International Journal of Enhanced Research in Science Technology & Engineering, ISSN: 2319-7463
Vol. 3 Issue 3, March-2014, pp: (127-131), Impact Factor: 1.252, Available online at: www.erpublications.com

Periodic Vibro-Impact Mode of Motion of Spherical Particles along the Sine Curve

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Abstract: Theoretical studies of periodic vibro-impact motion of a spherical particle at the surface of the small size separator deck constructed in the form of a gutter with a sinusoidal profile and supplied with a reflector in the lower part.

Keywords:Periodic motion, Particle oscillation on the profile of the deck,Reflective plate, Small-size deck.

1. Introduction

One of the ways to increase productivity of sieveless separators is to use a small size deck [1]. Technological characteristics of particle motion of the material to be processedwith flat surface decks studied in [2-5]. Designing deck surfaces asgutter with a curved profile will give an additional advantage - the ability to influence the process of centrifugal force, which is controlled by the shape of the profile (cross-sectional shape). This requires a study into the process of separation on the basis of a mathematical model of seed movement along curved surfaces.

2. Analysis of recent research

Supplyinga small-sized deck with a reflector and its periodic vibrationprovides an opportunity to use periodic vibro-impact mode of movement for sorting seedmixtureshigh stability[4]. Movement of seeds in this mode, in a rectilinear profile deck was studied in [2-5].The movement of seeds on the small size deck with a circle-shaped cross-section was analysed in [6-7].

3. The objective

The task of findingthe rational profile for the small size deck is related to the study and comparison of the characteristics of seed movement on deck surfaces with different cross-sectional shapes: circle, parabola, hyperbola, etc. In this paper, we shall study the case of sinusoidal profileof the cross section.

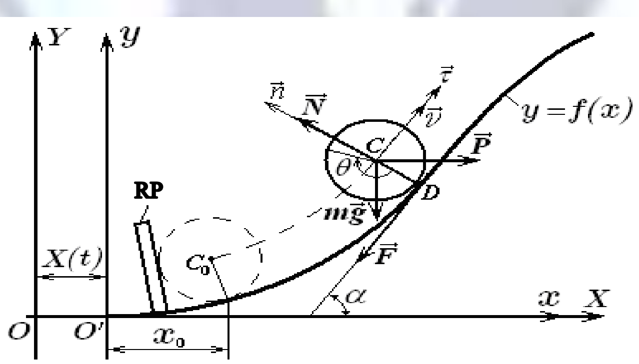

Fig. 1 : Computational scheme of particle motion.

4. Research Methods

The computational scheme of vibrational motion of a spherical particle is presented in (Fig. 1) The deck movement is measured from the XOY fixed coordinate system. The *xÓy* relative coordinate system is rigidly connected to the deck and moves together with it undergoing harmonic motion$X(t)=A\sin\omega t$, where:A,ω - amplitude and frequency of vibrations of the deck. The motion is carried out in the horizontal plane along the OX axis. The cross-sectional shape of the deck is given by the equation $y=f(x)$ (hereinafter - the sinusoid). The reflective plate [RP] is fixed in the lower part of the deck at a distance from the origin.Upon contact of a particle with the plate they collide, the particle reboundsand rolls along the deck until yet again another collision will not occur, etc. Moreover, in case of such periodic motion the interval between two sequential collisions of the deck equals to the oscillation period: T=2π/ω.

The particle moving in contact with the deck, in the relative coordinate system has an impact of the following forces (Fig.1): gravity $m\vec{g}$, normal reaction $\vec{N}$, inertial force $\vec{P}$ and frictional force $\vec{F}$ .Against this background differential equations of particle motion in natural coordinates $\vec{n}$, $\vec{\tau}$ can be written as follows:

$$\frac{mV^2}{\rho - r} = N - mg\cos\alpha - P\sin\alpha; \quad m\ddot{S} = P\cos\alpha - mg\sin\alpha - F; \quad I\ddot{\theta} = Fr\,. \tag{1}$$

where: ρ, α – the radius of curvature of the curve and tangent tiltat the contact point of the particle with the profile; r – the radius of the particle.

