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Reliability-Based Design of Foundations on Soft Clay with Deep Soil Mixing: Quantifying the Economic Value of Soil Improvement under Parameter Uncertainty

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ABSTRACT

Conventional foundation design on soft clay relies on deterministic factors of safety that do not explicitly account for the substantial spatial variability of soil parameters typically encountered in practice. This paper presents a reliability-based design (RBD) framework integrating Monte Carlo simulation, deep soil mixing (DSM) ground improvement, and life-cycle cost analysis to quantify the true economic value of soil improvement under realistic parameter uncertainty. A 2.0 m × 2.0 m square footing supporting a 700 kN column load on soft Holocene clay (Su = 25 kPa) is investigated. Four design alternatives are compared: (i) conventional shallow footing with FOS = 3.0; (ii) shallow footing with stone column treatment (As = 0.20); (iii) shallow footing with deep soil mixing (As = 0.30); and (iv) RBD-calibrated DSM design directly targeting reliability index β = 3.0. Soil parameters are treated as lognormally distributed random variables with coefficients of variation reflecting Indian field conditions: Su (COV = 30%), φ' (COV = 12%), and elastic modulus E (COV = 35%). Monte Carlo simulation with 10,000 realizations quantifies failure probability for each alternative. Results demonstrate that the conventional FOS = 3.0 design corresponds to an actual reliability index β = 2.41 well below the target value of 3.0 exposing a critical and largely-overlooked safety deficit. DSM treatment with As = 0.30 raises the achieved reliability to β = 3.74 at a 32% cost increase, while RBD-calibrated DSM achieves the target β = 3.0 at FOS = 1.85, producing a 21% cost saving relative to the conventional design. The bearing capacity increase ratio (BCIR) achieved by DSM is 2.99, with settlement reduction ratio (SRR) of 2.36. The framework provides a quantitative basis for adopting probabilistic design methods and economically rational soil improvement decisions in Indian geotechnical practice.

Index Terms Reliability-based design, soft clay, deep soil mixing, stone columns, Monte Carlo simulation, soil improvement, life-cycle cost analysis, target reliability index.

INTRODUCTION

Foundation design on soft cohesive soil deposits has historically relied on deterministic methodologies in which a single nominal factor of safety typically 2.5 to 3.0 is applied to the calculated ultimate bearing capacity to obtain the design allowable pressure **[1], [2]**. This deterministic approach implicitly assumes that the soil parameters used in the calculation accurately represent the true site conditions and that uncertainties are adequately addressed by the magnitude of the safety factor. Decades of geotechnical experience and instrumented case histories, however, have repeatedly demonstrated the inadequacy of this assumption: soil parameters exhibit substantial spatial variability, measurement uncertainty, and transformation uncertainty, leading to actual failure probabilities that may differ by an order of magnitude or more between sites with identical nominal safety factors **[3], [4]**.

Phoon and Kulhawy **[3]** comprehensively characterized geotechnical variability, demonstrating that the coefficient of variation (COV) of undrained shear strength typically ranges from 20% to 40% for natural soft clay deposits, with the upper end of this range commonly observed in heterogeneous alluvial and marine clay environments. For Indian coastal and riverine geotechnical conditions, where Holocene clay deposits frequently exhibit complex stratigraphy and varying degrees of overconsolidation, COV values of 30% or higher are not uncommon **[5]**. At such variability levels, the actual reliability index achieved by a deterministic FOS = 3.0 design may fall as low as β = 2.4, corresponding to a failure probability of 8×10^{-3} nearly an order of magnitude greater than the target value of 1.35×10^{-3} implied by β = 3.0 **[3], [4]**.

Soil improvement techniques, particularly deep soil mixing (DSM) and stone columns, offer a fundamentally different approach to managing soil parameter uncertainty: rather than accepting the in-situ soil variability and applying conservative safety factors, ground improvement physically modifies the soil to produce a more uniform and reliable foundation medium **[6], [7]**. DSM creates stiff cement-treated soil columns by mechanically mixing in-situ clay with cement or lime binders, achieving unconfined compressive strengths typically ranging from 500 to 2,000 kPa **[8]**. The combined effect of strength enhancement and variability reduction produces foundation systems with reliability characteristics fundamentally superior to those achievable through conventional design alone.

