PI controller for Multiple-Time Delays systems

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ABSTRACT

In this paper, a PI controller design is proposed for multivariable linear time invariant (LTI) systems with multiple time-delays. The orthogonal collocation method is used to transform the infinite dimensional model of the delayed system described by a set of linear partial differential equations to a finite dimensional model described by a set of linear ordinary differential equations. An SOF transformation for such systems is developed, transforming the multi-loop PI control problem to static output feedback stabilization (SOFS) problem and then solved via an iterative linear matrix inequality (ILMI) approach. A numerical example is provided to illustrate the practicality and the effectiveness of the proposed approach. A comparative study is also established to prove the superiority of our approach compared to a related one.

Keywords: PI controller, multivariable systems, multiple time-delay systems, static output feedback stabilization (SOFS), iterative linear matrix inequality (ILMI), orthogonal collocation method.

1. INTRODUCTION

PID controllers [1, 2] have been at the heart of control engineering practice for several decades. They are widely used in industrial applications as no other controllers match the simplicity, clear functionality, applicability and ease of use. To deal with the crucial problem of tuning multi-loop PID controllers, several new techniques have been recently emerged; see for example [3, 4, 5, 6] and the references therein.

SOF controller design is a well-studied field in the Linear Matrix Inequality (LMI) framework. One advantage of using LMIs or Iterative LMIs (ILMIs) is its convenience to include different specifications for the controller design. Therefore, various design specifications may be remodeled into the LMIs and the resulting LMI constraints can be efficiently solved by using recently developed convex optimization algorithms. For solving the static output feedback Stabilization (SOFS) problem, LMI tools have been introduced in [7, 8] and later used to solve design problems of multi-loop PID controllers [9, 10, 11].

In the same framework, the ILMI approaches was proposed in [12] to solve the SOFS problem and then extended to the PID control design [13, 14, 15]. For the last methods, the basic idea is to transform a PID controller into an equivalent SOFS one. This can be realized by augmenting, using some new state variables, the dimension of the PID controller system. The iterative algorithm in [13], for example, tried to find a sequence of additional variables such that the relevant sufficient conditions are close to the necessary and sufficient ones. In [6], a comparative analysis of three ILMI approaches for PID design using specific criteria is presented.

On the other hand, nowadays, there has been raised interest in accentuating the limitations that the process imposes on control designs. One of the well-known limitations is the presence of time delays in the system which appears frequently within many control systems, either in the states of the plant or in the control inputs [16, 17]. Many useful techniques for SISO PID control design for single time-delay systems are raised recently [18, 19, 20, 21, 22]. However, control design of MIMO linear time invariant (LTI) systems with multiple-time delays is of theoretical and practical significance and the control problem is much more complex. In this framework, there are currently only few available results in the literature [23, 24]. However, most research papers are not concerned by PID controllers [25]. It seems that PID controllers for MIMO processes with multiple time-delays are yet an open new direction for practical implementation.

In this paper, we propose a novel approach to address multi-loop PI controller design for LTI systems with multiple time-delays. The basic idea in our approach is to transform the problem of PI controllers for LTI systems with multiple delays to an SOFS problem design. To this purpose, equations involving delays in the state and the control variables are approximated using the orthogonal collocation method. An SOF transformation for LTI systems with multiple delays is established to transform the problem of PI controllers for such system to an SOFS problem design. Then, an algorithm based on ILMI approach is provided. Finally, the industrial scale polymerization (ISP) reactor is used to illustrate the design, application and merit of the proposed approach. The proposed approach is also compared to a related one.

The paper is organized as follows: The problem considered is formally stated in Section 2. The main results are detailed in Section 3. Section 4 is devoted to multivariable control system performance and robustness study. Simulation results are shown in Section 5 where a comparative study with a related approach is also provided.

