each edge $e = uv$ in E labeled by,

$$f^*(e) = \begin{cases} \dfrac{f(u)+f(v)}{2} & if \;\; f(u)+f(v) \; is \; even \\ \dfrac{f(u)+f(v)+1}{2} & if \; f(u)+f(v) is \; odd \end{cases}$$

Then f is said to be super mean labeling if the set

$$f(v) \cup \{f^*(e): e\epsilon E\} = \{1,2, \dots, p+q\}.$$

The graph which admits super mean labeling is called super mean graph.

In this paper we have found out the ***Super Mean Labeling*** *of Line and middle Graph of Pathand Cycle.*

Keywords: Super Mean Labeling, Line graph of path with n vertices - $L(P_n)$, Line Graph of cycle with nvertices - $L(C_n)$, Middle Graph of path with n vertices - $M(P_n)$,Middle Graph of cycle with n vertices - $M(C_n)$.

INTRODUCTION

A graph G is a combination of vertices and edges which are connected to each other.Let G (V,E)be a graph with vertex set V and the edge set E, respectively. By a graphG = (V,E) we mean a finite undirected graph with neither loops nor multiple edges. In this paper $V(G)$ is the vertex setand $E(G)$ is the edge set of a graph G.

The number of vertices of G is called order of G and it is denoted by p. The number of edges ofG is called size of G and it is denoted by q. A(p, q)graph G is a graph with p vertices and q edges.Terms and notations no defined here are used in the sense of Harary[1].

A graph labeling is the assignment of labels traditionally represented by integers,to verticesand/or edges of a graph. We have different methods of labeling.

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling was madewhich refers to Rosa's research and that was held in 1967[6].

In 2003, Somasundaram and Ponraj [2] have introduced the notation of Mean Labelingof graphs. The concept of Super Mean Labeling was introduced by Ponraj and Ramya [3].Furthermoresome results on super mean graphs are discussed in[7-11].

B. Gayathri and M. Tamilselvi[12,13] extended super mean labeling to k-super mean labeling.
In 2018, NurInayah, I.WayanSudarsanaSelvyMusdalifah, and Nurhasanah Daeng Mangesa haveintroduced the notion of Super Mean Labeling for Total Graph of Path and Cycle[4].

Preliminaries:

Definition 1:

Let G be a (p,q) graph. A graph G is called a **mean graph** if there is an injective function f from the vertices of G to $\{0,1,2,\dots,q\}$. Such that when each edge $e = uv$ is labeled with

$$f^*(e) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

Then the resulting edge labels are distinct.

Definition 2:

Let $f: V \to \{1,2,\dots.p+q\}$ be an injection on G. For each edge $e = uv$ and an integer $m \geq 2$, the induced Smarandachely edge m-labeling $f^*$is defined by

$$\boldsymbol{f^*(e = uv) = \frac{f(u)+f(v)}{m}}.$$

Then $f$is called a **Smarandachely super m-mean labeling** if

$f(v) \cup \{f^*(e): e \epsilon E\} = \{1,2,\dots,p+q\}$.

A graph that admits super mean m-labeling is called Smarandachely m-mean graph.Particularly, if$\boldsymbol{m = 2}$, we know that,

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & if\ f(u)+f(v) is even \\ \dfrac{f(u)+f(v)+1}{2} & if\ f(u)+f(v) is odd \end{cases}$$

Such a labeling f is called super mean labeling of G if

$$f(V) \cup \{f^*(e): e \in E\} = \{1,2,..,p+q\}.$$

A graph that admits super mean labeling is called super mean graph.

Definition 3:

The **Line Graph** L(G) of G is the graph with vertices are the edges of G with two vertices ofL(G) adjacent whenever the corresponding edges of G are adjacent[5].

Definition 4:

The Middle Graph M(G) is defined as follows[5]. The vertex set M(G) is V(G)UE(G). Twovertices X,Y in the vertex set M(G)are adjacent in M(G) in case one of the following holds:

i. X , Y are in E(G) and X,Y are adjacent in G .
ii. X is in V(G), Y is in E(G), and X,Y are incident in G.

On Super Mean Labeling for Line Graph of Path:

The theorem proposed in this section deals with the super mean labeling for line graph of pathon n vertices, $L(P_n)$.

Theorem 1:

The line graph of path on n vertices, $L(P_n)$ is a super mean graph for all $n \geq 3$.

