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Nonlinear Landau-Zener tunneling for the interaction of the mixture of two different particles

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Abstract: The nonlinear Landau–Zener tunneling of a Bose–Einstein condensate (BEC) of two different of particles in an accelerating optical lattice is investigated. The analytical eigenstates and the tunneling probability are obtained. It is shown that the eigenstates and the tunneling probability are modified dramatically by the interaction of the two different particles.

Introduction

Since the first experimental preparation of Bose-Einstein condensates (BECs) in dilute atomic gases, the study of their dynamical properties has become a very active field of research. Investigation covered different aspects of boson Josephson junctions [1, 6], population of topological states [7, 8], tunneling processes [9, 15], transport of condensate [6, 16-22], and other topics. However one of the most striking phenomena that were proposed recently, is the adiabatic tunneling of BEC, where avoided crossing of energy levels leads to the splitting of degenerate energy levels forming a tiny energy gap. Around the avoided crossing point of the two levels, the Landau-Zener tunneling (LZT) scenario describes the tunneling under the assumption that the energy difference between the two levels varies linearly with time. However the adiabatic tunneling may be broken down for two reasons. The first reason, when the sweep rate between the two bias levels increases, and the second reason when the nonlinear interaction exceeds the avoided crossing of energy and, the opportunity of the emergence of nonlinear eigenstates increases. An interesting extension of the tunneling problem involves Bose condensates of two interacting species. The main issue is how the interspecies interaction affects the tunneling process, and particularly the quantum coherence as the two condensates mix together. Previous studies of the general properties of two-component Bose condensates have emphasized the important role of the interspecies interaction, which leads to novel features, such as the components separation [23, 24], cancellation of the mean field energy shift [25], and the suppression of quantum phase diffusion [26]. However, the investigation of the impact of interspecie's interaction on tunneling dynamics has only just begun [27, 28].

In this paper, the main issue is to study the nonlinear (LZ) tunneling properties for the accelerating 1D optical lattice with the interaction of two species particles. The interaction of two species particles can be considered as two independent systems but with new fluctuation of energy for each one. The interaction of identical particles decreases the energy band gap and, as a result the chance of the nonlinear LZ tunneling increases, while the mutual interaction between two species of particles produces a new route of tunneling of particles in additional to the transition of particles in their own eigenstates. So, one of the main goals of this work is to investigate the influence of fluctuation of energy produces by the mutual interaction of particles on the eigenstates of each group of identical particles.

Theory of the nonlinear and Landau-Zener Model

The motion of a Bose–Einstein condensate in an accelerated 1D optical lattice is described by the Gross–Pitaevskii equation [29, 30]

$$i\frac{\partial \Psi_1}{\partial t} = -\frac{1}{2m}\left[\hbar\frac{\partial}{\partial x} - ima_l t\right]^2 \Psi_1 + V_o \cos(2k_l x)\,\Psi_1 + C_{11}\left|\Psi_1\right|^2 \Psi_1 + C_{12}\left|\Psi_2\right|^2 \Psi_1 \qquad (1)$$

$$i\frac{\partial \Psi_2}{\partial t} = -\frac{1}{2m}\left[\hbar\frac{\partial}{\partial x} - ima_l t\right]^2 \Psi_2 + V_o \cos(2k_l x)\,\Psi_2 + C_{22}\left|\Psi_2\right|^2 \Psi_2 + C_{12}\left|\Psi_1\right|^2 \Psi_2 \qquad (2)$$

where m is the mass of the component, k_l is the wave number of the laser light, V_o is the strength of the periodic potential that is proportional to the laser density. The absolute square of the wave function $\Psi_j\,(j=1,2)$ is the number densities of two-component condensates respectively at position x and time t. A force of ma_l is represented in the vector potential gauge, which may stand for either the inertial force in the commoving frame of an accelerating lattice or the gravity force $C_{ij}=\dfrac{4\pi\hbar^2}{m}a_{ij}$, and a_{ij} are scattering length of the respective interactions, it will be assumed that they are all positive. For convenience, we cast Eq. (1), and Eq. (2) into the dimensionless

$$i\frac{\partial\Psi_1}{\partial t}=-\frac{1}{2}(\frac{\partial}{\partial t}-i\alpha\,t)^2\Psi_1+V\cos(x)\Psi_1+C_{11}\left|\Psi_1\right|^2\Psi_1+C_{12}\left|\Psi_2\right|^2\Psi_1 \qquad (3)$$

$$i\frac{\partial\Psi_2}{\partial t}=-\frac{1}{2}(\frac{\partial}{\partial t}-i\alpha\,t)^2\Psi_2+V\cos(x)\Psi_1+C_{22}\left|\Psi_2\right|^2\Psi_2+C_{12}\left|\Psi_1\right|^2\Psi_2 \qquad (4)$$

where the variables are scaled as

$$t=\frac{4\hbar^2k_l^2}{m}t,\; x=2xk_l,\; \alpha=\frac{m^2a_l}{8\hbar^2k_l^2},\; \mathrm{V}=\frac{m\mathrm{V}_0}{4\hbar^2k_l^2},\;\text{and } \Psi=\frac{\Psi}{\sqrt{2k_l}}$$