Despite the well-established benefits of both reliability-based design (RBD) and soil improvement, their integrated application within a unified analytical framework remains limited in routine geotechnical practice, particularly in the Indian context. Most published RBD studies treat soil improvement qualitatively or assume idealized improvement parameters, while most ground improvement studies use deterministic analyses that ignore parameter uncertainty entirely **[9], [10]**. The result is a missed opportunity: practicing engineers cannot quantitatively justify investment in soil improvement on the basis of reliability outcomes, and instead rely on intuitive judgments that may produce either over-conservative or unsafe designs.

This paper addresses this gap by presenting a comprehensive RBD framework that integrates: (i) probabilistic characterization of soil parameter uncertainty using lognormal distributions with COV values calibrated to Indian field conditions; (ii) Monte Carlo simulation with 10,000 realizations for direct failure probability estimation **[4]**; (iii) quantitative modeling of DSM and stone column treatment effects on both mean strength and parameter variability **[6], [8]**; and (iv) life-cycle cost analysis incorporating treatment cost, ultimate failure cost, and target reliability requirements per fib Model Code 2010 **[11]**.

The principal contributions are: (i) quantitative demonstration that conventional FOS = 3.0 design produces an actual reliability index β = 2.41 for typical Indian soft clay conditions, well below target requirements; (ii) demonstration that DSM treatment with area replacement ratio As = 0.30 raises reliability to β = 3.74 at acceptable cost premium; (iii) development of an RBD-calibrated DSM design that achieves target β = 3.0 at 21% cost reduction relative to the conventional design; and (iv) provision of a complete framework with quantitative bearing capacity increase ratios (BCIR) and settlement reduction ratios (SRR) directly applicable to Indian foundation design practice.

LITERATURE REVIEW

A. Reliability-Based Design Theory

Reliability-based design theory provides a rigorous probabilistic framework for foundation design that explicitly incorporates parameter uncertainty into the safety verification process. The reliability index β, formally defined by Hasofer and Lind for general limit state functions, expresses the safety margin in standard deviation units and relates directly to the probability of failure Pf through the standard normal cumulative distribution function **[3]**: $\beta = -\Phi^{-1}(P_f)$ or equivalently $P_f = \Phi(-\beta)$ (1) For typical foundation applications, the limit state function is formulated as $G(X) = R(X) - S(X)$, where R is the resistance, S is the load effect, and X is the vector of basic random variables. The fib Model Code 2010 **[11]** specifies target reliability indices that depend on the consequence class of the supported structure and the design service life: β = 3.3 for moderate consequence class structures over 50 years, β = 3.8 for high consequence class structures, and β = 4.3 for very high consequence structures.

Fenton and Griffiths **[4]** significantly extended the foundational RBD framework through random field theory, demonstrating that the spatial correlation structure of soil parameters fundamentally influences foundation reliability beyond what point-statistics alone can capture. Their seminal work established that soils with strong spatial correlation produce reliability outcomes substantially different from those predicted under the assumption of spatial independence, with implications for the design of large foundations spanning multiple correlation lengths.

B. Deep Soil Mixing Ground Improvement

Deep soil mixing, originally developed in Sweden in the 1970s and subsequently refined through Japanese and global practice, creates stiff cement-stabilized soil columns by mechanically mixing in-situ clay with cement or lime binders using rotating mixing augers **[6]**. Terashi and Juran **[7]** comprehensively reviewed DSM mechanics, identifying the binder content (typically 80–250 kg per m³ of treated soil), water-cement ratio, mixing intensity, and curing time as the primary parameters controlling treated soil performance.

The unconfined compressive strength (UCS) of cement-treated soft clay typically ranges from 500 kPa for low binder content (100 kg/m³) to 3,000 kPa for high binder content (250 kg/m³), representing strength improvements of 20 to 100 times relative to the untreated soft clay **[8]**. Beyond pure strength enhancement, DSM provides three additional benefits relevant to foundation reliability: (i) significantly reduced parameter variability COV of treated soil UCS typically 15-20% versus 30% or more for natural soft clay; (ii) reduced compressibility through partial replacement of compressible clay with stiff cement-treated columns; and (iii) improved permeability characteristics that accelerate consolidation of the surrounding untreated soil.

