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Realization and Evaluation of prony signal modeling

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Abstract: **Proliferation of nonlinear loads like modern frequency power converters generate non-characteristic harmonic and inter harmonics. The power quality is now a days an isssue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric equipment. Power Signal modeling is concerned with the representation of signals. The modeled power signal consists of parameters, using which the original signal can be reconstructed or recovered. When once it is possible to accurately model a signal, then it becomes possible to perform important signal processing tasks such as signal compression, interpolation, prediction (extrapolation). This work investigates in minimizing the error using Prony method and results are compared with that of Pade Approximation method. The models used are AR (Auto Regressive) or All-Pole model, MA (Moving Average) or All-Zero model, ARMA (Auto Regressive Moving Average) or Pole-Zero model.**

Keywords: Pade Approximation method,Prony method,and power quality.

I. INTRODUCTION

The voltage waveform is likely to be a clean sinusoidal with a given frequency and amplitude. Present power electronic equipments cause a wide spectrum of harmonic components which depreciate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. The estimation of the components is very important for control and protection tasks. Power Signal modeling is an important problem. A waveform $x(n)$ consisting of N data values can be transmitted across a communication channel in two methods. One method is to process the signal on a point-by-point basis. The other method is to accurately model the signal with a small number of parameters k, where k<<N, and transmit these parameters instead of the signal values. In general, there are two steps in the modeling process. The first is to choose the appropriate parametric form for the model and the next step is to find the model parameters that provide the best approximation to the given signal.

Consider a deterministic signal $x(n)$ to be modeled as the unit sample response of a linear shift-invariant filter having system function $H(z)$ with an input $v(n)$ and output $\hat{x}(n)$ as shown in Fig.1. The intended goal is to find the filter $H(z)$ that makes $\hat{x}(n)$ as close as possible to $x(n)$.

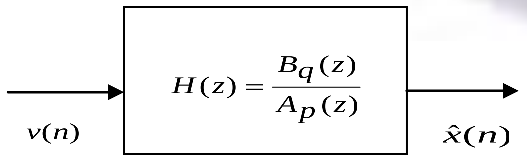

Fig.1. Modeling a signal $x(n)$ as the response of a linear shift-invariant filter to an input $v(n)$ with an output $\hat{x}(n)$.

II. APPROACHES

There are a number of approaches to signal modeling: Least Squares (Direct) Method, Pade Approximation Method and Prony Method [1].

A. Least Squares (Direct) Method

Consider a deterministic signal $x(n)$ as the unit sample response $\delta(n)$ of a linear time-invariant filter, having system function $H(z)$ [2],

$$H(z) = \frac{B_q(z)}{A_p(z)} = \frac{\sum_{k=0}^{q} b_q(k) z^{-k}}{\sum_{k=1}^{p} a_p(k) z^{-k}} \qquad (1)$$

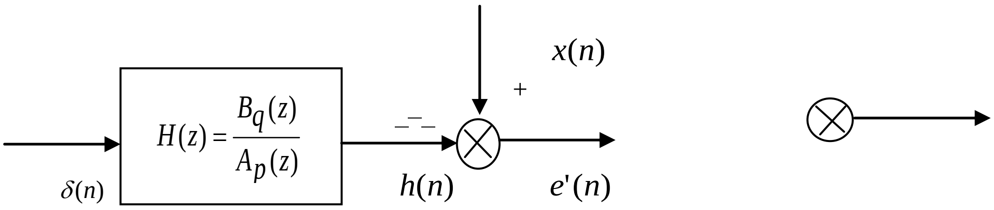

Fig.2. The direct method of signal modeling for approximating a signal $x(n)$ as the unit sample response of a linear shift-invariant system having p poles and q zeros.

As shown in Fig.2, $e'(n)$ is the modeling error which is defined as the difference between the signal $x(n)$ and the output of the filter $h(n)$.

