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A Study on Theoretical Investigation on Mathematical Modelling on Population Growth

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ABSTRACT

This research include, A concise overview of human population dynamics, determinants of population growth, stable and stationary populations, and mathematical modeling has been provided. This research also includes a population growth model, some basic concepts, and mathematical methods. This chapter also includes a thorough analysis of the literature on age-structured and non-age-structured population modeling in human population dynamics.

INTRODUCTION

Mathematical Modeling

A model is an intentional depiction of reality. Modeling is the process of examining processes and physical things and then using those objects to simulate the behavior of the systems being studied in different physical environments. In general, it is rare for a real-world situation to be converted into a mathematical problem, even when it is possible. The ensuing mathematical problem can be unsolvable. As a result, the problem must be idealized, simplified, or approximated by another problem that is mathematically solvable and very similar to the original problem. Mathematical modeling is an experimental method that solves a problem and continuously improves it over time to become more accurate, efficient, or quick. This is a formal component of the mathematics curriculum and a "scientific investigation process." Actually, mathematical modeling is not a brand-new subject. It has existed since antiquity. Mathematicians, physicists, engineers, statisticians, and astronomers use mathematical models to examine a variety of topics.

The goal of the multidisciplinary academic area of bio-mathematical modeling is to use applied mathematics approaches to model biological and natural processes. The study of population dynamics is becoming more and more popular in the early twentieth century. The study of population dynamics, which combines the disciplines of mathematics, demography, social sciences, ecology, population genetics, and epidemiology, aims to provide a straightforward, mechanistic explanation of how the size and makeup of biological populations—such as those of humans, animals, plants, or microorganisms—change over time. Although its scope has expanded recently, it is the study of long-term and marginal changes in the number of individuals in one or the other, individual weight, and age structure, with a history spanning more than 220 years. In the study of population dynamics and issues in the ecological and environmental sciences, mathematical and computational methods offer strong instruments and methods.

The topic has a long history and is linked to dynamical system theory and development statistics. These mathematical and computational methods are today thought to be the most effective means of teaching natural phenomena. An interest in the study of survival and interactions between live organisms and their surroundings has been sparked by these methods, which have been widely used and have offered a framework for the synthesis and analysis of such biological models. From a mathematical standpoint, there are basically two primary methods for modeling population dynamics:

i. Using ordinary and partial differential equations in a continuous time method
ii. A discrete time method that is more directly tied to a population's demographic makeup.

Both strategies make extensive use of qualitative theory of dynamical systems techniques. According to the continuous time perspective, the population's size keeps fluctuating over time, and the most widely used modeling framework pertains to the kinds of biotic interspecific and species-environment interactions. Models are developed to describe population census data using the discrete time approach. They are closed in terms of time and the manner in which data on population growth is received.

Human Population Dynamics

The study of human population dynamics monitors variables like life expectancy, migration, fertility, mortality, birth rate, and death rate that are associated with population shifts. Because demographic trends have an impact on the social, economic, and environmental systems, it is crucial to forecast population changes. The quality of natural resources such as biodiversity, air, land, and water can be affected by an increase in the human population. Political, economic, and cultural interests all benefit from the research of population growth in a particular area, which will be revived in this community to support its future. Trends in the human population are crucial to environmental research because they aid in assessing how human activity affects the environment. Natural resources like land, water, and energy supplies are under more stress as the population grows. In addition to producing greater trash, humans are also using more resources and producing pollutants such as greenhouse gas emissions and contamination of the air and water. According to Md. Burak and Mustafa, the dynamic population model, which is based on knowledge creation, is used to determine the Earth's potential in terms of population projections and to assess the efficacy of productivity initiatives. The Earth's carrying capacity is defined as a time-level working knowledge level rather than a constant in the dynamic population model. According to the findings, the world's population is expected to reach a maximum of 12 billion people within the next century. After that, the population will begin to decline at varying rates, which is thought to be a result of environmental degradation. India's ecology is suffering as a result of the country's fast population growth and expansion of development activities. Due to unchecked urbanization, industrialization, and increased human activity, the natural environment is deteriorating.