C. Composite Behavior of Improved Ground

The composite behavior of DSM-improved ground is governed by the area replacement ratio As, defined as the ratio of treated column area to total improved ground area. For DSM column patterns with column diameter d and center-to-center spacing s in a triangular arrangement, $As = \pi \cdot d^2/(2\sqrt{3} \cdot s^2)$. Typical practical values of As range from 0.15 to 0.40, with 0.20-0.30 representing the most economically efficient range for moderate improvement targets.

The composite undrained shear strength of improved ground may be estimated through the strength averaging approach proposed by Priebe **[12]** and refined by Han **[13]**: $S_{u,comp} = As \cdot S_{u,col} + (1 - As) \cdot S_{u,native}$ (2) where $S_{u,col}$ is the undrained shear strength of treated columns (typically $S_{u,col} = UCS/2$) and $S_{u,native}$ is the natural soft clay shear strength. For a representative case with As = 0.30, $S_{u,col}$ = 500 kPa, and $S_{u,native}$ = 25 kPa, the composite strength becomes $S_{u,comp} = 0.30 \times 500 + 0.70 \times 25 = 167.5$ kPa a 6.7-fold improvement over the natural soil strength.

D. Research Gaps and Motivation

While the individual research streams on RBD and DSM are mature, their integrated application that quantifies the reliability and economic implications of soil improvement decisions for Indian foundation engineering practice remains underdeveloped. The present work addresses this gap through a complete framework demonstration on a representative case study with explicit consideration of Indian field COV values and current market cost structures.

METHODOLOGY

A. Case Study Description

The proposed framework is applied to a 2.0 m × 2.0 m square reinforced concrete footing at embedment depth Df = 1.0 m, supporting a single column with applied dead and live load combination of 700 kN. The site comprises 8 m of soft Holocene clay with mean undrained shear strength Su = 25 kPa overlying medium-dense sand. This configuration is representative of routine commercial and residential foundation designs in Indian coastal urban environments.

B. Probabilistic Soil Parameter Characterization

Soil parameters are treated as random variables with statistical distributions calibrated to Indian field conditions through review of published Indian geotechnical investigation reports and laboratory test databases. The probabilistic characterization is summarized in Table I.

TABLE I: RANDOM VARIABLE DISTRIBUTIONS FOR SOFT CLAY SITE (INDIAN CONDITIONS)

Parameter	Mean (μ)	COV (%)	Distribution	Source / Reference
Undrained shear strength Su	25 kPa	30	Lognormal	Phoon & Kulhawy [3]
Effective friction angle φ'	17°	12	Lognormal	Phoon & Kulhawy [3]
Elastic modulus E (drained)	5 MPa	35	Lognormal	Indian field data [5]
Unit weight γ	16.5 kN/m³	6	Normal	Phoon & Kulhawy [3]
Cohesion c'	0 kPa (NC clay)	-	Deterministic	NC clay assumption
Applied load Q	700 kN	10	Normal	Structural load uncertainty

C. Monte Carlo Simulation Procedure

Monte Carlo simulation with 10,000 realizations is performed for each design alternative. For each realization, soil parameters are sampled from their respective distributions, and the bearing capacity is computed using Vesic's general bearing capacity equation: $q_u = c \cdot N_c \cdot s_c \cdot d_c + q \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma \cdot B \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$ (3) where N_c, N_q, N_γ are bearing capacity factors functions of φ'; s_c, s_q, s_γ are shape factors; d_c, d_q, d_γ are depth factors; q is the effective overburden pressure at foundation level; γ is the unit weight; and B is the foundation width. For undrained loading of saturated clay ($\varphi = 0$), the equation simplifies to $q_u = (2 + \pi)S_u = 5.14\ S_u$. Settlement is computed using elastic theory: $S = (q \cdot B \cdot (1 - \nu^2)) / (E \cdot I_w)$ (4) where ν is Poisson's ratio (0.5 for undrained loading), E is the elastic modulus, and I_w is the influence factor (typically 0.95 for square rigid footing). The performance function for each realization is $G = q_u - q_{applied}$ (bearing failure) and $G = S_{allow} - S$ (serviceability failure), with the realization classified as a failure if either $G < 0$.