We assume that the non- linear term does not break the periodic symmetry, so that the band structure remains. In the neighbor-hood of k = 1/2, the Brillouin zone edge, the wavefunction can be approximated by [9].

$$\Psi_1=a_1(t)\,e^{ikx}+b_1\,(t)\,e^{i(k-1)x} \qquad (5)$$

$$\Psi_2=a_2(t)\,e^{ikx}+b_2\,(t)\,e^{i(k-1)x} \qquad (6)$$

Since we are dealing with two different species of particles, the total probabilities are $n_1=\left|a_1\right|^2+\left|b_1\right|^2=1$, and $\mathrm{n}_2=\left|a_2\right|^2+\left|b_2\right|^2=1$. The physically interesting information of the state is completely represented by the population difference

$$S_1=\left|b1\right|^2-\left|a_1\right|^2, and\;\; S_2=\left|b_2\right|^2-\left|a_2\right|^2 \qquad (7)$$

Substituting Eqs. (5) and (6) into the dimensionless form of Eqs. (3) and (4), and comparing the coefficient of $e^{ikx},\; and\;\; e^{i(k-1)x}$, One gets

$$i\frac{\partial}{\partial t}\begin{pmatrix}a_1\\ b_1\\ a_2\\ b_2\end{pmatrix}=\mathrm{H}\begin{pmatrix}a_1\\ b_1\\ a_2\\ b_2\end{pmatrix}, \qquad (8)$$

where H is the Hamiltonian matrix, and is given by

$$H=\begin{pmatrix}\frac{\lambda}{2}+\frac{C_{11}}{2}\left(\left|b_1\right|^2-\left|a_1\right|^2+3n_1\right)+C_{12}n_2 & \frac{V}{2}+C_{12}a_2b_2^{*} & 0 & 0\\ \frac{V}{2}+C_{12}a_2^{*}b_2 & -\frac{\lambda}{2}-\frac{C_{11}}{2}\left(\left|b_1\right|^2-\left|a_1\right|^2+3n_1\right)+C_{12}n_2 & 0 & 0\\ 0 & 0 & \frac{\lambda}{2}+\frac{C_{22}}{2}\left(\left|b_2\right|^2-\left|a_2\right|^2+3n_2\right)+C_{12}n_1 & \frac{V}{2}+C_{12}a_1b_1\\ 0 & 0 & \frac{V}{2}+C_{12}a_1^{*}b_1 & -\frac{\lambda}{2}-\frac{C_{22}}{2}\left(\left|b_2\right|^2-\left|a_2\right|^2+3n_2\right)+C_{12}n_1\end{pmatrix} \quad \ldots\ (9)$$

where $\lambda = \alpha\, t$ is the sweep rate. For a solution with nonzero amplitudes, One imposes the determinant condition

$$H = \begin{pmatrix} \frac{\lambda}{2}+\frac{C_{11}}{2}\left(|b_1|^2-|a_1|^2+3n_1\right)+C_{12}n_2-\varepsilon_1 & \frac{V}{2}+C_{12}a_2 b_2^* & 0 & 0 \\ \frac{V}{2}+C_{12}a_2^* b_2 & -\frac{\lambda}{2}-\frac{C_{11}}{2}\left(|b_1|^2-|a_1|^2+3n_1\right)+C_{12}n_2-\varepsilon_1 & 0 & 0 \\ 0 & 0 & \frac{\lambda}{2}+\frac{C_{22}}{2}\left(|b_2|^2-|a_2|^2+3n_2\right)+C_{12}n_1-\varepsilon_2 & \frac{V}{2}+C_{12}a_1 b_1 \\ 0 & 0 & \frac{V}{2}+C_{12}a_1^* b_1 & -\frac{\lambda}{2}-\frac{C_{22}}{2}\left(|b_2|^2-|a_2|^2+3n_2\right)+C_{12}n_1-\varepsilon_2 \end{pmatrix} \tag{10}$$

Eqn.(10) can be written as two independent determinants

$$\det\begin{pmatrix} \frac{\lambda}{2}+\frac{C_{11}}{2}\left(|b_1|^2-|a_1|^2+3n_1\right)+C_{12}n_1-\varepsilon_1 & \frac{V}{2}+C_{12}\,a_2\,b_2^* \\ \frac{v}{2}+C_{12}\,a_2^*\,b_2 & -\frac{\lambda}{2}-\frac{C_{11}}{2}\left(|b_1|^2-|a_1|^2+3n_1\right)+C_{12}n_1-\varepsilon_1 \end{pmatrix}\times$$
$$\det\begin{pmatrix} \frac{\lambda}{2}+\frac{C_{11}}{2}\left(|b_2|^2-|a_2|^2+3n_2\right)+C_{12}n_2-\varepsilon_1 & \frac{V}{2}+C_{12}\,a_1\,b_1^* \\ \frac{v}{2}+C_{12}\,a_1^*\,b_1 & -\frac{\lambda}{2}-\frac{C_{11}}{2}\left(|b_2|^2-|a_2|^2+3n_2\right)+C_{12}n_2-\varepsilon_1 \end{pmatrix}=0 \tag{11}$$