Authors provide a variety of models and techniques to simulate the dynamics of the human population. In general, there are two types of population models: age-structured and non-age-structured. In his 1798 work "An essay on the principle of population," Thomas Malthus proposed the first model in the non-age-structured category, known as the Malthus model or exponential growth model. He asserts that food production occurs at an arithmetic pace and population increase occurs at an exponential rate. He believed that there are no restrictions on the resources needed to survive and that the only factors influencing the rate of population change are birth and death rates. Dean Hathout discusses the various properties and traits of hyperbolic and exponential modeling. Pierre Verhulst created the logistic growth model in 1845 by modifying the Malthus model to include the impact of natural resources on population change. He included the concept of carrying capacity in his model. As a result, population change depends not only on population size but also on how far this size deviates from its upper bound. This model was used by Pearl and Reed in 1920 to forecast the population of the United States from 1790 to 1910. Numerous scholars then examined and adjusted the Verhulst model in light of the variables influencing population growth. Kapur & Khan claim that the logistic model, which includes the S-curve with point of inflection and a limited population with zero growth rate, is the best model for understanding the populations of bacteria and humans [7]. Based on complicated stochastic median technology, Smanta and Chakarabarti investigated the impact of colorful noise on Gompertz and Logistic development models, as well as the stability and fluctuation in these models [2].

In his publications [3,1], Tsoularis discusses and analyzes a number of traits and characteristics that are generalized from the conventional logistic model and S-shaped curve. Harris researched the effects of the immigration and resistance environments on the diffusive logistic model. His area of study is how immigration affects the duration of crucial habitat that is required for population survival [6]. Grahm talked about how common logistics models have developed from three angles: how sensitive they are to initial conditions; how they relate to the difference equation model; and how to build a stochastic model with mean logistic growth. The findings show that while the logistic model is attractive due to its simplicity, its realism is questionable [5]. A method for creating a population growth model has been provided in [2]. Three concepts are included: the supply of resources for a population, the demand of the population for a resource field in the form of a border, and changes in resource availability with population growth as a variable. When population size and resource availability are estimated using quadratic and linear functions, respectively, the equation is in the structured form of a logistic equation.

Eguasa and Odion projected Nigeria's population using logistic and exponential models, and they utilized the least squares approach to check if the model's parameters were linear. In [6], the stability of the logistic model was investigated using the Lyapunov exponent and the Ishikawa iterative technique. It was found that the stability of the model also increased noticeably with greater values of the control parameters for specific alternatives. Using Poisson growth coefficients, Shaojuan examines the asymptotic behavior of stability of the logistic equation and modified stochastic logistic model [3]. The impact of carrying capacity variations on the logistic equation solution and the analysis of the asymptotic behavior of the model's solution is covered in [9]. A sub-solution and super solution for the corresponding solution are obtained. When age structure is taken into consideration, the logistic model and Malthus fail. Because fertility, mortality, and population age structure all have an impact on the population and have the potential to alter population growth patterns, age-structured models are crucial. Most age-structured models fall into one of two categories based on time: discrete or continuous. First, Sharpe & Lotka (11) addressed the age-distribution issue in population theory in [4]. He asserts that the population's age distribution varies somewhat. Its potential fluctuations are restricted, though. Some age distributions will not be feasible, and even if we were to arbitrarily alter a very uncommon form of another population's age distribution, the "irregularities" would undoubtedly persist over time. This suggests that there should be a limited "stable" type that has a different actual distribution and that returns to the disturbed through any agency. Lotka presented the Lotka integral equation in 1939 as an age-dependent, continuous time model for the female population. In [6], a convergent numerical approach was introduced to solve the Sharpe-Lotka age-dependent model using integral differential equation approximation techniques. Jenson introduced a density-dependent matrix model based on discrete time, and P.H. Leslie (1945) introduced the Leslie Matrix model, a discrete-time discrete-age-scale model that is widely used in demography and population ecology to determine the growth and age distribution in population with respect to time [9]. It is a modified Leslie matrix model that was connected to logistic, exponential, and life tables. Applying this model to population data for projection is easier and more straightforward [9]. The numerical solution of a non-linear-age-structured was covered in [9] using the variational iteration method (unwind scheme), and two numerical examples are provided to evaluate the precision and potency of the approaches.

Keyfiz described the Mckendrick Von Forester equation's solution and contrasted it with other discrete time models, such as the Lotka and Leslie models. He asserts that there are differences in their computational appropriateness, generalizability, and adaptation to various demographic goals and other biological applications. When the age and time intervals are short, all forms are the same; however, if the intervals are long, numerical values will differ [1]. Using the finite difference method in the case of a finite life span and a bounded or unbounded mortality function across all ages, [7] examines a linear Lotka-Mckendrick equation and demonstrates that all models are incompatible for various mortality functions. An age model is extended to a stochastic continuous time age-structure model by Chawdhay and Allen, who then compute solutions for two examples of age-dependent populations and compare them with the Monto Carlo methods [37]. In numerous additional authors changed the Lotka-Sharpe-Mckendrick model and used various variational techniques and numerical schemes to examine the behavior of the solution. Akamine and Suda claim that matrix models are irregular walk models in arbitrary scenarios. The mean of the maximum eigenvalues of matrices approaches the maximum eigenvalue of the mean matrix [13], and the average growth rate of the population (eigenvalue of the mean matrix) is significantly better than intrinsic rates determined by simulation and for all permutations.