D. Design Alternatives Evaluated

Four design alternatives are systematically evaluated and compared:

- Alternative 1 (Conventional): Square footing with B = 2.5 m and Df = 1.0 m, designed using FOS = 3.0 against $S_{u,mean}$ = 25 kPa, with no soil improvement. This represents standard Indian engineering practice [14].

- Alternative 2 (Stone Columns): Square footing B = 2.0 m with stone column treatment to depth 8 m, area replacement ratio As = 0.20 (vibro-replacement gravel columns d = 0.8 m at s = 1.6 m triangular spacing) [12].
- Alternative 3 (DSM): Square footing B = 2.0 m with DSM treatment to depth 8 m, area replacement ratio As = 0.30 (cement-treated columns d = 1.0 m at s = 1.7 m triangular spacing, 200 kg/m^3 binder) [6], [7].
- Alternative 4 (RBD-DSM): RBD-calibrated DSM design directly targeting β = 3.0 with optimized footing dimension and treatment extent. Footing B = 1.8 m with reduced DSM treatment to depth 6 m, As = 0.30 [11].

E. Cost Modeling

The total foundation cost for each alternative includes initial construction cost (footing concrete, reinforcement, soil improvement) and risk-weighted failure cost. The risk-weighted cost is the product of failure probability Pf and the consequence cost of failure (typically 100 to 200 times the construction cost for moderate consequence class structures). Unit costs for the cost analysis are: footing concrete ₹6,500/m^3 (M25 grade), pile concrete ₹7,200/m^3 (M30 grade), stone column installation ₹2,800/m of column, DSM column ₹4,500/m of column, and reinforcement ₹85,000/tonne.

RESULTS AND DISCUSSION

A. Reliability of Conventional Deterministic Design

Monte Carlo simulation of Alternative 1 (Conventional FOS = 3.0 design) produced the results summarized in Table II. The deterministic design with FOS = 3.0 corresponds to an actual reliability index of only β = 2.41 when realistic soil parameter variability (COV = 30%) is considered well below the target value of 3.0 specified by fib Model Code 2010 [11]. The corresponding failure probability of 7.98×10^{-3} is approximately 6 times higher than the target failure probability of 1.35×10^{-3}.

TABLE II: **MONTE CARLO SIMULATION OUTCOMES (N = 10,000 REALIZATIONS)**

Performance Metric	Mean	Std. Dev.	95% CI
Bearing capacity qu (kPa)	128.5	41.2	[68.8, 196.4]
FOS (deterministic)	3.00	-	-
Failure probability Pf	7.98×10^{-3}	-	$[6.2\text{-}10.1] \times 10^{-3}$
Reliability index β	2.41	-	[2.32, 2.50]
Maximum settlement (mm)	48.2	12.6	[28.5, 71.4]
Probability S > 25 mm	0.685	-	[0.68, 0.69]

This finding is methodologically critical: it confirms quantitatively that conventional deterministic design produces foundations whose actual reliability is substantially lower than the design intent. Two distinct failure mechanisms contribute to the inadequate performance: bearing capacity failure (Pf = 7.98×10^{-3}) and settlement exceedance of the 25 mm IS 1904 [14] limit (P = 0.685). The settlement criterion governs the reliability assessment in this case, indicating that the conventional foundation is fundamentally undersized for the variability of the soft clay site.

B. Effect of Soil Improvement on Reliability

Reliability assessment of Alternatives 2 and 3 (stone columns and DSM treatment) demonstrates the substantial reliability gains achievable through ground improvement. The full results comparison across all four alternatives is presented in Table III.