$$e'(n) = x(n) - h(n). \qquad (2)$$

In this method, the error measure that is to be minimized is the squared error,

$$\varepsilon_{LS} = \sum_{n=0}^{\infty} |e'(n)|^2 \qquad (3)$$

To minimize the squared error we need to calculate the partial derivative of ε_{LS} with respect to each of the coefficients $a_p{}^*(k)$ and $b_q{}^*(k)$ and then equating to zero,

$$\frac{\partial \varepsilon_{LS}}{\partial a_p{}^*(k)} = \frac{1}{2\Pi} \int_{-\Pi}^{\Pi} \left[X(e^{j\omega}) - \frac{B_q(e^{j\omega})}{A_p(e^{j\omega})} \right] \frac{B_q{}^*(e^{j\omega})}{\left[A_p{}^*(e^{j\omega})\right]^2} e^{jk\omega} d\omega = 0 \qquad (4)$$

$$\frac{\partial \varepsilon_{LS}}{\partial b_q{}^*(k)} = -\frac{1}{2\Pi} \int_{-\Pi}^{\Pi} \left[X(e^{j\omega}) - \frac{B_q(e^{j\omega})}{A_p(e^{j\omega})} \right] \frac{e^{jk\omega}}{A_p{}^*(e^{j\omega})} d\omega = 0 \qquad (5)$$

for $k = 0,1,2...q$

resulting to a set of non-linear equations. Therefore, this approach is not mathematically tractable and not amenable in real-time signal processing applications.

B. Pade Approximation Method

Unlike the least squares solution, Pade Approximation method requires solving a set of linear equations. The model that is formed from the Pade Approximation method will always produce an exact fit to the data over the interval [0, p+q], provided that the data matrix X_q is non-singular.

Let $x(n)$ be a signal that is to be modeled as the unit sample response of a casual linear shift-invariant filter, with a system function having p poles and q zeros. The development of Pade Approximation method begins by expressing equation (1) in time domain as,

$$h(n) + \sum_{k=1}^{p} a_p(k) h(n-k) = b_q(n) \qquad (6)$$

where, $h(n) = 0$, for n < 0

$b_q(n) = 0$, for n < 0 and n > q

To find the coefficients $a_p(k)$ & $b_q(k)$ that give an exact fit of the data to model over the interval [0, p+q], equate $H(z)$ and $x(n)$.

$$x(n) = \begin{cases} b_q(n) & n = 0, 1 \ldots q \\ 0 & n = q+1 \ldots q+p \end{cases} \qquad (7)$$

The above equation in the matrix form is be expressed as,

$$\begin{bmatrix} x(0) & 0 & \cdots & 0 \\ x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x(q) & x(q-1) & \cdots & x(q-p) \\ x(q+1) & x(q) & \cdots & x(q-p+1) \\ \vdots & \vdots & & \vdots \\ x(q+p) & x(q+p-1) & \cdots & x(q) \end{bmatrix} \begin{bmatrix} 1 \\ a_p(1) \\ a_p(2) \\ \vdots \\ \vdots \\ a_p(p) \end{bmatrix} = \begin{bmatrix} b(0) \\ b(1) \\ \vdots \\ b(q) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (8)$$

To find the coefficients $a_p(k)$ & $b_q(k)$ equation (8) is divided into two halves:

i. Upper part for solving denominator coefficient $a_p(k)$

ii. Lower part for solving numerator coefficient $b_q(k)$.

1) Determination of Denominator Coefficients:

Step 1: Consider the lower part of equation (8), separate the first column of X_q matrix (to make X_q a p×p matrix) and shift it to the right side then equation (8) becomes,

$$\begin{bmatrix} x(q) & x(q-1) & \cdots & x(q-p+1) \\ x(q+1) & x(q) & \cdots & x(q-p+2) \\ \vdots & \vdots & & \vdots \\ x(q+p-1) & x(q+p-2) & \cdots & x(q) \end{bmatrix} \begin{bmatrix} a_p(1) \\ a_p(2) \\ \vdots \\ \vdots \\ a_p(p) \end{bmatrix} = - \begin{bmatrix} x(q+1) \\ x(q+2) \\ \vdots \\ x(q+p) \end{bmatrix}$$

(9)

Step 2: In terms of matrix notation equation (9) is expressed as,

$$X_q * \bar{a}_p = -x_{q+1} \quad (10)$$

where, X_q is a Toeplitz matrix of order p × p .

Taking X_q to be a non-singular matrix, then X_q^{-1} exists and the coefficients of $A_p(z)$ becomes,

$$\overline{a_p} = -X_q^{-1} x_{q+1} \quad (11)$$

2) Determination of Numerator Coefficients

Step 1: Represent the upper part of equation (8) in matrix notation to obtain numerator coefficients $b_q(k)$

$$X_0 a_p = b_q \quad (12)$$

The unit sample response $h(n)$ is calculated using equation (1). The error $e'(n)$ is calculated by the equation (2).Thus, the coefficients of the filter are obtained that make, $h(n)$ the unit sample response of the filter as close as possible to $x(n)$, to minimize the error $e'(n)$.