Proof:

Let $V(L(P_n)) = \{u_i: 1 \leq i \leq n-1\}$ and $E(L(P_n)) = \{e_i, 1 \leq i \leq n-2\}$ with $e_i = u_i u_{i+1}$ for $1 \leq i \leq n-2$. Immediately, we have that the cardinality of the vertex set and the edge set of

$L(P_n)$ are $P = n-1$ and $q = n-2$ respectively, and so $p+q = 2n-3$. Define an injection $f: V(L(P_n)) \to \{12, \dots 2n-2\}$ for $n \geq 3$ as follows:

$$f(u_i) = 2i-1 \quad for \quad i = 1,2, \dots n-1$$

And so we have,

$$f^*(e_i) = 2i \qquad for \ i = 1,2, \dots, n-2$$

Next, we consider the following sets:

$$A_1 = \{f(u_i) = 2i-1 \ for \ i = 1,2, \dots n-1\} A_2 = \{f^*(e_i) = 2i \qquad for \ i = 1,2, \dots, n-2\}$$

It can be verified that $f\,(V(L(P_n))) \cup f^*\,(E(L(P_n))) = \bigcup_{i=1}^{2} A_i = \{1,2, \dots, n-1\}$ and so f is a super mean labeling of $L(P_n)$. Hence $L(P_n)$ is a super mean graph.

On Super Mean Labeling for Line Graph of Cycle:

The theorem proposed in this section deals with the super mean labeling for line graph of cycleof n vertices, $L(C_n)$.

Theorem 2:

The line graph of cycle on n vertices, $L(C_n)$, is a super mean graph if either n is odd and $n \geq 3$ or n is even and $n \geq 6$.

Proof:

Let $V(L(C_n)) = \{u_i : 1 \leq i \leq n\}$ and $E(L(C_n)) = \{e_i, 1 \leq i \leq n\}$ with

$$e_i = \begin{cases} u_i u_{i+1} & for \ 1 \leq i \leq n-1 \\ u_i u_1 & for \ i = n \end{cases}$$

Immediately, we have that the cardinality of the vertex set and the edge set of $L(C_n)$ are $p = n \ and \ q = n$ respectively, and so $p+q = 2n$.

Define an injection $f: V(L(C_n)) \to \{1,2, \dots, 2n\}$ for odd and $n \geq 3$ as follows.

$$f(u_i) = \begin{cases} 2i-1 & for\ i = 1,2, \ldots \frac{n+1}{2} \\ 2i & for\ i = \frac{n+1}{2}+1, \ldots, n \end{cases}$$

And also we have,

$$f^*(e_i) = \begin{cases} 2i & for\ i = 1,2, \ldots, \frac{n-1}{2} \\ 2i+1 & for\ i = \frac{n+1}{2}, \ldots n-1 \\ n+1 & for\ i = n \end{cases}$$

Next, we consider the following sets,

$$A_1 = \{f(u_i) = 2i-1 \ \ for\ i = 1,2, \ldots, \frac{n+1}{2}\}$$

$$A_2 = \{f(u_i) = 2i \ \ for\ i = \frac{n+1}{2}+1, \ldots, n\}$$

$$A_3 = \{f^*(e_i) = 2i \ \ for\ \ i = 1,2, \ldots, \frac{n-1}{2}\}$$

$$A_4 = \{f^*(e_i) = 2i+1 \ \ for\ \ i = \frac{n+1}{2}, \ldots n-1\}$$

$$A_5 = \{f^*(e_i) = n+1 \ \ for\ \ i = n\}$$

It can be verified that $f(V(L(C_n))) \cup f^*(E(L(C_n))) = \bigcup_{i=1}^{5} A_i = \{1,2, \ldots 2n\}$ and so f is a super mean labeling of $L(C_n)$. Hence $L(C_n)$ is a super mean labeling for line graph of cycle when n isodd and $n \geq 3$.