The nonlinear eigenstates are then defined as the solution of the time-independent version of Eq. (11), and thus One obtains

$$\left(\frac{\lambda}{2}+\frac{C_{11}}{2}(S_1+3n_1)+C_{12}n_2-\varepsilon_1\right)a_1=\left[\frac{-V}{2}+\delta_2\right]b_1 \tag{12.a}$$

$$\left(\frac{\lambda}{2}+\frac{C_{11}}{2}(S_1+3n_1)+C_{12}n_1-\varepsilon_1\right)b_1=\left[\frac{-V}{2}+\delta_2\right]a_1 \tag{12.b}$$

$$\left(\frac{\lambda}{2}+\frac{C_{22}}{2}(S_2+3n_2)+C_{12}n_2-\varepsilon_2\right)a_2=\left[\frac{-V}{2}+\delta_2\right]b_2 \tag{12.c}$$

$$\left(\frac{\lambda}{2}+\frac{C_{22}}{2}(S_2+3n_2)+C_{12}n_2-\varepsilon_2\right)b_2=\left[\frac{-V}{2}+\delta_2\right]a_2 \tag{12.d}$$

Where the fluctuation of energy due to the interaction of the two species of particles is given by $\delta_1=\frac{C_{12}}{2}\sqrt{1-S_2^2}$, $\delta_2=\frac{C_{12}}{2}\sqrt{1-S_1^2}$.

Since $n_1=1$, and $n_2=1$, the constant energy $\left(\frac{3}{2}C_{11}\,n_1+C_{12}\,n_1\right)$ and $\left(\frac{3}{2}C_{22}\,n_2+C_{12}\,n_2\right)$ can be dropped from Eqns. 12, and the solution of Eqs.(12) leads to the quartic algebraic equation for the eigenenergy,

$$\varepsilon_1^4+\varepsilon_1^3\,C_{11}^3+\varepsilon_1^2\left[\frac{C_{11}^2}{4}-\frac{1}{4}\left[V+\delta_1\right]^2-\frac{\lambda^2}{4}\right]-\frac{\left[V+\delta_1\right]^2 C_{11}\,\varepsilon_1}{4}-\frac{\left[V+\delta_1\right]C_{11}}{16}=0 \tag{13.a}$$

$$\varepsilon_2^4+\varepsilon_2^3\,C_{22}^3+\varepsilon_2^2\left[\frac{C_{22}^2}{4}-\frac{1}{4}\left[V+\delta_2\right]^2-\frac{\lambda^2}{4}\right]-\frac{\left[V+\delta_2\right]^2 C_{22}\,\varepsilon_2}{4}-\frac{\left[V+\delta_2\right]C_{22}}{16}=0 \tag{13.b}$$

Using Eqs. (13.a) and (13.b), One can analyze the influence of the interaction of the two different particles on the energy band structure.

Results and Discussion

Figures. (1-5) shows the energy levels ε_1 as a function of λ for ($V = 0.2$), and for different values of the interaction parameters C_{11} and C_{12}. Figure.1 shows that, for weak nonlinearity of $C_{11} = C_{22} = 0.05,\ C_{12} = 0$, the tunneling occurs between two fixed eigenstates and the perturbation energy is equal to zero. For this case the analytic solution of the quartic equations 11.a and 11.b will give tow real roots. While Fig..2 shows that for $C_{11} = C_{22} = 0.05,\ C_{12} = 0.5$, the influence of the fluctuation energy δ_i will increase the band energy between these two states. For $(C_{11}, C_{22} = 1,\ C_{12} = 0)$ the analytic solution of the quartic equation gives four real roots. The loop which appears at the tip of the lower level in Fig. 3 is produced due to the nonlinear term. This means that even in the adiabatic limit, the non linear term will cause LZ transmission. As C_{12} increases, the width of the loop in Figures (4, 5) decreases and this behavior can be explained according to the results reported in Ref. [29], where the width of the loop can be calculated from the relation, $\lambda_{width} = \left(C_{11}^{\frac{2}{3}} - (V + \delta_i)^{\frac{2}{3}} \right)^{\frac{3}{2}}$. For $C_{12} = 5$ as presented in Fig.6, the energy gap becomes very large and the opportunity of the nonlinear L-Z tunneling is prevented.

It is known that, for identical particles the nonlinear interaction C_{ii} will produce the nonlinear LZ tunneling between the states of these identical particles, while the increase of C_{ij} will increase the fluctuation energy δ_i and as a result, a new degeneracy of eigenstates is produced, that permits the identical particles to transmit into a new eigenstaes. Figure.7 shows the fluctuation energy δ_i versus λ for $C_{12} = 0.5$. One can conclude that the particles of the same kind have two routes for tunneling, the first one is the tunneling of particles in their own states, and the second route is the transmission of particles to the new degeneracy of eigenstates.