2. PROBLEM POSITION

Consider the infinite dimensional multivariable LTI system with multiple time-delays described by:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau_1) + B_0 u(t-\tau_2) + B_1 u(t-\tau_3)$$
$$y(t) = Cx(t) \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state vector, the control vector and the output vector, respectively. $A_0 \in \Re^{n\times n}$, $A_1 \in \Re^{n\times n}$, $B_0 \in \Re^{n\times m}$, $B_1 \in \Re^{n\times m}$ and $C \in \Re^{p\times n}$ are known constant matrices. τ_1, τ_2 and τ_3 are time-delays.

The objective is to design the finite dimensional PI controller described by:

$$u(t) = F_{1,PI} y(t) + F_{2,PI} \int_0^t y(t)dt \tag{2}$$

where $F_{1,PI}$, $F_{2,PI}$ are proportional and time integral gain matrices, respectively, that stabilize the system (1) under the following assumptions:

Assumption 1. τ_1, τ_2 and τ_3 are assumed to be known and constant delays.
Assumption 2. The PI controller (2) is well-posed.

The last control problem is very complex. To be relaxed, the infinite dimensional system (1) will be reduced to finite dimensional LTI system whereas the PI controller (2) will be transformed into an SOF controller.
Consider then a reduced finite dimensional LTI system obtained from system (1) using one of the approximation methods [26]. This system can be described by:

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$$
$$\tilde{y} = \tilde{K}\tilde{x} \tag{3}$$

and stabilized via the SOF controller:

$$u = \tilde{F}_{PI}\tilde{y} \tag{4}$$

where $\tilde{x} \in \Re^{4n+p}$, $u(t) \in \Re^m$, $\tilde{y} \in \Re^{2p}$ are the state vector, the control vector and the output vector of the approximated system, respectively. $\tilde{A}$, $\tilde{B}$, $\tilde{K}$ are matrices of appropriate dimensions. $\tilde{F}_{PI}$ is the SOF feedback gain matrix to be designed such that the closed loop dynamics $\dot{\tilde{x}} = (\tilde{A} + \tilde{B}\tilde{F}_{PI}\tilde{K})\tilde{x}$ are stabilized via the state feedback controller $\tilde{F}_{PI}$.

3. MAIN RESULTS

Each delayed variable in system (1) can be modeled as a distributed parameter system described by the following partial differential equation [26]:

$$\frac{\partial \omega(z,t)}{\partial t} = -\frac{1}{\tau}\frac{\partial \omega(z,t)}{\partial z} \tag{5}$$

with the boundary condition:

$$v(t) = \omega(0,t) \tag{6}$$

and the output equations :

$$v(t-\tau) = \omega(1,t) \tag{7}$$

where t and z are time and pseudo-space variables, respectively. As shown by Fig. 1, $v(t)$, $\omega(z,t)$ and $v(t-\tau)$ are the input, the state variable and the output of the delay block, respectively. τ is a constant time delay.

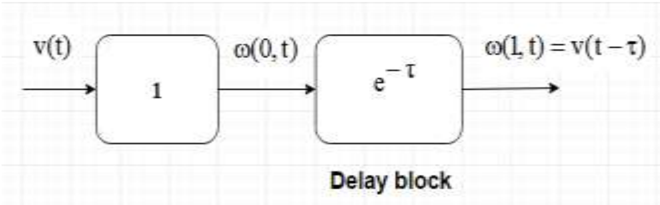

Figure 1. Delay block representation

Using the orthogonal collocation method, the following 3(N+1) finite dimensional equations can be obtained for the delayed vectors of the system (1) [27]:

$$\dot{\omega}_1(t) = -\frac{1}{\tau_1}\overline{A}_1\omega_1 + \frac{1}{\tau_1}\overline{B}_1 x(t) \quad (8)$$

$$\dot{\omega}_2(t) = -\frac{1}{\tau_2}\overline{A}_2\omega_2 + \frac{1}{\tau_2}\overline{B}_2 u(t) \quad (9)$$

$$\dot{\omega}_3(t) = -\frac{1}{\tau_3}\overline{A}_3\omega_3 + \frac{1}{\tau_3}\overline{B}_3 u(t) \quad (10)$$