Now define an injection $f_1: V(L(C_n)) \rightarrow \{1,2, \ldots 2n\}$ for n is even and $n \geq 6$ as follows

$$f_1(u_i) = \begin{cases} 2i-1 & for\ i = 1,2 \\ 3i-3 & for\ i = 3,4 \\ 2i+1 & for\ i = 5, \ldots, \frac{n}{2}+1 \\ 2i+2 & for\ i = \frac{n}{2}+2, \ldots, n-1 \\ 7 & for\ i = n \end{cases}$$

And also we have,

$$f_1^*(e_i) = \begin{cases} 2 & for\ i = 1 \\ 3i-1 & for\ i = 2,3 \\ 2i+2 & for\ i = 4, \ldots \frac{n}{2} \\ 2i+3 & for\ i = \frac{n}{2}+1, \ldots, n-2 \\ n+4 & for\ i = n-1 \\ 4 & for\ i = n \end{cases}$$

Next we consider the following sets,

$$A_1 = \{f_1(u_i) = 2i-1 \quad for\ i = 1,2\}$$
$$A_2 = \{f_1(u_i) = 3i-3 \quad for\ i = 3,4\}$$
$$A_3 = \{f_1(u_i) = 2i+1 \quad for\ i = 5, \ldots, \frac{n}{2}+1\}$$
$$A_4 = \{f_1(u_i) = 2i+2 \quad for\ i = \frac{n}{2}+2, \ldots, n-1\}$$
$$A_5 = \{f_1(u_i) = 7 \quad for\ i = n\}$$
$$A_6 = \{f^*(e_i) = 2 \quad for\ i = 1\}$$
$$A_7 = \{f_1^*(e_i) = 3i-1 \quad for\ i = 2,3\}$$
$$A_8 = \{f_1^*(e_i) = 2i+2 \quad for\ i = 4, \ldots . \frac{n}{2}\}$$
$$A_9 = \{f_1^*(e_i) = 2i+3 \quad for\ i = \frac{n}{2}+1, \ldots, n-2\}$$
$$A_{10} = \{f^*(e_i) = n+4 \quad for i = n-1\}$$
$$A_{11} = \{f_1^*(e_i) = 4 \ for\ i = n\}$$

It can be verified that $f_1(V(L(C_n))) \cup f_1^*(E(L(C_n))) = \bigcup_{i=1}^{11} A_i = \{1,2, \ldots 2n\}$ and so f_1 is a super mean labeling of $L(C_n)$. Hence $L(C_n)$ is a super mean labeling for line graph of cycle when n is evenand $n \geq 6$.

On Super Mean Labeling for Middle Graph of Path:

The theorem proposed in this section deals with the super mean labeling for middle graph ofpath on n vertices, $M(P_n)$.

Theorem 3:
The middle graph of path on n vertices, $M(P_n)$, is a super mean graph for all $n \geq 3$.

Proof:

Let $V(M(P_n)) = \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n-1\}$ and $E(M(P_n)) = \{e'_i, e''_i: 1 \leq i \leq n-1\} \cup \{e_i: 1 \leq i \leq n-2\}$ with $e_i = u_i u_{i+1}$ for $1 \leq i \leq n-2$, $e'_i = v_i u_i$ and $e''_i = u_i v_{i+1}\ for\ 1 \leq i \leq n-1$

Immediately, we have that the cardinality of the vertex set and the edge set of $M(P_n)$ are $p = 2n - 1$ and $q = 3n - 4$ respectively, and so $p + q = 5n - 5$.
Define an injection $f: V(M(P_n)) \to \{1,2, \dots 5n - 5\}$for $n \geq 3$ as follows:

$$f(v_i) = \begin{cases} 1 & for\ i = 1 \\ 5i - 5 & for\ i = 2, \dots n \end{cases}$$

Also, $f(u_i) = 5i - 2 \quad for\ 1 \leq i \leq n - 1$

And so we have,

$$f^*(e_i) = 5i + 1 \quad for \quad i = 1,2, \dots, n - 2$$

$$f^*(e'_i) = 5i - 3 \quad for \quad i = 1,2, \dots, n - 1$$

$$f^*(e''_i) = 5i - 1 \quad for \quad i = 1,2, \dots, n - 1$$

Next, we consider the following sets:

$A_1 = \{f(v_i) = 1 \ for \ i = 1\}$
$A_2 = \{f(v_i) = 5i - 5 \ for \ i = 2, \dots, n\}$
$A_3 = \{f(u_i) = 5i - 2 \ for \ i = 1,2, \dots n - 1\}$ $A_4 = \{f^*(e_i) = 5i + 1 \ for \ i = 1,2, \dots, n - 2\}$ $A_5 = \{f^*(e'_i) = 5i - 3 \ for \ i = 1,2, \dots, n - 1\}$
$A_6 = \{f^*(e''_i) = 5i - 1 \ for \ i = 1,2, \dots, n - 1\}$

It can be verified that $f(V(M(P_n))) \cup f^*(E(M(P_n))) = \bigcup_{i=1}^{6} A_i = \{1,2, \dots ,5n - 5\}$and so f is a super mean labeling of $M(P_n)$. Hence $M(P_n)$ is a super mean graph.