Another way to study the influence of the fluctuation energy in the LZ tunneling, is by calculating the tunneling probability, and according to the results of Ref [11], the tunneling probability can be easily obtained for two limiting cases.

When $\dfrac{C_{11}}{V + \delta} < 1$, the tunneling probability Γ is thus found to be

$$\Gamma \sim \exp\left(- \frac{q\,\pi V^2}{2\bar{\alpha}} \right) \tag{14}$$

where the factor in the exponent is given by

$$q = \int_0^{\beta} \left(1 + x^2\right)^{\frac{1}{4}} \left[\frac{1}{\left(1 + x^2\right)^{\frac{3}{2}}} - \frac{C_{11}}{V} \right]^{\frac{3}{2}} \partial x \tag{15}$$

and

$$\beta = \sum \left(\frac{V}{C_{11}} \right)^{\frac{2}{3}} - 1 \tag{16}$$

When $\dfrac{C_{11}}{V + \delta} >> 1$, the tunneling probability Γ is given by

$$\Gamma = 1 - \frac{\pi V^2}{2\bar{\alpha}} \tag{17}$$

where the factor $\bar{\alpha}$ satisfies

$$\bar{\alpha} = \alpha + 2C_{11} \left(\frac{V^2}{2} \right) \sqrt{\left(\frac{\pi}{\bar{\alpha}} \right)} \tag{18}$$

It is clear that when $C_{12} = 0$, the results given by Eqs. (14)–(18) reduce to the corresponding results of Ref. [9]. For the case of $\frac{C_{11}}{V + \delta} < 1$, the numerical results of the tunneling probability for different values of C_{11} are shown in Figs (8, 9) with $C_{12} = 0.05$ and $C_{12} = 0.5$ respectively. We can see that, the a diabetic tunneling increases as the fluctuation energy increases. Figure.9 shows the opposite effect of the level bias λ, where the increase of λ, will reduce the a diabetic tunneling.

For the case of $\frac{C_{11}}{V + \delta} > 1$, the tunneling probability given by the direct numerical results of Eq. (18) are presented in Figs. (10-12). The results indicate that, nonlinear L-Z probability tunneling decreases as the fluctuation energy increases, while nonlinear L-Z probability increases as the tunneling parameter λ increases.

Conclusion

The tunneling properties of BEC with the repulsive interaction between two species of particles immersed in one-dimensional accelerating optical lattice are investigated. With the mean-field theory and the approximation, the nonlinear eigenstates and the tunneling properties are investigated numerically. The results show that, the eigenstates are affected by the fluctuation energy produced by the repulsive interaction between the two different particles. The nonlinear interaction of identical particles will increase the opportunity of nonlinear LZ tunneling in their own eigenstates while the nonlinear interaction between the different species will create a new degeneracy of eigenstates, and the identical particles have another route to be tunneled. In other words, the fluctuation energy will decrease the probability of transition of particles on their eigenstates.

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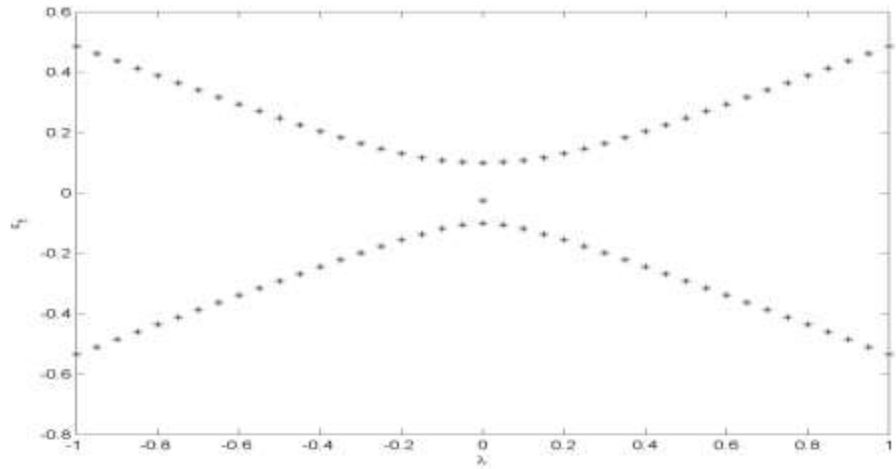

Figure. 1: Energy levels for different λ with V =0.2, C_{11} =0.05, C_{22}=0.05, C_{12}=0.0