The population of the age group is tracked by a new logistic population model that repeatedly multiplies a density-dependent matrix constructed from a native Leslie matrix, the model's selected carrying capacity, and the target stable-state age group. With the same beginning population and carrying capacity, the model's entire population is converted into a discrete logistic model, with a growth rate equal to the major eigenvalue of the Leslie matrix minus one [14]. Leslie and modified matrix models were applied in [2,10] to forecast population change. According to the appropriate hypothesis, the stability of the equilibrium point and oscillation behavior are also taken into consideration in [1], which develops a stage-structured model involving juvenile and mature phases employing discrete time lag. [2, 4, 3] explore the use of numerical techniques and the application of delay differential and integral equations, integro-differential equations, and their variational approaches. For population projection, Lingiang Fan provides an acceptable delay differential model with two time lags in [4]. The growth function, which is determined by the lowest and maximum age of females, is used to examine the population growth rate. In a polluted environment, the dynamical behavior and stability of time delay models with one or two discrete delays are examined in [5, 6,2]. In [2], the Euler method is used to explore the Hutchinson equation with distributed delay. The behavior of equilibria and stability of fixed points are also taken into consideration, and the results are analyzed by a number of numerical computations. Adamu Wakili examined the properties of continuous and dispersed time delays in his study, as well as the use of delay differential models in real-world scenarios [10].

Population Projection

Men were very interested in researching human population dynamics from a scientific perspective in the eighteenth century. The study of fertility, mortality, population growth, and future population composition, as well as the migratory patterns of the populace in various nations or regions, heavily relies on population projections. Furthermore, by analyzing the historical or current population growth trend, one may readily determine the size and makeup of the population on any given future data, which could aid planners in creating the nation's future policies.

Since the beginning of civilization, man has been interested in the future. He had occasionally experimented with various models and methods using the data that was available. Today, understanding the qualitative and quantitative characteristics of the population of the future is crucial [2, 9]. Projecting the population's size and growth can be done in a number of ways. The mathematical approach to population projection, which can be represented by logistic curves, exponential functions, logarithmic curves, or straight lines, is predicated on the idea that there is a fundamental relationship between population size and time. It is possible to see the present population patterns on a plot of the predicted and future populations. In addition, the ratio of the subnational population to the overall population and changes in its members can be expressed using this method. In addition, whereas many population growth models treat the population as a dependent variable, mathematical techniques treat it as an independent variable [4,1]. In [2,5], a number of population and security projection strategies are presented. Basher and Hoque Ahmad use logistic and exponential growth models to determine population growth factors and trends, such as birth and death rates and anticipated population growth [3,6]. Chiu (1990) developed new algorithms for studying nonlinear age-structured models and projected the U.N. population growth using these algorithms and numerical techniques. Augustus Wali projected the populations of Rwanda and Uganda by analyzing population growth and applying the Malthus and Logistic growth models. He identified the vital coefficient, carrying capacity, and the factor influencing population growth. Shilpa and Srinidhi (2014) look into India's demographic trends.

The population is projected until 2025 using a logistic model technique, and the time it will take to attain the carrying capacity is also examined [1, 6]. The birth rate, natural population growth, net reproduction rate, and net maternity function of the initial stable population can all be predicted if the population age distribution is available, according to a model for population estimation under sequential changes in reproduction schedule on the condition of stability using birth-trajectory introduced in [8]. Hal Caswell [1] asserts that matrix models are effective instruments for analyzing sensitive populations and investigating different population aspects including migration, fertility, mortality, births, and deaths in the projection. A Newton divided difference approach is used to estimate the population in [141] with a 10% percentage error because population censuses are done at irregular intervals. Additionally, population forecasting is done using logistic and exponential models. The following chapter of this thesis uses a mathematical model to model population growth or projections utilizing secondary data from the Indian population census. In addition to being important to demographers, population projection is now the foundation of both population mathematics and social studies. Censuses are typically conducted every ten years, and family and economic policies are influenced by population changes on a daily basis. Therefore, the population of a time period between two censuses is necessary for the creation and effective implementation of these policies, making projections unavoidable.