On Super Mean Labeling for Middle Graph of Cycle:

The theorem proposed in this section deals with the super mean labeling for middle graph ofcycle of n vertices, $M(C_n)$.

Theorem 4:

The middle graph of cycle on n vertices, $M(C_n)$, is a super mean graph if either n is odd and $n \geq 3$ or n is even and $n \geq 4$.

Proof:

Let $V(M(C_n)) = \{v_i, u_i: 1 \leq i \leq n\}$ and $E(M(C_n)) = \{e_i, e'_i, e''_i: 1 \leq i \leq n\}$ with

$$e_i = \begin{cases} u_i u_{i+1} & for\ 1 \leq i \leq n - 1 \\ u_i u_1 & for\ i = n \end{cases}$$

$$e'_i = u_i v_i \quad for\ i = 1,2, \dots n$$

$$e''_i = \begin{cases} v_i u_{i+1} & for\ 1 \leq i \leq n - 1 \\ v_i u_1 & for\ i = n \end{cases}$$

Immediately, we have that the cardinality of the vertex set and the edge set of $M(C_n)$ are $p = 2n \ and \ q = 3n$respectively, and so$p + q = 5n$.

Define an injection $f: V(M(C_n)) \to \{1,2, \dots ,5n\}$ for odd and$n \geq 3$ as follows.

$$f(v_i) = \begin{cases} 5i-2 & for\ i = 1,2,\dots\frac{n-1}{2} \\ 5i+1 & for\ i = \frac{n+1}{2} \\ 5i & for\ i = \frac{n+3}{2},\dots,n \end{cases}$$

$$f(u_i) = \begin{cases} 1 & for\ i = 1 \\ 5i-4 & for\ i = 2,3,\dots,\frac{n+1}{2} \\ 5i-2 & for\ i = \frac{n+3}{2},\dots,n \end{cases}$$

And also we have,

$$f^*(e_i) = \begin{cases} 4 & for\ i = 1 \\ 5i-1 & for\ i = 2,3,\dots\frac{n-1}{2} \\ 5i & for\ i = \frac{n+1}{2} \\ 5i+1 & for\ i = \frac{n+3}{2},\dots,n-1 \\ \frac{5n-1}{2} & for\ i = n \end{cases}$$

$$f^*(e_i') = \begin{cases} 2 & for\ i = 1 \\ 5i-3 & for\ i = 2,3,\dots,\frac{n-1}{2} \\ 5i-1 & for\ i = \frac{n+1}{2},\dots,n \end{cases}$$

$$f^*(e_i'') = \begin{cases} 5i & for\ i = 1,2,\dots,\frac{n-1}{2} \\ 5i-2 & for\ i = \frac{n+1}{2},\dots,n-1 \\ \frac{5n+1}{2} & for\ i = n \end{cases}$$