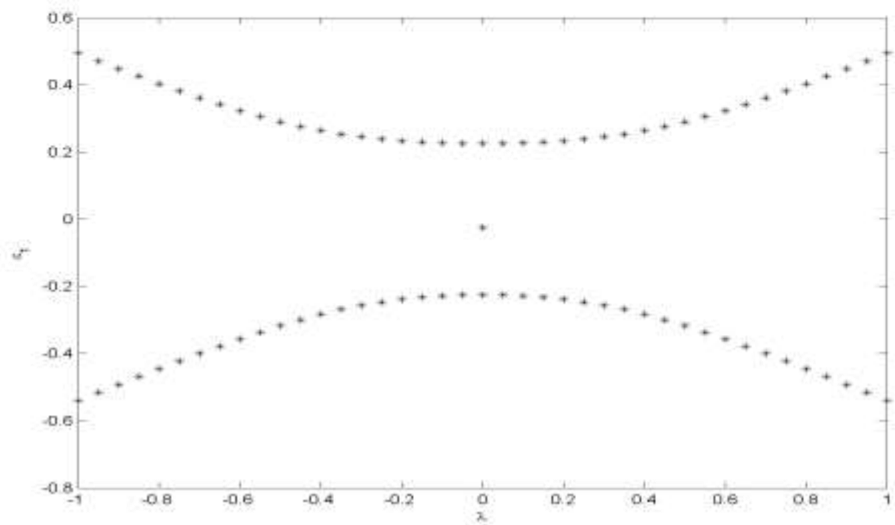

Figure. 2: Energy levels for different λ with V =0.2, C_{11} =0.05, C_{22}=0.05, C_{12}= 0.5

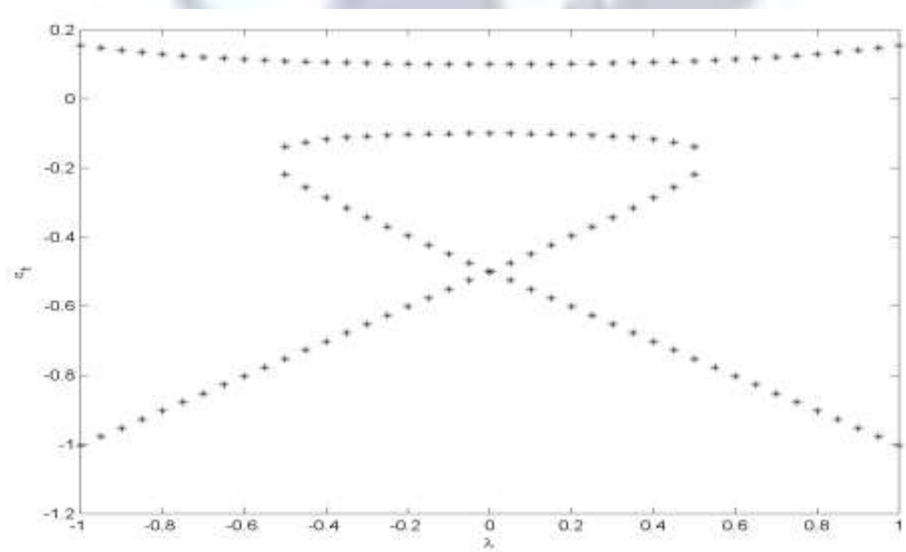

Figure. 3: Energy levels for different λ with V =0.2, C_{11} = 1, C_{22}=1, C_{12}= 0.0

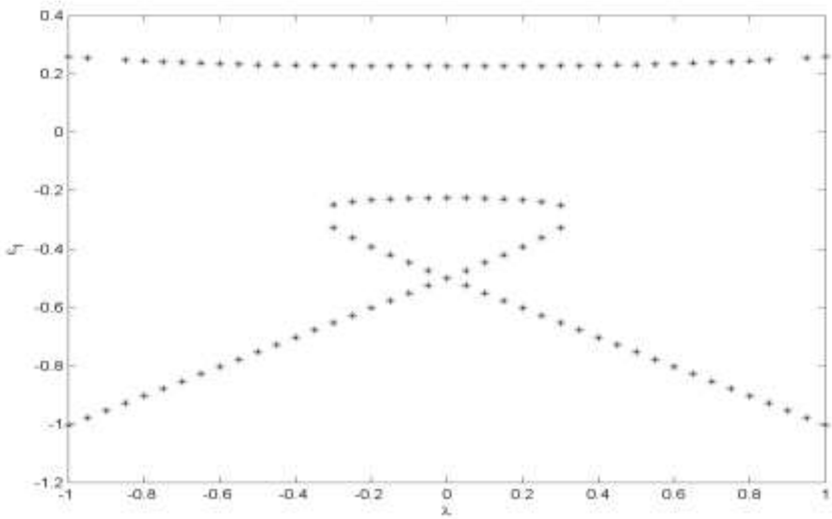

Figure.4 : Energy levels for different λ with V =0.2, C_{11} =1, C_{22}=1, C_{12}=0.5