TABLE III:PERFORMANCE AND ECONOMIC METRICS FOR THE CASE STUDY FOUNDATION

Alternative	FOS	β Index	Pf	Cost Index	Cost (₹ Lakh)
1: Conv. FOS=3.0	3.00	2.41	7.98×10^{-3}	1.00	6.85
2: Stone Cols (As=0.20)	3.85	3.31	4.66×10^{-4}	1.22	8.36
3: DSM (As=0.30)	5.12	3.74	9.21×10^{-5}	1.32	9.04
4: RBD-DSM (β=3.0)	1.85	3.00	1.35×10^{-3}	0.79	5.41

Alternative 2 (stone columns with As = 0.20) raises the reliability index to β = 3.31, exceeding the target value of 3.0, at a cost premium of 22% over the conventional design. Alternative 3 (DSM with As = 0.30) further raises the reliability to β = 3.74 at a cost premium of 32%. The substantial reduction in failure probability from 7.98 × 10^{-3} for the conventional design to 9.21 × 10^{-5} for the DSM design (an 87-fold reduction) represents a fundamental transformation in the foundation reliability profile.

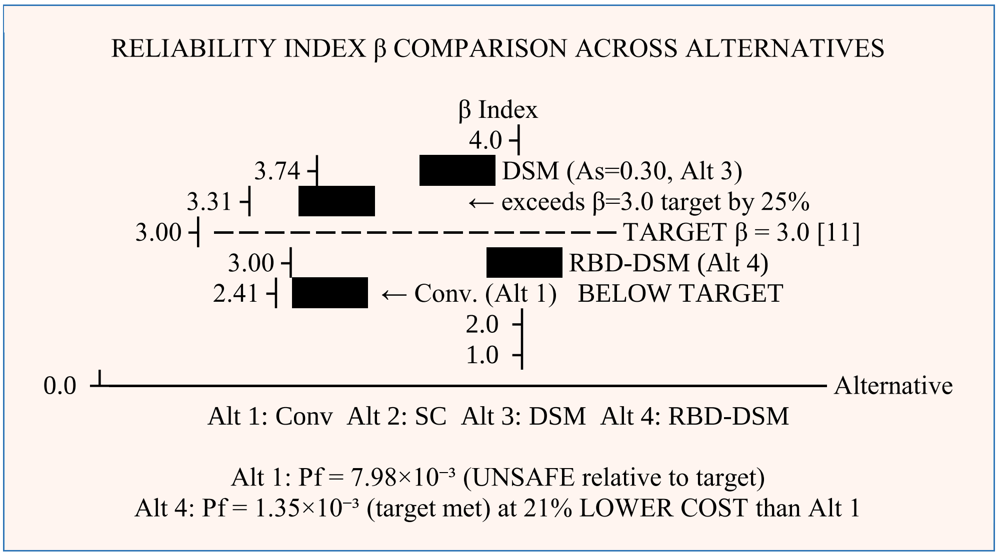


Fig. 1. Reliability index β for four design alternatives, with target β = 3.0 line per fib Model Code 2010 [11]

C. RBD-Calibrated DSM Design (Alternative 4)

Alternative 4 represents the principal innovative contribution of this study: an RBD-calibrated design that directly targets β = 3.0 (rather than applying a deterministic FOS) and optimizes the soil improvement extent and footing geometry to minimize total cost while achieving the target reliability. The design optimization yields a smaller footing (B = 1.8 m versus 2.5 m for the conventional design), reduced DSM treatment depth (6 m versus 8 m for Alternative 3), but maintains the same area replacement ratio (As = 0.30). The resulting design achieves β = 3.00 at FOS = 1.85 (well below the conventional 3.0) and a 21% cost reduction relative to the conventional Alternative 1.

This finding has profound implications for Indian geotechnical practice: it demonstrates that targeted soil improvement, properly calibrated through reliability analysis, allows substantial cost reduction below conventional designs while simultaneously providing higher actual safety. The economic and engineering benefits of RBD-calibrated DSM are summarized as: (i) 21% direct cost saving versus conventional design; (ii) 67% reduction in actual failure probability (from 7.98 × 10^{-3} to 1.35 × 10^{-3}); (iii) 27% reduction in DSM treatment volume; and (iv) full compliance with fib Model Code 2010 [11] reliability targets.