The advantage in this approach is to solve a set of linear equations, while the limitation is that there is no guarantee on how accurate the model will be for values of n outside the interval [0, p+q].

C. Prony Method

Prony method is a technique for modelling sampled data as a linear combination of exponentials. Although it is not a spectral estimation technique. Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. The limitation with the Pade Approximation method is that, it only uses values of the signal $x(n)$ over the interval [0, p+q] to determine the model parameters, and over this interval, it models the signal without error. This method is developed, to produce a better approximation of the signal for all values of n i.e. for all $n \geq 0$. The autocorrelation method is used to find the denominator coefficients and the Pade Approximation method is used to find the numerator coefficients.

Let $x(n)$ be a signal that is to be modeled as the unit sample response of a linear shift-invariant filter, with a system function $H(z)$ having p poles and q zeros.

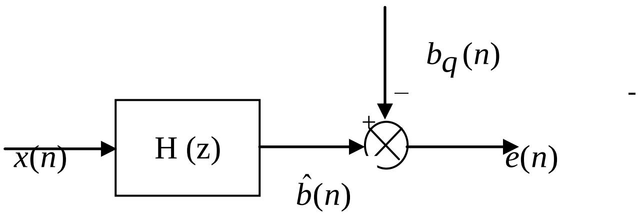

Fig.3**:** System interpretation of Prony method for signal modeling

From Fig.3 a new error e(n) is defined,

$$e(n) = a_p(n) * x(n) - b_q(n) = \hat{b}_q(n) - b_q(n) \qquad (13)$$

$$e(n) = \begin{cases} x(n) + \sum_{l=1}^{p} a_p(l) x(n-l) - b_q(n) & \text{n=0, 1… q} \\ x(n) + \sum_{l=1}^{p} a_p(l) x(n-l) & \text{n > q} \end{cases} \qquad (14)$$

1) Determination of Mean Square Error & denominator Coefficient

Step 1: The mean square error is defined as, $\varepsilon_{p,q} = \sum_{n=q+1}^{\infty} \left| e(n)^2 \right|$

$$= \sum_{n=q+1}^{\infty} e(n) \left| x(n) + \sum_{l=1}^{p} a_p(l) x(n-l) \right|^*$$

(15)

In order to minimize the squared error, set the partial derivatives of $\varepsilon_{p,q}$ with respect to $a_p*(k)$ equal to zero,

$$\frac{\partial \varepsilon_{p,q}}{\partial a_p*(k)} = \sum_{n=q+1}^{\infty} \frac{\partial [e(n)e*(n)]}{\partial a_p*(k)} = \sum_{n=q+1}^{\infty} e(n) \frac{\partial e*(n)}{\partial a_p*(k)} = 0;$$

for k=1, 2, ……………p (16)

Since the partial derivative of e*(n) with respect to $a_p*(k)$ is $x*(n-k)$ equation (17) becomes,

$$\boxed{\sum_{n=q+1}^{\infty} e(n) x*(n-k) = 0;} \qquad \text{K=1, 2, 3 …p} \qquad (17)$$

and is known as the orthogonality principle.

Step 2: Substituting equation (14) in equation (17), autocorrelation function $r_x(k,l)$ can be written as,

$$\sum_{l=1}^{p} a_p(l) r_x(k,l) = -r_x(k,0); \quad \text{k=1, 2… p} \qquad (18)$$

where $r_x(k,l) = \sum_{n=q+1}^{\infty} x(n-l)x^*(n-k)$

this is referred as Prony's Normal Equation.

Step 3: Equation (18) in matrix notation is expressed as,

$$\begin{bmatrix} r_x(1,1) & r_x(1,2) & \cdots & \cdots & r_x(1,p) \\ r_x(2,1) & r_x(2,2) & \cdots & \cdots & r_x(2,p) \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ r_x(p,1) & r_x(p,2) & \cdots & \cdots & r_x(p,p) \end{bmatrix} \begin{bmatrix} a_p(1) \\ a_p(2) \\ \vdots \\ \vdots \\ a_p(p) \end{bmatrix} = \begin{bmatrix} r_x(1,0) \\ r_x(2,0) \\ \vdots \\ \vdots \\ r_x(p,0) \end{bmatrix}$$

i.e. $R_x \bar{a}_p = -r_x \quad \Rightarrow \quad \overline{a_p} = -R_x^{-1} r_x$ (19)

(R_x is assumed to be a non-singular matrix.)