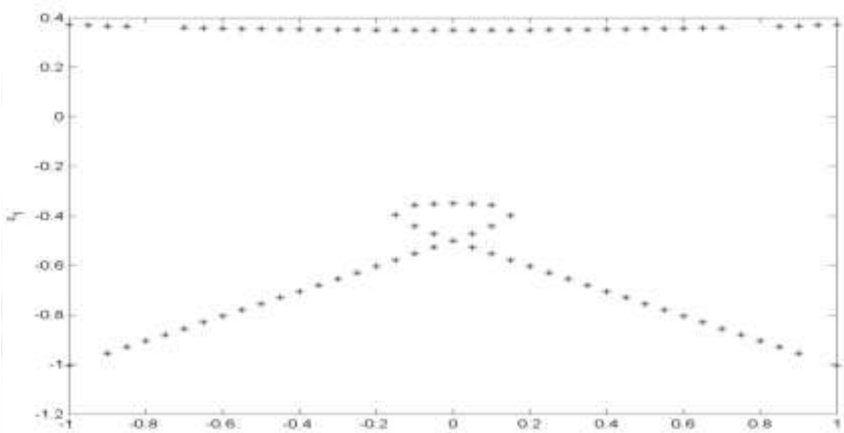

Figure. 5: Energy levels for different λ with V = 0.2, C_{11} =1, C_{22}=1, C_{12}=1

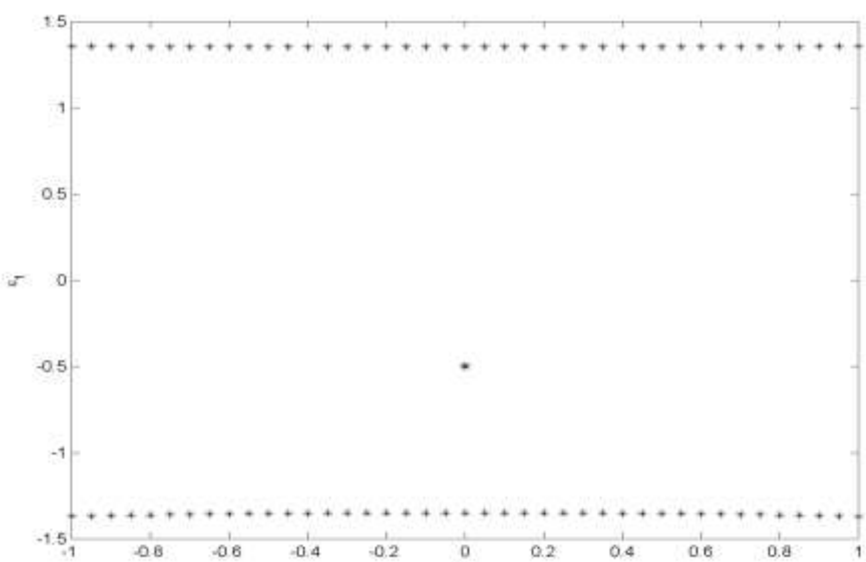

Figure. 6: Energy levels for different λ with V = 0.2, C_{11} =1, C_{22}=1, C_{12}=5

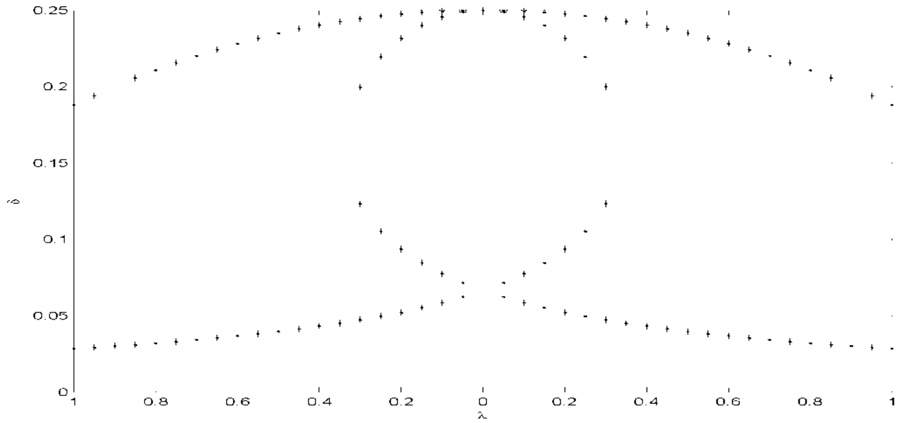

Figure. 7: The fluctuation energy for different λ with V = 0.2, C_{11} =1, C_{22}=1, C_{12}=0.5