augmented by the following outputs :

$$x(t-\tau_1) = \omega_1(1,t) = \overline{C}_1\omega_1(t) \quad (11)$$

$$u(t-\tau_2) = \omega_2(1,t) = \overline{C}_2\omega_2(t) \quad (12)$$

$$u(t-\tau_3) = \omega_3(1,t) = \overline{C}_3\omega_3(t) \quad (13)$$

where N is the number of the collocation points $z_0, z_1, \ldots, z_{N+1} \in [0,1]$ considered, in this paper, as the zeros of the (N+2) th order Jacobi polynomials [28]. For k=1,2,3

$$\overline{A}_k = \left[a_{k,ij}\right] = \left.\frac{dL_j(z)}{dz}\right|_{z=z_i} \in \Re^{n\times n},\ i, j = 1,\ldots,N+1,\ k=1,2,3.$$

$$\overline{B}_1 = \left[\left.\frac{dL_0(z)}{dz}\right|_{z=z_i} \quad \cdots \quad \left.\frac{dL_0(z)}{dz}\right|_{z=z_i}\right] \in \Re^{n\times m}, i = 1,\ldots,N+1$$

$$\overline{B}_2 = \overline{B}_3 = \left[\left.\frac{dL_0(z)}{dz}\right|_{z=z_i} \quad \cdots \quad \left.\frac{dL_0(z)}{dz}\right|_{z=z_i}\right] \in \Re^{n\times m}, i = 1,\ldots,N+1$$

$$\overline{C}_1 = \left[c_{1,ij}\right]^T \in \Re^{n\times n}, c_{i,j} = \begin{cases} 0 & \text{if} \quad i = 1,\ldots,N \\ 1 & \text{if} \quad j = N+1 \end{cases}$$

$$\overline{C}_2 = \overline{C}_3 = \left[c_{2,ij}\right]^T = \left[c_{3,ij}\right]^T \in \Re^{m\times n}, c_{2,ij} = c_{3,ij} = \begin{cases} 0 & \text{if} \quad i = 1,\ldots,N \\ 1 & \text{if} \quad j = N+1 \end{cases}$$

and where $L_j(z)$ are the Nth order Lagrange interpolation polynomials [27, 28].

Using the equations (11), (12) and (13), the system (1) can be written as:

$$\dot{x}(t) = A_0 x(t) + A_1\overline{C}_1\omega_1(t) + B_0\overline{C}_2\omega_2(t) + B_1\overline{C}_3\omega_3(t) \quad (14)$$

Let now:

$$\tilde{x} = \left[\tilde{x}_1{}^T \quad \tilde{x}_2{}^T\right]^T \in \Re^{4n+p}$$

where:

$$\tilde{x}_1 = \left[x(t) \quad \omega_1(t) \quad \omega_2(t) \quad \omega_3(t)\right] \in \Re^{4n}$$

$$\tilde{x}_2(t) = \int_0^t y(t)dt$$

and let:

$$\tilde{y} = [\tilde{y}_1 \quad \tilde{y}_2]^T = \tilde{K}\tilde{x}$$

where:

$$\tilde{y}_1 = y = Cx = [C \quad 0 \quad 0 \quad 0 \quad 0]\tilde{x}$$

$$\tilde{y}_2 = \int_0^t y(t)dt = [0 \quad 0 \quad 0 \quad 0 \quad I]\tilde{x}$$

The state space of a new augmented system controlled via an SOF controller is then deduced as:

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}u \\ \tilde{y} &= \tilde{K}\tilde{x} \\ u &= \tilde{F}_{PI}\tilde{y} \end{aligned} \tag{15}$$

where

$$\tilde{A} = \begin{pmatrix} A_0 & A_1\bar{C}_1 & B_0\bar{C}_2 & B_1\bar{C}_3 & 0 \\ \frac{1}{\tau_1}\bar{B}_1 & -\frac{1}{\tau_1}\bar{A}_1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_2}\bar{A}_2 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_3}\bar{A}_3 & 0 \\ C & 0 & 0 & 0 & 0 \end{pmatrix} \in \Re^{(4n+p)\times(4n+p)}, \quad \tilde{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_2}\bar{B}_2 \\ \frac{1}{\tau_3}\bar{B}_3 \\ 0 \end{pmatrix} \in \Re^{(4n+p)\times m}$$

$$\tilde{K} = [\tilde{K}_1 \quad \tilde{K}_2]^T \in \Re^{2p\times(4n+p)}$$

$$\tilde{K}_1 = [C \quad 0 \quad 0 \quad 0 \quad 0] \in \Re^{p\times(4n+p)}$$

$$\tilde{K}_2 = [0 \quad 0 \quad 0 \quad 0 \quad I_{p\times p}] \in \Re^{p\times(4n+p)}$$

From (2), the control law can be then expressed as:

$$u = \tilde{F}_{1,PI}\tilde{y}_1 + \tilde{F}_{2,PI}\tilde{y}_2 \tag{16}$$

On the other hand, we have from (15):

$$u = \tilde{F}_{PI}\tilde{y} \tag{17}$$

we can deduce that once the matrix $\tilde{F}_{PI} = [\tilde{F}_{1,PI} \quad \tilde{F}_{2,PI}] \in \Re^{m\times 2p}$ is designed such that the closed loop system (3)-(4) is asymptotically stable and considering the analogy between (16) and (17), the original PI gains can be recovered as:

$$[\tilde{F}_{1,PI} \quad \tilde{F}_{2,PI}] = [F_{1,PI} \quad F_{2,PI}] \tag{18}$$

Theorem

The multivariable LTI system with multiple delays (1) is stabilizable via the PI controller (2) if there exist a constant matrix $\tilde{F}_{PI} = [\tilde{F}_{1,PI} \quad \tilde{F}_{2,PI}] \in \Re^{m\times 2p}$ and a symmetric positive definite matrix $P = P^T > 0, P \in \Re^{(4n+p)\times(4n+p)}$ satisfying the following matrix inequality:

$$\tilde{A}^T P + P\tilde{A} - P\tilde{B}\tilde{B}^T P + (\tilde{B}P + \tilde{F}_{PI}\tilde{K})^T(\tilde{B}P + \tilde{F}_{PI}\tilde{K}) < 0 \tag{19}$$

such that:

$$\begin{aligned} F_{1,PI} &= \tilde{F}_{1,PI} \\ F_{2,PI} &= \tilde{F}_{2,PI} \end{aligned} \tag{20}$$

Proof.

The proof that the closed loop dynamics $\dot{\tilde{x}} = (\tilde{A} + \tilde{B}\tilde{F}_{PI}\tilde{K})\tilde{x}$ are asymptotically stable if the condition (20) is satisfied is parallel to this found in [12]. Condition (20) is already proved in (18).

Remark 1.

Note that the well-posedness of the MIMO PI controller is guaranteed if all closed-loop transfer matrix is well-defined and proper. In our case, this is guaranteed if $F_{1,PI} \neq 0$, $F_{2,PI} \neq 0$.

Algorithm

Step 1: Define the orthogonal collocation optimal parameters as chosen in [29].
Step 2 : Transform the infinite dimensional system (1) to a finite dimensional system (8)-(13) by computing matrices $\bar{A}_1$, $\bar{A}_2$, $\bar{A}_3$, $\bar{B}_1$, $\bar{B}_2$, $\bar{B}_3$, $\bar{C}_1$, $\bar{C}_2$, $\bar{C}_3$.
Step 3: Apply the SOF transformation to derive a system's state space realization ($\tilde{A}$, $\tilde{B}$, $\tilde{K}$) . If it does proceed to Step 4.
Step 4: Set i=1 and choose X_1 such as $X_1 \geq 0$
Step 5: Solve the optimization problem OP1 for P_i, $\tilde{F}_{PI}$ and α_i:
OP1: Minimize α_i subject to the following LMI constraints:

$$\begin{bmatrix} \Sigma_{1i} & (\tilde{B}^T P_I + \tilde{F}_{PI}\tilde{K})^T \\ \tilde{B}^T P_I + \tilde{F}_{PI}\tilde{K} & -I \end{bmatrix} < 0, P_i > 0 \tag{21}$$

where $\Sigma_{1i} = \tilde{A}^T P_i + P_i^T \tilde{A} - X_i^T \tilde{B}\tilde{B}^T P_i - P_i^T \tilde{B}\tilde{B}^T X_i + X_i^T \tilde{B}\tilde{B}^T X_i - \alpha_i P_i$. Denote by α_i^* the minimized value of α_i .
Step 6: If $\alpha_i^* \leq 0$, the feedback matrix gains are $F_{1,PI} = \tilde{F}_{1,PI}$, $F_{2,PI} = \tilde{F}_{2,PI}$. Stop. Otherwise, go to step7.
Step 7: Solve the optimization problem OP2 for P_i and $\tilde{F}_{PI}$.
OP2: Minimize tr (P_i) subject to LMI constraints (21) with $\alpha_i = \alpha_i^*$, where tr stands for the trace of a square matrix. Denote by P_i^* the optimal P_i. The feedback matrix gains are $F_{PI} = \tilde{F}_{PI}$.
Step 8 : If $\left\| X_i\tilde{B} - P_i^*\tilde{B} \right\| < \delta$, where δ is a prescribed tolerance , go to step 9 ; otherwise, set i:= i+1 , $X_i = P_i^*$ and go to step 5.
Step 9: It cannot be decided by this algorithm whether the SOF problem is not solvable. Stop.

Remark 2. The transfer matrix of the MIMO PI controller is described by:

$$K_{PI}(s) = F_{1,PI} + \frac{F_{2,PI}}{s} \tag{22}$$

4. PERFORMANCE AND ROBUSTNESS CRITERIA

To evaluate the closed loop performances of the proposed method, many performance criteria can be used [30]. In this paper, we select the following criteria [31]:

A. Integral absolute error index (IAE)

The integral absolute error (IAE) criterion is defined as:

$$IAE = \int_0^T |e(t)| dt \tag{23}$$

where T is a finite chosen for the integral approach steady-state value.

B. Total Variation (TV)

To evaluate the magnitude of the manipulated input usage, the total up and down movement of the control signal is considered as

$$TV = \sum_{k=1}^{T} |u(k+1) - u(k)| \tag{24}$$

TV is a good measure of the smoothness of controller output and should be small.

C. Robustness study

The robustness of the controller is evaluated by inserting a perturbation uncertainty of $\pm$ 10% into the parameters of the actual process, simultaneously.

5. APPLICATION: THE ISP REACTOR

To illustrate the effectiveness of the proposed approach, the typical example of the Industrial Scale Polymerization (ISP) reactor [32] is used. Three case studies are considered and results are evaluated using IAE and TV indices. The three case studies are as follows 1) Set-point tracking, 2) disturbance rejection and 3) parametric uncertainties. Note that Sedumi and Yalmip Toolboxes [33] are used to solve ILMIs.

For the orthogonal collocation method, optimal parameters are chosen for N=3. The following matrices are then obtained:

$$\bar{A}_1 = \begin{pmatrix} 10.3923 & 1.1547 & -1.1547 & 0.8038 \\ -4.6188 & -0.0000 & 4.6188 & -3 \\ 1.1547 & -1.1547 & -10.3923 & 11.1962 \\ -1.4291 & 1.3333 & -19.9043 & 19 \end{pmatrix}, \bar{B}_1 = \begin{pmatrix} -11.1962 & -11.1962 & -11.1962 & -11.1962 \\ 3 & 3 & 3 & 3 \\ -0.8038 & -0.8038 & -0.8038 & -0.8038 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \bar{C}_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\bar{A}_2 = \begin{pmatrix} 10.3923 & 1.1547 & -1.1547 & 0.8038 \\ -4.6188 & -0.0000 & 4.6188 & -3 \\ 1.1547 & -1.1547 & -10.3923 & 11.1962 \\ -1.4291 & 1.3333 & -19.9043 & 19 \end{pmatrix}, \quad \bar{B}_2 = \begin{pmatrix} -11.1962 & -11.1962 \\ 3 & 3 \\ -0.8038 & -0.8038 \\ 1 & 1 \end{pmatrix}, \quad \bar{C}_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\bar{A}_3 = \begin{pmatrix} 10.3923 & 1.1547 & -1.1547 & 0.8038 \\ -4.6188 & -0.0000 & 4.6188 & -3 \\ 1.1547 & -1.1547 & -10.3923 & 11.1962 \\ -1.4291 & 1.3333 & -19.9043 & 19 \end{pmatrix}, \bar{B}_3 = \begin{pmatrix} -11.1962 & -11.1962 \\ 3 & 3 \\ -0.8038 & -0.8038 \\ 1 & 1 \end{pmatrix} \text{ and } \bar{C}_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transfer matrix of the ISP reactor system is described by [32]:

$$G(s) = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix} \tag{25}$$

Applying Gilbert method [34], the state space representation can be deduced as:

$$A_0 = \begin{pmatrix} -0.2187 & 0 & 0 & 0 \\ 0 & -0.5534 & 0 & 0 \\ 0 & 0 & -0.46 & 0 \\ 0 & 0 & 0 & -0.5552 \end{pmatrix}, B_0 = \begin{pmatrix} 5.066 & 0 \\ 0 & 0 \\ 2.1569 & 0 \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 0 & 0 \\ 0 & -6.4416 \\ 0 & 0 \\ 0 & 3.2204 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

with $\tau_2 = 0.2h$ and $\tau_3 = 0.4h$.

The time scales are in hours, so the process dynamic's responses are quite slow.

To solve the ILMIs (21), we choose $X_1 = I_{18} + 0.002$, δ = 0.1 and α = 0.7549 which gives the following PI gains:

$$F_{1,PI} = \begin{pmatrix} 0.2059 & 0.3061 \\ 0.2062 & 0.3060 \end{pmatrix} \text{ and } F_{2,PI} = \begin{pmatrix} 0.0302 & 0.0302 \\ 0.0292 & 0.0335 \end{pmatrix}$$

In order to prove the performances of the proposed method, a comparison study is established with the PID controller designed via the approach given in [13]. The filter $1/(\tau_d s+1)$ with $\tau_d > 0$ is also applied to the derivative action to attenuate the noise in high frequencies. The PID controller is described by:

$$K_{PID}(s) = F_{1,PID} + \frac{F_{2,PID}}{s} + F_{3,PID} \times \frac{s}{\tau_d s+1} \tag{26}$$

Solving the ILMIs given in [13] for the system reduced using the orthogonal collocation method, the MIMO PID gains are given by:

$$F_{1,PID} = \begin{pmatrix} 0.0187 & 0.2988 \\ 0.0192 & 0.2989 \end{pmatrix},\ F_{2,PID} = \begin{pmatrix} -0.0132 & 0.0812 \\ -0.0135 & 0.0812 \end{pmatrix},\ F_{3,PID} = \begin{pmatrix} 0.0151 & 0.0661 \\ 0.0151 & 0.0660 \end{pmatrix}$$

and we choose $\tau_d = 0.1$.

The transfer matrices of the PI and the PID controllers are given in Table 1.

Case study 1: Set-point tracking

For a sequential unit step change in the set-points at t=0 and t=600h, Figure 2 compares the closed-loop responses afforded by the two controllers. One can see that the proposed PI controller has a faster rising time and settling response over the PID controller. Table 2 shows the performances indices for each approach. It is clear that the PI controller has better performances.

