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Deriving a New Averaged Error Minimization Method For Dynamic Matrix Control

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Abstract: This paper presents a new and robust measuring mechanism to the least-squares approach used in Dynamic Matrix Control by minimizing distinctively upcoming errors. This approach entails individual recommendation in which a subsequent move is an average of all individual recommendations. This leads to control the set-point and zone where the differential equation yields an overdamped solution. This solution is then used to approximate the time constants and the analytical solution. Time constants are ultimately derived from the quadratic characteristic equation.

Key–Words: Dynamic Matrix Control, Minimizing Errors, Least-squares alternatives.

1 Introduction

Minimizing the sum of the squares of future errors is an essential and highly desirable step in control theory. This work presents an alternative to the least squares optimization used in Dynamic Matrix Control (DMC). Traditionally, calculating future moves is accomplished by minimizing the sum of the squares of the future errors (least squares). Here, each future error is individually minimized. Each minimization results in an individual recommendation for the lone future move and the actuated move would be an average of all the individual recommendations. This paper presents an analytical study of the closed-loop dynamics of the method and it is used here mainly to estimate the closed-loop time constants. The performance of the method is illustrated and compared to a DMC benchmark via simulation.

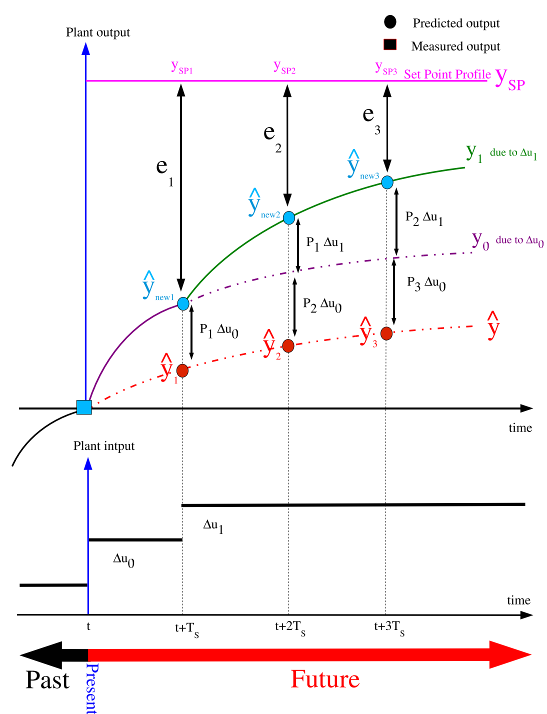

Figure 1: Minimizing Future Errors

1.1 Fromulation

Considering a sampling period Δt and a current time $t = n\Delta t$ leads to a prediction horizon of $T = N\Delta t$ depicted in **Figure 1**. For a simple first order plant situation, we have:

$$\dot{p}(t) + p(t) = 1 \tag{1}$$

So,

$$p(t) = 1 - e^{-t} \tag{2}$$

When sampling $p(t)$ in a unit step open loop test, the notation $P_k = p(k\Delta t) = 1 - e^{k\Delta t}$ is used and the measured output is:

$$y_m^n = \sum_{i=0}^{n-1} \Delta u_i p((n-i)\Delta t) + \ noise \tag{3}$$

Also, the predicted output is then described by:

$$\hat{y}_j^n = \sum_{i=0}^{n-1} \Delta u_i p((n-i+j)\Delta t) \text{and } j = 0, 1, 2... \tag{4}$$

The main result of this paper is presented in the next statement.

Theorem 1 *For a simple first order plant, minimizing individual future error is equivalent to applying the Least - Square apprach.*

2 Averaging Method

In the considered application type, it is then desirable to minimize each future error e_k^n and to average all the recommendations.

$$\Delta u_n = \frac{1}{N} \sum_{k=1}^{N} \frac{\gamma e_k^n}{p_k} \tag{5}$$

where $\gamma \in [0, 1]$.

$$e_k^n = Setpoint - \hat{y}_k^n = 1 - \sum_{i=0}^{n-1} \Delta u_i p((n-i+k)\Delta t) \tag{6}$$

and

$$\Delta u_n = \frac{\gamma}{N} \sum_{k=1}^{N} \frac{1 - \sum_{i=0}^{n-1} \Delta u_i p((n-i+k)\Delta t)}{p(k\Delta t)} \tag{7}$$

$$= \frac{\gamma}{N} \Big[\sum_{k=1}^{N} \frac{1}{p(k\Delta t)} - \tag{8}$$

$$\sum_{k=1}^{N} \frac{\sum_{i=0}^{n-1} \Delta u_i p((n-i+k)\Delta t)}{p(k\Delta t)} \Big] \tag{9}$$

To transform the previous equations from the discrete domain into a continuous one, we apply the following chnage of variables:

$$t = n\Delta t, y = i\Delta t, z = k\Delta t, T = N\Delta t \tag{10}$$

So $\Delta y = \Delta z = \Delta t$.