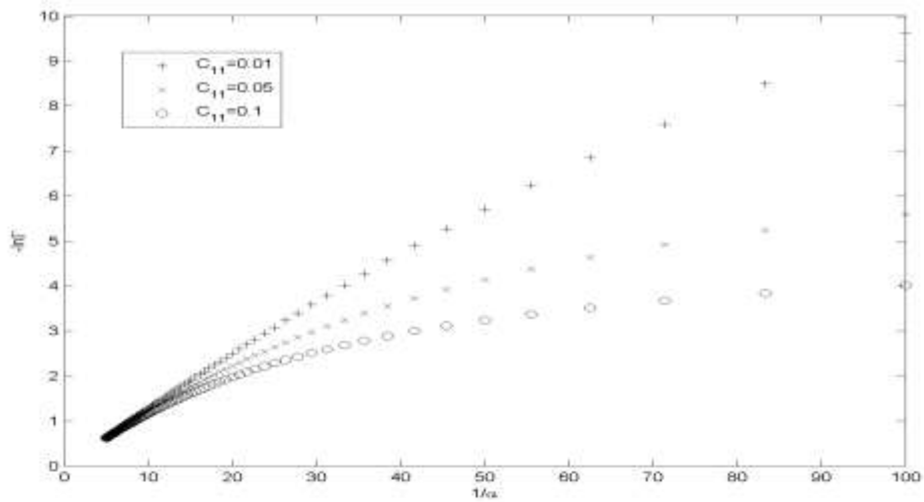

Figure. 8: Numerical results of the exponential dependence of the tunneling probability as a function of 1/α for different interaction strength, with V=0.2, C_{12}=0.05, λ=0.05

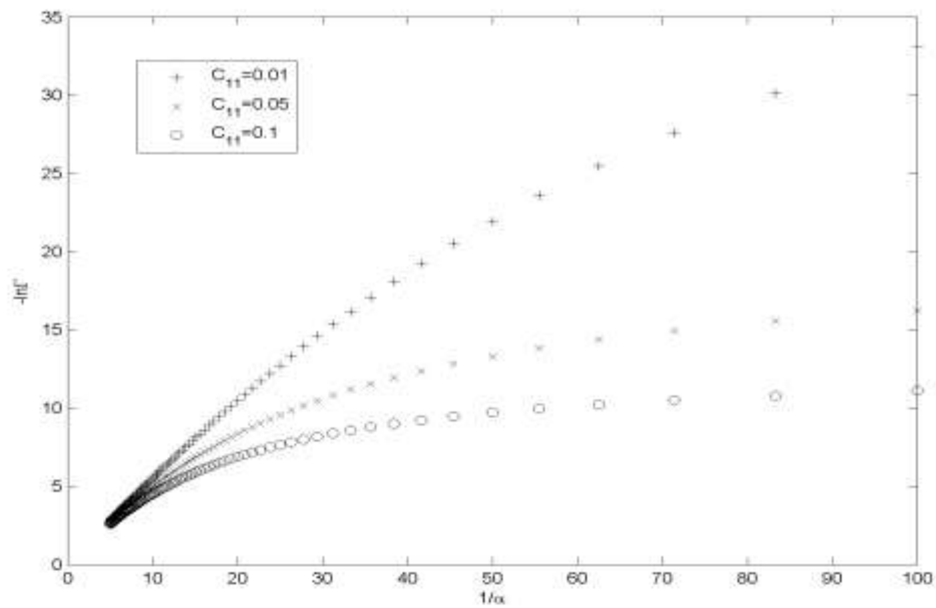

Figure. 8:Numerical results of the exponential dependence of the tunneling probability as a function of 1/α for different interaction strength, with V=0.2, C_{12}=0.5, λ=0.05

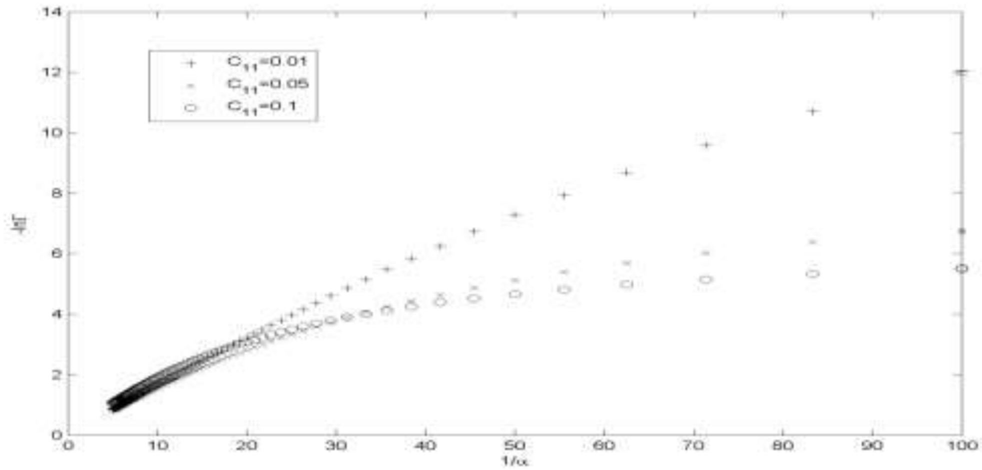

Figure. 9: Numerical results of the exponential dependence of the tunneling probability as a function of 1/α for different interaction strength, with V=0.2, C_{12}=0.5, λ=0.5