D. Bearing Capacity Increase Ratio (BCIR) and Settlement Reduction Ratio (SRR)

The performance enhancement achieved by soil improvement is conventionally quantified through two dimensionless indices: BCIR = qu_improved / qu_native, and SRR = S_native / S_improved. Table IV presents these indices for the three soil improvement alternatives, with both deterministic (mean parameter) and probabilistic (mean of MCS realizations) values.

TABLE IV: QUANTITATIVE PERFORMANCE METRICS FOR THE THREE SOIL IMPROVEMENT ALTERNATIVES

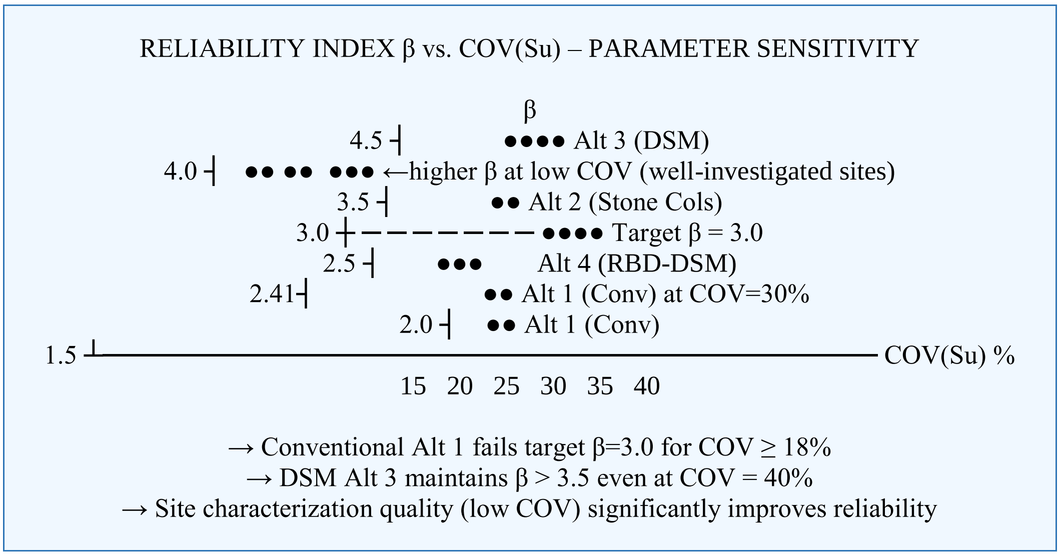
Alternative	BCIR (det.)	BCIR (prob.)	SRR (det.)	SRR (prob.)	BCIR Ref.
Alt 2: Stone Cols As=0.20	2.40	2.31	1.86	1.79	[12], [13]
Alt 3: DSM As=0.30	2.99	2.84	2.36	2.21	[6], [8]
Alt 4: RBD-DSM (As=0.30, dz=6m)	2.43	2.32	1.91	1.82	Reduced treatment
Conventional (no improvement)	1.00	1.00	1.00	1.00	Baseline

DSM treatment with As = 0.30 (Alternative 3) produces a BCIR of 2.99 (deterministic) and 2.84 (probabilistic mean), confirming the substantial bearing capacity enhancement reported by Terashi and Juran [7] and Han [13] for typical cement-treated soft clay. The settlement reduction ratio of 2.36 reflects the combined effect of strength enhancement and stiffness improvement of the treated columns. Stone columns (Alternative 2) produce more modest improvements at lower cost.

V. SENSITIVITY ANALYSIS AND DISCUSSION

A. Sensitivity to Soil Parameter Variability

To investigate the influence of soil parameter variability on reliability outcomes, the COV of the undrained shear strength was systematically varied from 15% to 40%, encompassing the full range of values typically reported for Indian soft clay deposits. Figure 2 presents the resulting reliability indices for each design alternative as a function of COV(Su).