Step 4: The Prony's normal equation in different approach

Consider a data matrix X_q consisting of p-infinite column vector as,

$$X_q = \begin{bmatrix} x(q) & x(q-1) & \cdots & \cdots & x(q-p+1) \\ x(q+1) & x(q) & \cdots & \cdots & x(q-p+2) \\ x(q+2) & x(q+1) & \cdots & \cdots & x(q-p+3) \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \end{bmatrix} \tag{20}$$

The autocorrelation matrix and its vectors can be expressed in terms of X_q as,

$$R_x = X_q^H X_q$$

$$r_x = X_q^H x_{q+1} \tag{21}$$

Substituting equation (21) in equation (19) we can obtain,

$$\bar{a}_p = (X_q^H X_q)^{-1} X_q^H x_{q+1} \tag{22}$$

Step 5: To find the minimum error, substitute equation (17) in equation (15) and solve $\varepsilon_{p,q}$ which results in,

$$\boxed{\varepsilon_{p,q} = r_x(0,0) + \sum_{k=1}^{p} a_p(k) r_x(0,k)} \tag{23}$$

Step 6: Substitute equation (23) in equation (19) then

$$a_p = R_x^{-1} \varepsilon_{p,q} u_1 \tag{24}$$

where R_x is a Hermitian matrix having p+1 rows and p+1 columns and u_1 is a unit vector.

2) Determination of Numerator Coefficients and Error

The numerator coefficients are obtained using equation (7) and the error $e'(n)$ is found out using the equation (2) where the impulse response $h(n)$ is calculated using equation (6).

III. SIMULATION

Let the signal $x(n)$ be,

$x(n) = [5, 2.2, 0.125, 0.0625, 0.01325]^T$. Pade and Prony methods are compared for AR, MA, and ARMA models. For these models, inputs are $x(n)$ p and q and the outputs are a, b and error in the estimated signal. An error is generated if p+q>=length of $x(n)$.So, the values of p and q must be selected such that their sum is less than the length of $x(n)$.

A. Realization of Pade Approximation Method

This method is realized using MATLAB. The values of p and q chosen are: AR model: p=2 and q=0, MA model: p=0 and q=2, ARMA model: p=1 and q=1. Using the convolution matrix, the data matrix X_q and the coefficients are calculated after substituting the p and q values. Pade approximation $\hat{x}(n)$ is computed by evaluating the unit sample response of $H(z)$ and then the error is calculated. The results are shown in Table 1and Fig. 4 to 6

Table 1: Evaluation of pade and prony methods

Method	Values of p and q	Coefficients		Error = $x(n)\ h(n)$
Pade	p=2 q=0	$a = \begin{bmatrix} 1 \\ -0.5 \\ 0.225 \end{bmatrix}$	$b = [5]$	$err = [0;0;0;0.562;0.3094]$
	p=0 q=2	$a = [1]$	$b = \begin{bmatrix} 5 \\ 2.5 \\ 0.125 \end{bmatrix}$	$err = [0;0;0;0.0625;0.0313]$
	p=1 q=1	$a = \begin{bmatrix} 1 \\ -0.05 \end{bmatrix}$	$b = \begin{bmatrix} 5 \\ 2.25 \end{bmatrix}$	$err = [0;0;0;0.0063;0.003]$
Prony	p=2 q=0	$a = \begin{bmatrix} 1 \\ -0.4805 \\ 0.1719 \end{bmatrix}$	$b = [5]$	$err = [0.097;0.169;0.333;0.212]$
	p=0 q=2	$a = [1]$	$b = \begin{bmatrix} 5 \\ 2.5 \\ 0.125 \end{bmatrix}$	$err = [0;0;0;0.0625;0.313]$
	p=1 q=1	$a = \begin{bmatrix} 1 \\ 0.051 \end{bmatrix}$	$b = \begin{bmatrix} 5 \\ 2.243 \end{bmatrix}$	$err = [0;0;-0.0035;0.0559;0.0309]$

B. Realization of Prony's Method

This method is realized using MATLAB. The values of p and q chosen are: AR model: p=2 and q=0, MA model: p=0 and q=2, ARMA model: p=1 and q=1. Using the convolution matrix, the data matrix X_q is calculated after

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substituting the p and q values. The denominator and numerator coefficients $a_p(k)$ and $b_q(k)$ are calculated using autocorrelation and Pade Approximation method respectively. Then, the error is calculated using equation (2) for various values of p and q. The results are shown in Table 1 and Fig. 4 to 6.