Population Growth and Its Determinants

An estimated 83 million people, or 1.1% of the world's population, are added each year. Between 1800 and 2018, the world's population grew from 1 billion to 7.616 billion. With an anticipated 8.6 billion people by 2030, 9.8 billion by 2050, and 11.2 billion by 2100, the population is predicted to continue to grow. Living standards are generally low in many nations with fast population expansion, while they are high in many nations with slower population increase. With a population of over 1.271 billion people, or 17.5% of the global population, India is the second most populous country in the world. Up until 1991, India's population grew at a very high pace both annually and decadally. After that, the country's high absolute growth caused a little reduction in both yearly growth rates and decadal population growth. There are still many people on the planet, which strains its resources and leads to major environmental issues. The population was just about 234.8 million at the start of the 20th century. Over the course of a century, India's population grew by more than four times, reaching 1210 million in 2011. It's interesting to note that the Indian population climbed threefold in the last fifty years of the twentieth century, compared to 1.5 times in the first fifty.

The decadal growth from 2001 to 2011 showed a substantial fall since independence, which is important. India's decadal growth was 23.87% in 1991, 2.33% over the next decades, and 21.54% in 2001. India's decadal growth in 2011 was 17.64%, with a further decline of 3.90%. The decreasing trend of population growth indicates that India's attempts to slow the rate of population expansion have been successful and outstanding.

Three significant factors that influence population increase are migration, the birth rate, and the death rate. The number of people who die per unit of time is known as the death rate, while the maximum number of new individuals produced per unit of time under ideal conditions is known as the birth rate. Even though fertility is a physiological process, the socioeconomic status of a nation can be inferred from its birth rate data during the population growth process. Similar to the death rate, migration (including immigration and emigration) has little bearing on population growth when compared to birth and death rates. Population growth is typically the result of migrants arriving in a country with the intention of settling there. In a similar vein, emigrants—people who relocate permanently to other nations—are likely to cause a decline in the population [112]. The following list includes some more significant factors that influence population increase in addition to the birth rate, death rate, emigrants, and immigration.

Fecundity and Fertility

Fertility is the ability to have live births or the quantity of births that occur, while fecundity is typically used to refer to the physical ability to bear children. Put another way, any married woman who is capable of becoming fertilized—including departing maidens, widows, divorced women, and women using contraceptives—is considered infecund; if she is able to conceive, she is considered fecund. Alongside this fertility comes the ability to conceive or give birth to a live kid. The study of fertility measurement is crucial to human population dynamics because it examines a number of interrelated factors that either directly or indirectly influence the birth rate and population growth, including migration, housing, public health, low per capita income, and a low standard of living [7, 1]. Fertility is influenced by a wide range of factors, including biological, physiological, social, and economic ones. Age and sex are two biological elements that affect fecundity or fertility. In addition to socioeconomic factors including caste, religion, race, education, and economic status, physiological factors can affect a woman's reproductive lifespan in terms of sterility. The following are some techniques for calculating fertility and birth rate:

Crude Birth –Rate

This is referred to as "per thousand per year" and is the ratio of "children-born" to "mid-year population" in a given year. In terms of mathematics

$$\text{Crude Birth-Rate} = \frac{B}{P} \times k$$

where k is a constant, usually 1000, and B and P stand for "number of children born" and "mid-year population," respectively.

General fertility rate / Ratio

This is the proportion of live births to the total number of mothers. Only women who are of childbearing age—that is, between the ages of 13 and 45—are included here. This is determined by

$$\text{G.F.R} = \frac{B}{P_{13-45}} \times k$$

where P13–45 is the mid-year female population in the age group of 13–45 years in that year, B is the total number of live births in the year, and k is the constant, or 1000.

Age-Specific Birth-Rate

Age-related The birth rate is typically computed in relation to women's ages. It can be computed mathematically by

$$\text{Age-specific birth-rate} = \frac{b_i}{P_i} \times k,$$

where Pi is the themed-year population of women in ith age interval, k (=1000) is constant, and b is the number of births in ith age interval.

Total Fertility Rate

It is the sum of the birth rates by age. It can be calculated by adding up the birth rates for each age group over the course of the childbearing years.

$$\text{Total Fertility rate} = \sum_{13}^{45} \left(\frac{b_i}{P_i} \right) \times k,$$

where every symbol has the same meaning as the definition given above.