$$\Delta u_n = \frac{\gamma}{N} \Big[\sum_{z=\Delta t}^{T} \frac{1}{p(z)} - \tag{11}$$

$$\sum_{z=\Delta t}^{T} \frac{\sum_{t=0}^{T-\Delta t} \Delta u_i p(t+z-y)}{p(z)} \Big] \tag{12}$$

$$= \frac{\gamma}{N} \Big[\sum_{z=\Delta t}^{T} \frac{1}{p(z)} - \tag{13}$$

$$\sum_{z=\Delta t}^{T} \frac{1}{p(z)} \sum_{t=0}^{T-\Delta t} \Delta u_i p(t+z-y) \Big] \tag{14}$$

$$= \frac{\gamma}{N} \Big[\sum_{z=\Delta t}^{T} \frac{1}{p(z)} \frac{\Delta z}{\Delta z} - \tag{15}$$

$$\sum_{z=\Delta t}^{T} \frac{1}{p(z)} \Big(\sum_{t=0}^{T-\Delta t} \Delta u_i p(t+z-y) \frac{\Delta y}{\Delta y} \Big) \frac{\Delta z}{\Delta z} \Big] \tag{16}$$

Multiplying both sides by Δz and rearranging the inner sum, one gets:

$$\Delta u_n \Delta z = \frac{\gamma}{N} \Big[\sum_{z=\Delta t}^{T} \frac{1}{p(z)} \Delta z - \tag{17}$$

$$\sum_{z=\Delta t}^{T} \frac{1}{p(z)} \Big(\sum_{t=0}^{T-\Delta t} \frac{\Delta u_i}{\Delta y} p(t+z-y)\Delta y \Big) \Delta z \Big] \tag{18}$$

As $\Delta t \to dt$, we get $\Delta y \to dy$, $\Delta z \to dz$, and $\Delta u_n \to du$. Also note that $dy = dz = dt$.
The continuous form is then given by:

$$dudz = \frac{\gamma}{N} \Big[\int_{dt}^{T} \frac{1}{p(z)} dz - \tag{19}$$

$$\int_{dt}^{T} \frac{1}{p(z)} \Big(\int_{0}^{T-dt} \frac{du}{dy} p(t+z-y)dy \Big) dz \Big] \tag{20}$$

Or equivalently:

$$\frac{N}{\gamma} dudz = \int_{dt}^{T} \frac{1}{p(z)} dz - \tag{21}$$

$$\int_{dt}^{T} \frac{1}{p(z)} \Big(\int_{0}^{T-dt} \dot{u}(y) p(t+z-y)dy \Big) dz \tag{22}$$

$$\tag{23}$$

Define A to be constant:

$$A = \int_{dt}^{T} \frac{1}{p(z)} dz \tag{24}$$
$$= \int_{dt}^{T} \frac{1}{1-e^{-z}} dz \tag{25}$$
$$= \int_{dt}^{T} \frac{-e^{-z}}{-e^{-z}+(e^{-z})^2} dz \tag{26}$$
$$\tag{27}$$

Then:

$$A = T - dt + \ln \frac{p(T)}{p(dt)} \tag{28}$$
$$= T - dt + \ln \frac{P_N}{P_1} \tag{29}$$

Now define $D(t)$ to be

$$D(t) = \int_{dt}^{T} \frac{1}{p(z)} \Big(\int_0^{T-dt} \dot{u}(y) p(t+z-y) dy \Big) dz \tag{30}$$

$$= \int_{dt}^{T} \frac{1}{p(z)} \Big(\int_0^{T-dt} \dot{u}(y)(1-e^{-(t+z-y)}) dy \Big) dz \tag{31}$$

$$= \int_{dt}^{T} \frac{1}{p(z)} \Big(\int_0^{T-dt} \dot{u}(y) dy \Big) dz \tag{32}$$
$$- \int_{dt}^{T} \frac{1}{p(z)} \Big(\int_0^{T-dt} \dot{u}(y) e^{-(t+z-y)} dy \Big) dz \tag{33}$$

$$= [u(t) - u(0)] \overbrace{\int_{dt}^{T} \frac{1}{p(z)} dz}^{A} \tag{34}$$
$$- \underbrace{\Big[\int_{dt}^{T} \frac{e^{-z}}{p(z)} dz \Big]}_{B} \underbrace{\Big[\int_0^{t} \dot{u}(y) e^{-(t-y)} dy \Big]}_{I(t)} \tag{35}$$

Then:

$$D(t) = [u(t) - u(0)]A - B\, I(t) \tag{36}$$

With $B = ln(\frac{P_N}{P_1})$ and A is $T - dt + B$. For $I(t)$ we have:

$$I(t) \quad = e^{-t} \textstyle\int_0^t \dot{u}(y) e^{y} dy \tag{37}$$

Which leads to:

$$\dot{I}(t) \quad = -I(t) + \dot{u}(t) \tag{38}$$

Therefore, the last equation becomes:

$$\frac{Ndz}{\gamma} du = A - [u(t) - u(0)]A + B\, I(t) \tag{39}$$

Starting the system from rest is mathematically equivalent to $u(0) = 0$. Mutliplying and dividing by dt the left hand side, one obtains:

$$\overbrace{\frac{Ndzdt}{\gamma}}^{q} \frac{du}{dt} = A - u(t)A + B\, I(t) \tag{40}$$