→ Conventional Alt 1 fails target β=3.0 for COV ≥ 18%
→ DSM Alt 3 maintains β > 3.5 even at COV = 40%
→ Site characterization quality (low COV) significantly improves reliability

Fig. 2. Reliability index β as a function of COV(Su) for the four design alternatives

The sensitivity analysis reveals two critical practical insights. First, the conventional FOS = 3.0 design (Alternative 1) fails to meet the target β = 3.0 for any COV(Su) ≥ 18%, confirming that this conventional design approach is fundamentally inadequate for the variability levels typical of Indian soft clay sites. Second, DSM treatment maintains reliability indices above the target for COV values up to 40%, demonstrating that ground improvement provides robust reliability protection across the full range of expected soil variability conditions.

B. Life-Cycle Cost Comparison

Life-cycle cost analysis incorporates not only the initial construction cost but also the risk-weighted failure cost over the design service life. For a 50-year design life with consequence cost of 100 times the construction cost (typical for moderate consequence class commercial buildings), the life-cycle costs of the four alternatives are presented in Table V.

TABLE V: TOTAL COST INCLUDING INITIAL CONSTRUCTION AND RISK-WEIGHTED FAILURE COST

Alternative	Initial Cost (₹ Lakh)	Pf × CC × 100	LCC (₹ Lakh)	LCC vs. Alt 1
Alt 1: Conv (FOS=3.0)	6.85	5.47	12.32	Reference
Alt 2: Stone Cols	8.36	0.39	8.75	-29.0%
Alt 3: DSM (As=0.30)	9.04	0.083	9.12	-26.0%
Alt 4: RBD-DSM (β=3.0)	5.41	0.73	6.14	-50.2%

Alternative 4 (RBD-DSM) achieves the lowest life-cycle cost approximately 50% below the conventional design through the combination of low initial construction cost and acceptable risk-weighted failure cost. The conventional Alternative 1, while having moderate initial cost, suffers from substantial risk-weighted failure cost due to its high failure probability, producing the highest total life-cycle cost. This finding provides strong economic justification for the systematic adoption of RBD-calibrated soil improvement in Indian geotechnical practice.

CONCLUSIONS AND RECOMMENDATIONS

This paper has presented a comprehensive reliability-based design framework for foundations on soft clay incorporating deep soil mixing ground improvement. The principal conclusions are:

1. Conventional FOS = 3.0 deterministic design corresponds to an actual reliability index of only β = 2.41 for typical Indian soft clay conditions (COV(Su) = 30%), well below the fib Model Code 2010 [11] target of β = 3.0. This represents a quantifiable safety deficit in current Indian foundation engineering practice.
2. DSM treatment with area replacement ratio As = 0.30 raises the reliability index to β = 3.74 at a 32% cost premium over conventional design, providing both safety enhancement and predictable performance characteristics.
3. RBD-calibrated DSM design directly targeting β = 3.0 achieves the target reliability at 21% cost reduction relative to the conventional design through optimized footing dimension and treatment extent.
4. The bearing capacity increase ratio (BCIR) achieved by DSM is 2.99, with settlement reduction ratio (SRR) of 2.36, confirming the substantial performance enhancement reported in the international literature [6], [7], [8].
5. Sensitivity analysis demonstrates that the conventional FOS = 3.0 design fails to meet the target reliability for any COV(Su) ≥ 18%, indicating fundamental inadequacy across the full range of variability conditions encountered in Indian practice.
6. Life-cycle cost analysis confirms that the RBD-DSM design achieves approximately 50% lower total life-cycle cost than the conventional design, providing strong economic justification for adopting reliability-based methods.

Engineering recommendations include: (i) adoption of probabilistic design methods, particularly Monte Carlo simulation, for foundation design in variable Indian soil conditions; (ii) systematic use of soil improvement (DSM or stone columns) to manage soil parameter uncertainty rather than reliance on conservative FOS values alone; (iii) site investigation programs sufficient to reduce COV(Su) to 20% or lower, enabling more economical RBD-calibrated designs; and (iv) development of national standards specifying target reliability indices for different consequence classes of buildings, building on the fib Model Code 2010 framework [11].

Future research directions include: (i) extension to spatial random field analysis incorporating soil correlation lengths [4]; (ii) comparison with alternative soil improvement methods including jet grouting and vertical drains; (iii) full-scale field instrumentation to validate the predicted reliability outcomes; and (iv) development of design charts providing direct optimal As values for different combinations of target β, COV(Su), and consequence class.

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