Next, we consider the following sets,

$$A_1 = \{f(v_i) = 5i-2 \quad for\ i = 1,2,\dots,\frac{n-1}{2}\}$$

$$A_2 = \{f(v_i) = 5i+1 \quad for\ i = \frac{n+1}{2}\}$$

$$A_3 = \{f(v_i) = 5i \quad for\ i = \frac{n+3}{2}$$

$2 \quad , \ldots , n\}$

$$A_4 = \{f(u_i) = 1 \quad for\ i = 1\}$$

$$A_5 = \{f(u) = 5i - 4 \quad for\ i = 2,3, \ldots, \frac{n+1}{2}\}$$

$$A_6 = \{f(u_i) = 5i - 2 \quad for\ i = \frac{n+3}{2}$$

$$A_7 = \{f^*(e_i) = 4 \ \ for \ \ i = 1\}$$

$$A_8 = \{f^*(e_i) = 5i - 1 \ \ for \ \ i = 2,3, \ldots, \frac{n-1}{2}\}$$

$$A_9 = \{f^*(e_i) = 5i \ \ for \ \ i = \frac{n+1}{2}\}$$

$$A_{10} = \{f^*(e_i) = 5i + 1 \ \ for \ \ i = \frac{n+3}{2}, \ldots, n-1\}$$

$$A_{11} = \{f^*(e_i) = \frac{5n-1}{2} \quad for \ \ i = n\}$$

$$A_{12} = \{f^*(e'_i) = 2 \ \ for \ \ i = 1\}$$

$$A_{13} = \{f^*(e'_i) = 5i - 3 \ \ for \ \ i = 2,3, \ldots, \frac{n-1}{2}\}$$

$$A_{14} = \{f^*(e'_i) = 5i - 1 \ \ for \ \ i = \frac{n+1}{2}, \ldots, n\}$$

$$A_{15} = \{f^*(e''_i) = 5i \ \ for \ \ i = 1,2, \ldots, \frac{n-1}{2}\}$$

$$A_{16} = \{f^*(e''_i) = 5i + 2 \ for \ \ i = \frac{n+1}{2}, \ldots, n-1\}$$

$$A = \{f^*(e'') = \frac{5n+1}{} \quad for \ \ i = n\}$$

17 i 2

It can be verified that $f(V(M(C_n))) \cup f^*(E(M(C_n))) = \bigcup_{i=1}^{17} A_i = \{1,2,\dots 5n\}$ and so f is a super mean labeling of $M(C_n)$. Hence $M(C_n)$ is a super mean labeling for line graph of cycle when n is odd and $n \geq 3$.

Now define an injection $f_1 : V(M(C_n)) \to \{1,2,\dots 5n\}$ for n is even and $n \geq 6$ as follows,

$$f_1(v_i) = \begin{cases} 1 & for\ i = 1 \\ 5i-5 & for\ i = 2,3,\dots \frac{n}{2} - 1 \\ 5i-2 & for\ i = \frac{n}{2}, \dots, \frac{n}{2} + 1 \\ 5i-3 & for\ \frac{n}{2} + 2, \dots, n \end{cases}$$

n

Next we consider the following sets,

$$A_1 = \{f_1(v_i) = 1 \ \ for\ i = 1\}$$

$$A_2 = \{f_1(v_i) = 5i-5 \ \ for\ i = 2,3,\dots, \frac{n}{2} - 1\}$$

$$A_3 = \{f_1(v_i) = 5i+2 \ \ for\ i = \frac{n}{2}, \dots, \frac{n}{2} + 1\}$$

$$A_4 = \{f_1(v_i) = 5i-3 \ \ for\ i = \frac{n}{2} + 2, \dots, n\}$$

$$f_1(u_i) = \begin{cases} 5i-2 & for\ i = 1,2,\dots, \frac{n}{2} - 1 \\ 5i-5 & for\ i = \frac{n}{2} \\ 5i & for\ \frac{n}{2} + 1, \dots, n \end{cases}$$

And also we have,

$$f_1^*(e_i) = \begin{cases} 5i+1 & for\ i = 1,2,\dots \frac{n}{2} - 2 \\ 5i-1 & for\ i = \frac{n}{2} - 1 \end{cases}$$

ISSN: 2319-7463, Vol. 7 Issue 9, September-2018, Impact Factor: 4.059

$$f^*(e_i^{'}) = \begin{cases} 5i & for\ i = \frac{n}{2} \\ 5i+3 & for\ i = \frac{n}{2}+1,\dots,n-1 \\ \frac{5n+4}{2} & for\ i = n \\ \frac{5i-1}{2} & for\ i = 1 \\ 5i-3 & for\ i = 2,\dots,\frac{n}{2} \\ 5i-1 & for\ i = \frac{n}{2}+1,\dots,n \end{cases}$$

$$f^*(e_i^{''}) = \begin{cases} 5i-1 & for\ i = 1,\dots,\frac{n}{2}-2\ and\ \frac{n}{2}+1,\dots,n-1 \\ 5i+1 & for\ i = \frac{n}{2}-1\ and\ \frac{n}{2} \\ \frac{5n+2}{2} & for\ i = n \end{cases}$$