Taking the first derivative of both sides and using Equation 38 for $\dot{I}(t)$, a second order ordinary differential equation in u is obtained:

$$q\ddot{u} + (A + q - B)\dot{u} + Au = A \tag{41}$$
$$q\ddot{u} + (T - dt + q)\dot{u} + Au = A \tag{42}$$

A steady-state of $u_{ss}(t) = 1$ in the last equation shows that the new formulation provides control to the set-point. In the zone where the differential equation yields an overdamped solution we can approximate the time constants and the analytical solution. The time constants are found from the quadratic characteristic equation $q\lambda^2 + (T - dt + q)\lambda + A = 0$. So $\Delta = (T-dt+q)^2 - 4qA = (T-dt+q)^2 - 4q(T-dt+B)$. As $dt << 1$, by taking $T - dt + B \approx T + B$, Δ can be approximated by:

$$\Delta = T^2[1 + \frac{Tdt^2}{\gamma^2} - \frac{2dt}{\gamma}(1 + \frac{2B}{T})] \tag{43}$$

As $[\frac{Tdt^2}{\gamma^2} - \frac{2dt}{\gamma}(1 + \frac{2B}{T}) << 1]$, $\sqrt{\Delta}$ can be approximated as:

$$\sqrt{\Delta} = T\big[1 + \frac{1}{2}[\frac{Tdt^2}{\gamma^2} - \frac{2dt}{\gamma}(1 + \frac{2B}{T})]\big] \tag{44}$$

Approximate solutions of the quadratic are:

$$\lambda_1 = \frac{\gamma}{2T} - \frac{q}{4} - \frac{\gamma}{dt} - \frac{B}{T} \tag{45}$$

$$\lambda_2 = \frac{\gamma}{2T} + \frac{q}{4} - 1 - \frac{B}{T} \tag{46}$$

This provides fast and slow time constants defined by: $\tau_{fast} = 1/\lambda_1$ and $\tau_{slow} = 1/\lambda_2$.

The solution of $q\lambda^2 + (T - dt + q)\lambda + A = 0$ is then:

$$u(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \tag{47}$$

where

$$c_1 = \frac{-\dot{u}(0) - \lambda_2}{\lambda_2 - \lambda_1} \tag{48}$$

$$c_2 = \frac{\dot{u}(0) + \lambda_1}{\lambda_2 - \lambda_1} \tag{49}$$

and $\dot{u}(0)$ can be approximated by $\frac{\Delta u_0 + \Delta u_1}{\Delta t}$. Clearly, the very first control move is $\Delta u_0 = \frac{\gamma A}{T}$. The second control move and Δu_1 can be approximated as follow:

$$\begin{aligned}
\Delta u_1 &= \frac{\gamma}{N} \sum_{k=1}^{N} \frac{1 - \Delta u_0 p((k+1)\Delta t)}{p(k\Delta t)} \\
&= \frac{\gamma}{N} \sum_{k=1}^{N} \frac{1}{p(k\Delta t)} - \\
&\quad \frac{\gamma \Delta u_0}{N} \sum_{k=1}^{N} \frac{p((k+1)\Delta t)}{p(k\Delta t)} \\
&\approx \Delta u_0 - \frac{\gamma \Delta u_0}{N} \sum_{k=1}^{N} 1 \\
&= \Delta u_0 (1 - \gamma)
\end{aligned} \tag{50}$$

And an approximation of $\dot{u}(0)$ is then:

$$\dot{u}(0) = \frac{\Delta u_0 (2 + \gamma)}{dt} \tag{51}$$

3 Conclusion

In this paper we presented a new measuring mechanism and proved that is can be used as an alternative to the least-squares approach used in Dynamic Matrix Control. We accomplished this by minimizing distinctively upcoming errors. This new method entailed individual recommendation in such a way that every subsequent move is identified as an average of all individual recommendations. This approach resulted into controlling the set-point and zone where the differential equation yields an overdamped solution. Time constants and the analytical solution are then approximated using the overdamped solution as derived from the quadratic characteristic equation.

References:

[1] H. Bellout, J. Neustupa and P. Penel, On the Navier-Stokes Equation with Boundary Conditions Based on Vorticity, *Math. Nachr.* 269–270, 2004, pp. 59–72.

[2] V. Girault and P.–A. Raviart, *Finite Element Methods for Navier–Stokes Equations,* Springer –Verlag, Berlin–Heidelberg–New York–Tokyo 1986

[3] E. Hopf, Über die Anfangswertaufgabe für die Hydrodynamischen Grundgleichungen, *Math. Nachr.* 4, 1951, pp. 213–231.

[4] T. Kato, Non–stationary flows of viscous and ideal fluids in $\mathbb{R}^3$, *J. Func. Anal.* 9, 1972, pp. 296–305.

[5] J. Serrin, On the interior regularity of weak solutions of the Navier–Stokes equations, *Arch. Rat. Mech. Anal.* 9, 1962, pp. 187–195.