Case study 2: Disturbance rejection

As in [31], a disturbance model G_d is taken as $G_d = \begin{bmatrix} \dfrac{-4.243e^{-0.4s}}{3.445s+1} & \dfrac{-0.601e^{-0.4s}}{1.982s+1} \end{bmatrix}^T$. As shown by Figure 3, a unit step changes in the disturbance were also made to the 1st and 2nd loops at t=0 and t=600h, respectively. It is clear from Table 2, Figures 2 and 3 that the proposed PI controller provides superior performances than the PID controller in such case study.

Case study 3: Parametric uncertainties

The robustness of the controller is also evaluated by inserting a perturbation uncertainty of $\pm 10\%$ in the process gain, time constant and time delay, simultaneously, whereas the controller settings are those provided for the nominal process. As shown by Table 3, the controller settings of the proposed PI method provide superior performances for both case studies: Set-point and disturbances changes.

Table 1: Controllers parameters for the ISP Reactor

Tuning method	Controller parameters
PI	$K_{PI}(s) = \begin{bmatrix} \dfrac{0.2059s+0.0302}{s} & \dfrac{0.3061s+0.0332}{s} \\ \dfrac{0.2062s+0.0292}{s} & \dfrac{0.306s+0.0335}{s} \end{bmatrix}$
PID (with filter)	$K_{PID}(s) = \begin{bmatrix} \dfrac{0.01697s^2+0.01738s-0.0132}{0.1s^2+s} & \dfrac{0.09598s^2+0.3069s+0.0812}{0.1s^2+s} \\ \dfrac{0.01702s^2+0.01785s-0.0135}{0.1s^2+s} & \dfrac{0.09598s^2+0.307s+0.0812}{0.1s^2+s} \end{bmatrix}$

Table 2: Performance indices for the ISP Reactor

Tuning method	Set-point		Disturbance	
	IAE	TV	IAE	TV
PI	174.42	1.97	224.96	1.33
PID (with filter)	440.57	4.93	417.66	1.39