$$A_5 = \{f_1(u_i) = 5i-2\ for\ i = 1,2,\dots,\frac{n}{2}-1\}$$

$$A_6 = \{f_1(u_i) = 5i-5\ for\ i = \frac{n}{2}\}$$

$$A_7 = \{f_1(u_i) = 5i\ for\ i = \frac{n}{2}+1,\dots,n\}$$

$$A_8 = \{f^*(e_i^1) = 5i+1\ for\ i = 1,2,\dots\frac{n}{2}-2\}$$

$$A_9 = \{f^*(e_i^1) = 5i-1\ for\ i = \frac{n}{2}-1\}$$

$$A_{10} = \{f^*(e_i^1) = 5i\ for\ i = \frac{n}{2}\}$$

$$A_{11} = \{f^*(e_i^1) = 5i+3\ for\ i = \frac{n}{2}+1,\dots,n-1\}$$

$$A = \{f^*(e\ ) = \frac{5n+4}{}\ for i = n\}$$

12 1 i $\overline{}$ 2

$A \quad = \{f^*(e') = \frac{5i-1}{2} \quad for\ i = 1\}$

13 1 i 2

$A \quad = \{f^*(e') = 5i - 3 \quad for\ i = 2, \dots, \frac{n}{2}\}$

14 1 i

$A \quad = \{f^*(e') = 5i - 1 \quad for\ i = \frac{n}{2} + 1, \dots, n\}$

15 1 i

$A \quad = \{f^*(e'') = 5i - 1 \quad for\ i = 1, \dots \frac{n}{2} - 2 \ and\ \frac{n}{2}\}$

16 1 i

$A \quad = \{f^*(e') = 5i + 1 \quad for\ i = \frac{n}{2} - 1\ and\ \frac{n}{2} + 1, \dots, n-1\}$

17 1 i

$A \quad = \{f^*(e') = \frac{5n+2}{2} \quad for\ i = n\}$

18 1 i

It can be verified that $f_1(V(M(C_n))) \cup f^*(E(M(C_n))) = \bigcup_{i=1}^{18} A_i = \{1,2, \dots 5n\}$

a super mean labeling of $M(Cn)$. Hence $M(Cn)$ is a super mean labeling for line graph of cycle when n is even and $n \geq 6$.

Summary and Remarks:

Here we propose new results corresponding to super mean labeling for line and middle graph of path and cycle. All results reported here are in line and middle graph of path and cycle, $L(Pn)$, $L(Cn)$, $M(Pn)$ and $M(Cn)$. In future, it is not only possible to investigate some more results corresponding to other graph families but also Smarandachely super m-mean labeling in general as well.

REFERENCES

[1]. F.Harary, “Recent results on generalized Ramsey theory for graphs”, in Graph Theory and Applications, vol.303 of Lecture Notes in Mathematics, pp.125-138, Springer Berlin Heidelberg, Berlin, Heidelberg, 1972.

[2]. S. Somasundaram and R. Ponraj, “mean labeling of graphs”, National Academy of science letters, vol.26,no.7-8,pp.210-213,2003

[3]. R. ponraj and D. Ramya, “On super mean graphs of order”, Bulletin of Pure and Applied Sciences, vol.25, no.1,pp.143-148,2006

[4]. NurInayah, “On super mean labeling for total graph of path and cycle, International Journal of Mathematics and mathematical sciences, vol.2018.

[5]. K. Rajam, “On Harmonious Colouring of Line Graph of Star Graph Families”, International Journal of Statistika and Mathematika. ISSN 2277.2790E-ISSN:2249-8605, vol. 7, 2013 pp33-36.

[6]. Rosa, Symposium, Rome, July 1996), Gordon and Breach On certain valuations of the vertices of a graph Theory of graphs (Internet , N.Y. and Duhod, Paris (1967)349-355.

[7]. P. Jeyanthi and D. Ramya, Super mean labeling of some classes of graphs, International J.Math. Combin.,1(2012) 83-91.

[8]. P. Jeyanthi, D. Ramya and P.thangavelu, On super mean graphs, AKCE J. Graphs Combin.,6 No.1 (2009) 103-112.

[9]. D. Ramya, R. Ponraj and P. Jeyanthi, super mean labeling of graphs, ArsCombin., 112 (2013)65-72.

[10]. P. Sugirtha, R. Vasuki and J.Venkateswari, Some new super mean graphs, International Journal of Mathematics Trends and Technology, Vol. 19 No. 1 March 2015.

[11]. R. Vasuki and A. Nagarajan, Some results on super mean graphs, International J. Math. Combin.,3(2009) 82-96.

[12]. B. Gayathri, M. Tamilselvi, M. Duraisamy, k-super mean labeling of graphs, In: Proceedings of the International Conference On Mathematics and Computer Sciences, Loyola College, Chennai (2008), 107-111.

[13]. B. Gayathri and M. Tamilselvi, k-super mean labeling of some trees and cycle related graphs, Bulletin of Pure and Applied Sciences, Volume 26E(2)(2007) 308-311.