Let us use a known relation $\ddot{S} = r\ddot{\theta}$ and let $\mu = (r_i/r)^2$. Then the last of equations (1)yield $F = m\mu\ddot{S}$. Let us superpose time origin with the point of particle collision with the plate.Then the force of inertia can be written as $P = mA\omega^2\sin\omega(t + t_y)$,where: (r_i –radiusofinertiaof a particle, $\omega t y$ – impactphase.Further, excluding the F friction force from the second equation of system (1), we obtain:

$$(1+\mu)\ddot{S} = A\omega^2\sin\omega\left(t + t_y\right)\cos\alpha - g\sin\alpha\,. \tag{2}$$

Here, the angle of slope α of a tangent (see Fig. 1) can be expressed in terms of abscissa of the point of contact of particle D with the profile through derivatives:

$$\cos\alpha = 1\Big/\sqrt{1+y'^2}\,; \quad \sin\alpha = y'\Big/\sqrt{1+y'^2}\,. \tag{3}$$

To completely pass into equation (2) from natural coordinate S to x coordinate, we shall express S through the length $l$of a curve $y = f(x)$ between the initial ($x = x_0$) and the current points of contact of a particle with the profile. Particle motion at any specific timecan be considered as instant along circumferential curvature, sharing curvature,common tangent and sense of curvaturewiththe curve $y = f(x)$ at the point of contact D . Given this, the following relation holds:

$$\frac{dS}{\rho - r} = \frac{dl}{\rho}\,. \tag{4}$$

The radius of curvature of the arc and the differential curve can be defined in a standard way:

$$\rho = \left(1+y'^2\right)^{\frac{3}{2}}\Big/\left|y''\right|; \quad dl = \sqrt{1+y'^2}\,dx\,. \tag{5}$$

Given relations (4), (5) we sequentially obtain:

$$S = \int_{x_0}^{x}\left(\sqrt{1+y'^2} - \frac{y''r}{1+y'^2}\right)dx\,; \quad \dot{S} = \left(\sqrt{1+y'^2} - \frac{y''r}{1+y'^2}\right)\dot{x}\,; \tag{6}$$

$$\ddot{S} = \left(\frac{y'y''}{\sqrt{1+y'^2}} - \frac{y'''(1+y'^2) - 2y'y''^2}{(1+y'^2)^2}r\right)\dot{x}^2 + \left(\sqrt{1+y'^2} - \frac{y''r}{1+y'^2}\right)\ddot{x}\,. \tag{7}$$

We consider that within the given interval $y''>0$ – the sinusoid is concave.

We can use expressions(2), (3) and (7)to make a differential equation describing the position change of the contact point D with time:

$$\left(1+y'^2 - \frac{y''r}{\sqrt{1+y'^2}}\right)\ddot{x} + \left(y'y'' - \frac{y'''(1+y'^2) - 2y'y''^2}{(1+y'^2)^{3/2}}r\right)\dot{x}^2 + \frac{1}{1+\mu}\left(g\,y' - A\omega^2\sin\omega\left(t + t_y\right)\right) = 0\,. \tag{8}$$

Almost all the values in this equation (except μ and y') are dimensional. This causes certain inconveniences when using numerical methods for its solution. Hencewe shall use the dimensionless time τ , which is expressed as a fraction of the oscillation period and the dimensionless coordinates deck χ, η - in fractions of the oscillation amplitude. To denote the derivatives of the dimensionless coordinate dimensionless time instead of "points" we shall use the "asterisk". Thus, for the transition to dimensionless variables we shall use the following notations:

$$t = T\tau, \quad x = A\chi, \quad y = A\eta \tag{9}$$

and their derived relations:

$$\dot{x} = \frac{dx}{dt} = \frac{A\,d\chi}{T\,d\tau} = \frac{A}{T}\overset{*}{\chi}; \qquad \ddot{x} = \frac{d\dot{x}}{dt} = \frac{A}{T}\cdot\frac{d\overset{*}{\chi}}{T\,d\tau} = \frac{A}{T^2}\overset{**}{\chi}\;; \tag{10}$$

$$y' = \frac{dy}{dx} = \frac{A\,d\eta}{A\,d\chi} = \frac{d\eta}{d\chi} = \eta'; \qquad y'' = \frac{dy'}{dx} = \frac{d\eta'}{A\,d\chi} = \frac{1}{A}\eta''; \qquad y''' = \frac{dy''}{dx} = \frac{1}{A}\frac{d\eta''}{A\,d\chi} = \frac{1}{A^2}\eta'''. \tag{11}$$

Then the differential equation (8) can be rewritten in order to make it more convenient for use:

$$\overset{**}{\chi} = \frac{1}{1+\eta'^2-\frac{\eta'' r_A}{\sqrt{1+\eta'^2}}}\left[\left(\frac{\eta'''\left(1+\eta'^2\right)-2\eta'\eta''^2}{\sqrt{\left(1+\eta'^2\right)^3}}r_A-\eta'\eta''\right)\overset{*}{\chi}^2+\frac{4\pi^2}{1+\mu}\left(\sin 2\pi\left(\tau+\tau_y\right)-\frac{\eta'}{K}\right)\right], \tag{12}$$

where $r_A = r/A$ – the ratio of the particle radius to oscillation amplitude.