Mortality

Mortality is the state of dying. "The permanent disappearance of all evidence of life at any time after birth has taken place" is the definition of death. Therefore, stillbirths and abortions are not considered deaths. Measurements of mortality are just as crucial to the study of human population dynamics as fertility. It is crucial for establishing population policies and determining insurance prices. It has a significant direct impact on both the population growth and substitution rates. The population will decline if mortality rises, while the population will rise if mortality falls. Numerous factors influence mortality or fatalities. From a demographic perspective, they could be biological, physiological, environmental, etc.; mortality is correlated with an individual's age and sex.

The mortality rate was extremely high in the past due to famines and food shortages, the spread of diseases, unhygienic conditions, and protracted and frequent battles. Following World War II, both the birth rate and mortality rates in affluent nations fell so precipitously that many nations now face the challenge of very slow population growth. Disease control medications, public health initiatives, medical facilities, the expansion of education, women's status, food supply, and life expectancy are further variables contributing to the drop in mortality rates in emerging nations such as India [160]. Here are a few techniques for calculating mortality:

Crude death rate

It is the proportion of all recorded fatalities to the nation's total population in a given year. This is sometimes referred to as death per thousand annually.

$$\text{Mathematically, Crude Death Rate} = \frac{D}{P} \times k,$$

where P is the population at the halfway point of the year, D is the total number of recorded deaths in a year, and k is the constant (chosen as 1000).

Age-Specific Death-Rate

This is the population to death ratio for a specific age group, sex, caste, religion, or location. In terms of mathematics

$$\text{Age-specific death-rate} = \frac{d_i}{P_i} \times k,$$

where di is the death toll. K is constant, while Pi is the mid-year population in the ith age group.

Life Expectancy

It measures how long an organism (human) is projected to live on average, taking into account their current age, year of birth, and other demographic characteristics like gender.

Life Table

The foundation for calculating a society's life expectancy is a life table, a mathematical sample that provides an overview of deaths in a nation. It provides information about a person's likelihood of dying at a specific age or surviving to a specific age.

Stable and Stationary Population

fictitious population model. The stable population is the long-term structure that would eventually exist in a hypothetical population if the presumptive age-specific birth and death rates continued to exist unchanged. It does not rely on the makeup of any specific population and is entirely derived from these birth and death rates [5, 6]. It is also closed against migration. Age-specific birth and death rates determine the stable population, which helps determine the growth rate for various age and sex compositions. The life table, which represents the history of a fictitious population, is restricted to movement in or out and subject to change with specific mortality and fertility rates.

CONCLUSION

The corresponding age group is the focus of these alterations [1, 83]. In a similar vein, the population size becomes stable if age-specific fertility and mortality rates remain constant and migration has no impact on them. The average presence in each age group will therefore initially be roughly equal to the cohort. Fertility and mortality are successfully combined in a single self-sufficient plan by the stable population. Since it is based on both the population of reproductive age and the birth rate, the new cohort, in contrast to the stagnant population, has no set size. Therefore, even though the assumptions are fixed, the population may change. The age distribution of a stable population is determined by the stationary birth and death rate table. As a result, it aids in understanding the various stages of development and age structure under specific circumstances. Both stationary and stable population models are built on the basis of age structure and are predicated on the fundamental premise of fixed and definite fertility and mortality rates [4, 5].

Pietro Cerone conducted research on population stability and created a model utilizing the Lotka-Sharp model to analyze the steady and fluctuating migratory flow in the population over time. He also created a model to examine the fertility patterns of both the local and immigrant populations. He solved it using the Laplace transform approach [2]. Juha Altho presented an open stable population model in [8] that shows the number of births as a function of net migration in the population. It also discusses how net migration, which is regarded as a migration survivor function, affects the population's stability, growth rate, and age distribution [5,15]. Lukevenkov and Belokurov used the Lyapunov stability approach to propose a model for studying human population stability. He examines the limit of population growth and the amount of time needed to reach it, and the model's solution is a stable stationary solution [4]. "Immigration for population stationarity leads to a gross inefficiency in terms of size and age distribution of the stationary population and immigration-based population stationarity is inefficient as the stationary population results in having a larger size than that obtained bringing the fertility rates to the whole natural level of equilibrium," according to research by Anna Maria in [15]. In [8,3], population momentum was investigated. In these papers, population aging is measured as a function of population momentum, and it is explained that the characteristics of the current population determine the lower limit of the ultimate population. Numerous facets of age distribution, mortality, and fertility in stationary and stable populations are examined and discussed.

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