methods

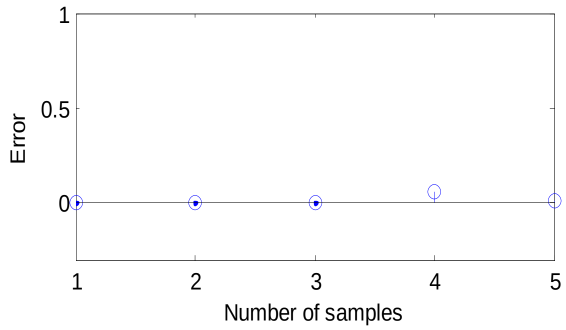

Fig. 4(a): Error obtained using Pade Approximation method with p=1 and q=1

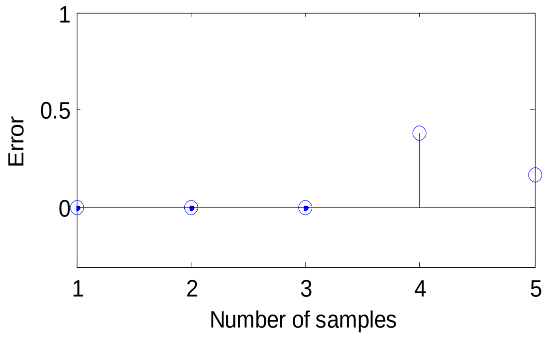

Fig. 5(a): Error obtained using Pade Approximation method with p=2 and q=0

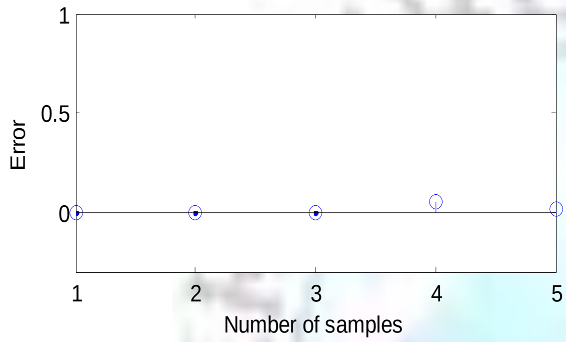

Fig. 4(b): Error obtained using Prony method with p=1 and q=1

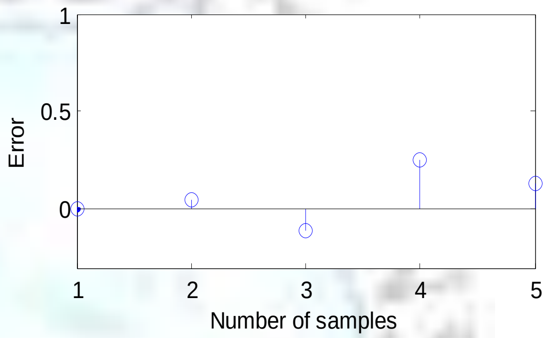

Fig. 5(b): Error obtained using Prony method with p=2 and q=0

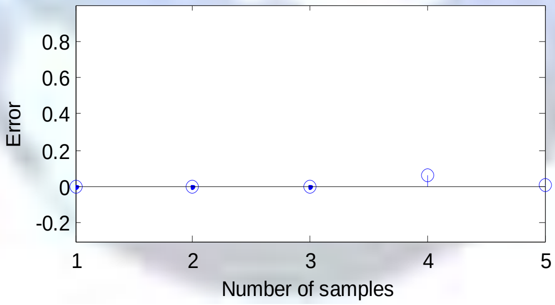

Fig. 6(a): Error obtained using Pade Approximation method with p=0 and q=2

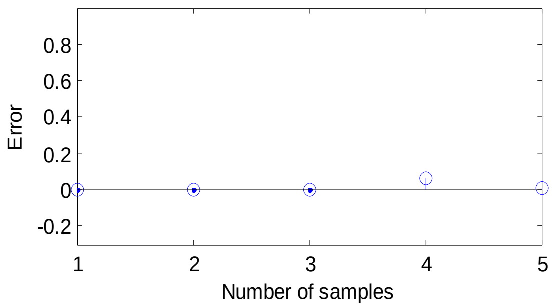

Fig. 6(b): Error obtained using Prony method with p=0 and q=2

IV. CONCLUSION

It is observed from the simulation results that Prony method of signal modeling is an improved method than that of Pade Approximation method

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