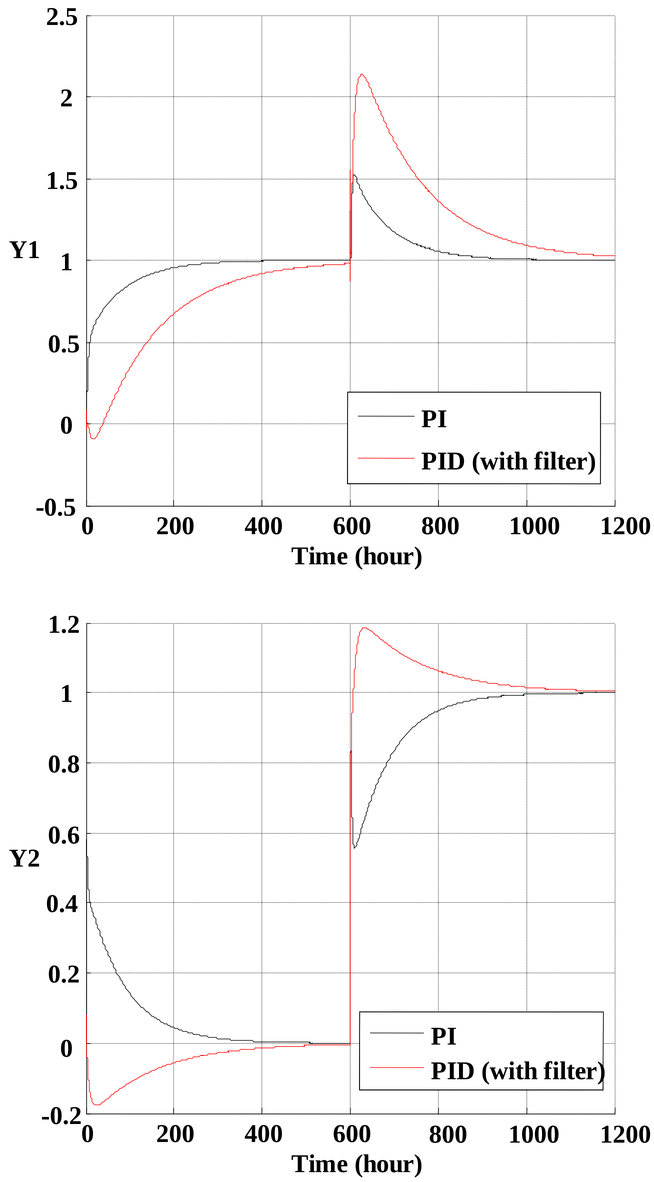

Figure 2. Closed loop responses to unit step changes in the set-point

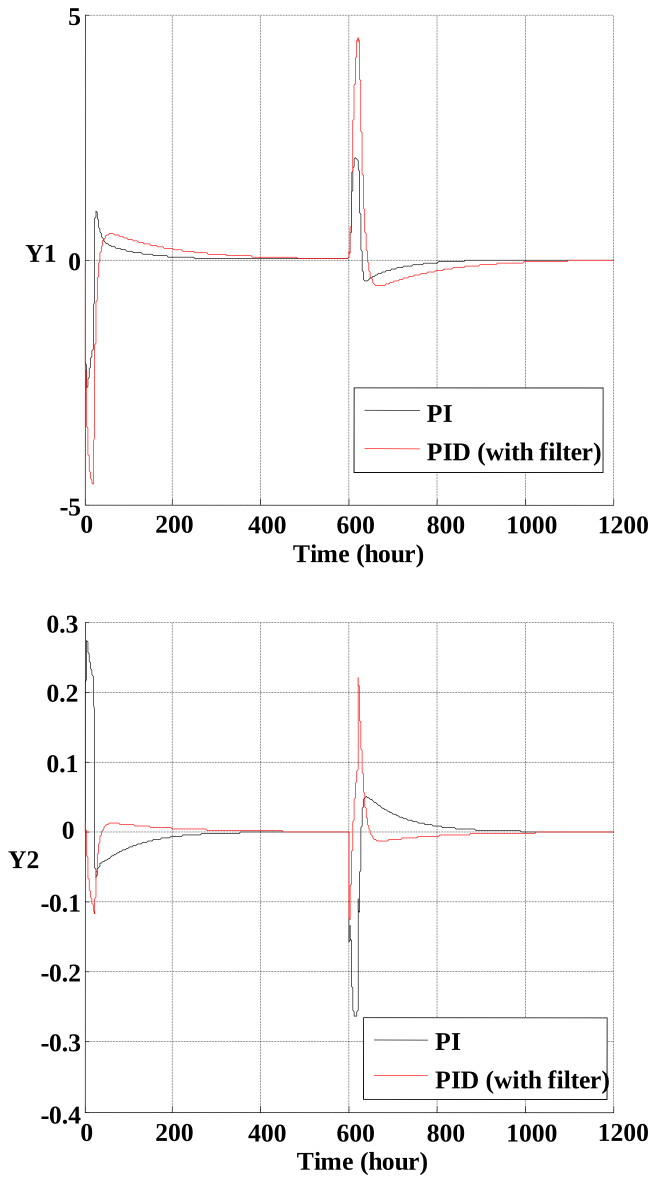

Figure 3. Closed loop responses to unit step changes in the disturbance

Table 3: Robustness analysis under ± 10% parametric uncertainties in all parameters for the ISP Reactor

Tuning method	ISP (+10%)				ISP (-10%)			
	Set-point		Disturbance		Set-point		Disturbance	
	IAE	TV	IAE	TV	IAE	TV	IAE	TV
PI	158.68	1.95	224.06	1.33	193.50	1.94	225.85	1.33
PID (with filter)	404.24	4.82	415.56	1.28	482.68	4.94	418.84	1.52

CONCLUSION

This paper addresses a PI controller design method for multivariable processes with multiple-time delays. A comparative study is established between the proposed approach and a related one for the ISP reactor considering different case studies (set point tracking, disturbance rejection and parametric uncertainties) and using different performance indices. Simulation results prove the superiority of the proposed PI controller over a related approach.

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