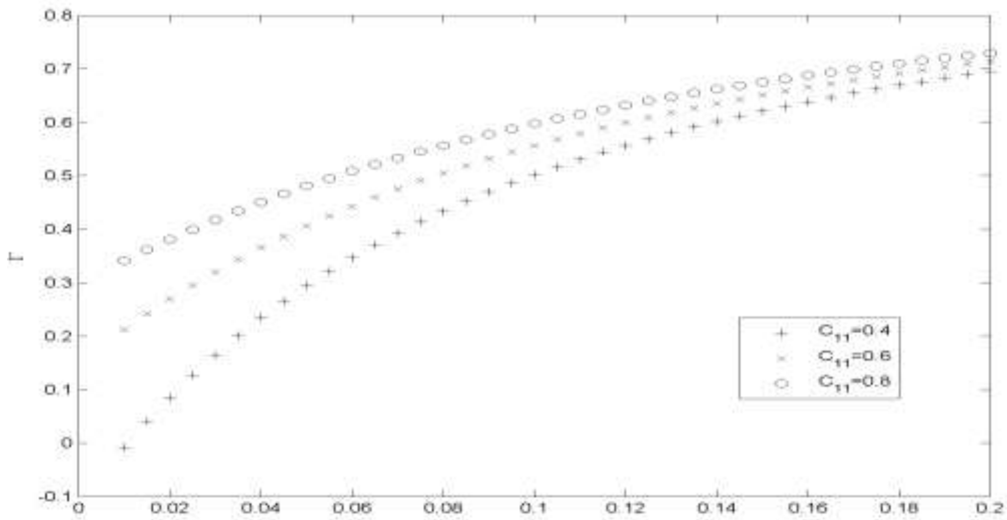

Figure. 10: Tunneling probability as a function of α for different interaction strength C_{11} with V=0.2, C_{12}=0.05, λ =0.05

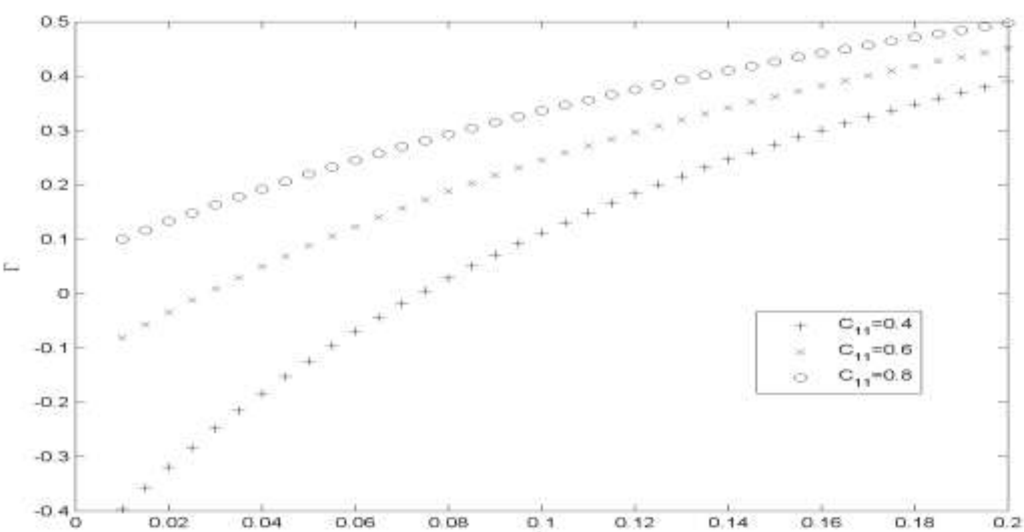

Figure. 11: Tunneling probability as a function of α for different interaction strength $C_{11}$with V=0.2, C_{12}=0.5, λ=0.05

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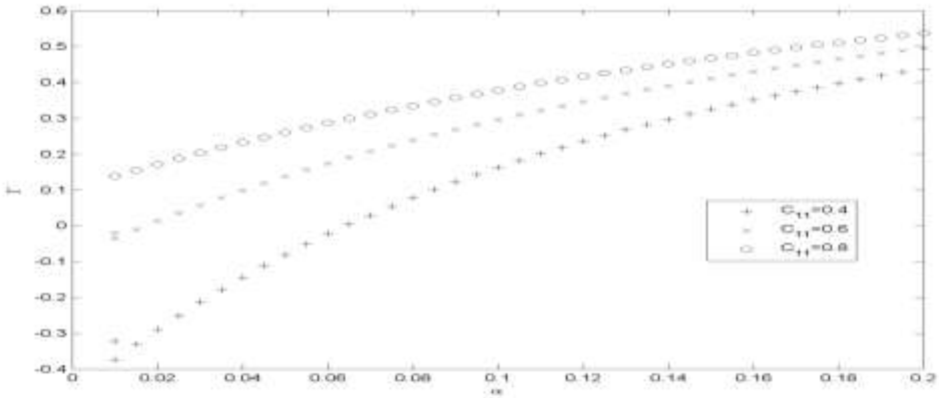

Figure. 12: Tunneling probability as a function of α for different interaction strength C_{11} with V=0.2, C_{12}=0.5, λ=0.5.