$K = A\omega^2/g$ – coefficient of kinematic mode of the deck motion.

With the use of equations(1)and notations (9) - (11)we canget the expression for the friction force and the normal reaction also in dimensionless form:

$$F^* = \frac{F}{mg} = \frac{\mu}{(1+\mu)\sqrt{1+\eta'^2}}\left[K\sin 2\pi\left(\tau-\tau_y\right)-\eta'\right], \tag{13}$$

$$N^* = \frac{N}{mg} = \frac{1}{\sqrt{1+\eta'^2}}\left[\frac{K\eta''}{4\pi^2}\left(1-\frac{\eta'' r_A}{\sqrt{(1+\eta'^2)^3}}\right)\overset{*}{\chi}^2+K\eta'\sin 2\pi\left(\tau+\tau_y\right)+1\right]. \tag{14}$$

Values F^*, N^* should be used to control the motion of a particle mode. In this case we can apply a technologically justified non-separatedmotion mode in the absence of slippage. This is achieved if $N^* > 0$ – movement without interruption and if $\left|F^*\right| < F^*_{\max} = f\,N^*$ – without slip(f – coefficient of friction).

Equation (12) can be solved if we know the shape of the deck profile set in dimensionless coordinates: $\eta = \Phi(\chi)$. We shall consider a sinusoidal profile for which we can write:

$$\eta = \kappa_1 \sin\left(\kappa_2\chi - \pi/2\right) + \kappa_1, \tag{15}$$

where κ_1, κ_2 - dimensionless coefficients.

It is clear that the substitution of equations (15) into (12)-(14) will lead to their usual form and simplify the numerical treatment. However, their main advantage is lost - applicability to profiles of any shape. In addition, the availability of symbolic processors, now used in many software like (Mathcad, Mathematica, Maple, etc.), allows, with the exception of built-in/firmware, enter the user functions and record their generic terminto the program text.

On this basis/Given this, with consideration to relations (12)-(14), we have developed a Mathcad-program for the study of periodic motion of a particle on the small size vibrating deck with an arbitrary cross-sectional profile. When running this program, it is only required to enter the equation of the test profile.

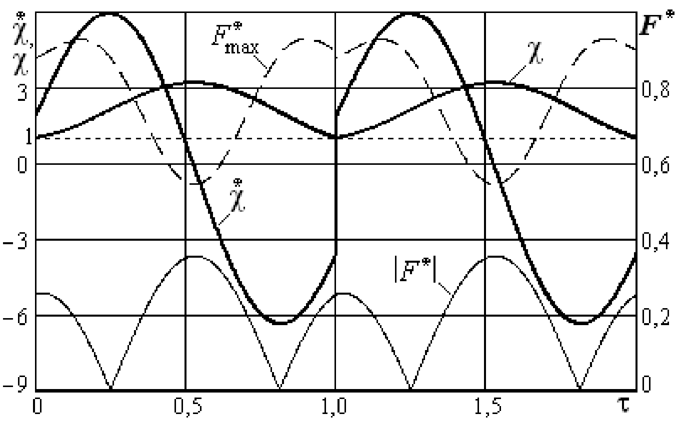

Figure 2. Characteristics of particle motion

5. Results

Figure (2) displays the characteristics of particle motion in a sinusoidal profile(15). The particle velocity before and after the collision with the reflective plate was connectedby the knownrelation$V = -RU$ (R – coefficient of restitution speed).To make calculations we assumed: $\chi_0 = 1$; $r_A = 0,2$; $\kappa_1 = 3$; $\kappa_2 = 0,18$; $f = 0,8$; $R = 0,5$; $K = 1$; $\mu = 0,4$. As can be seen, for the accepted values of parameters of a condition $|F^*| < F^*_{\max}$ and $N^* > 0$ are fulfil and hence, the particle moves along the deck in a periodical non-separated mode without slip.

The most 'unsafe' area of motion,where slippageand even separation of the particle from the deck surface may occur, is the area of maximum distanceof the particle from the reflective plate where the particle changes the direction of motion, and the reaction N^*isat least value). This is because in the given area particle velocity is close to zero andthe inertia of the centrifugal force that presses the particle towards the surface of the deck, has almost no effect. Deviation of the particle χ from the neutral position reaches the maximal possible value about half way through the end of the course of particle movement. At the end of each period the deviation graph has a break, and the speed $\overset{*}{\chi}$ demonstrates a sudden change of value which is a consequence of collision.

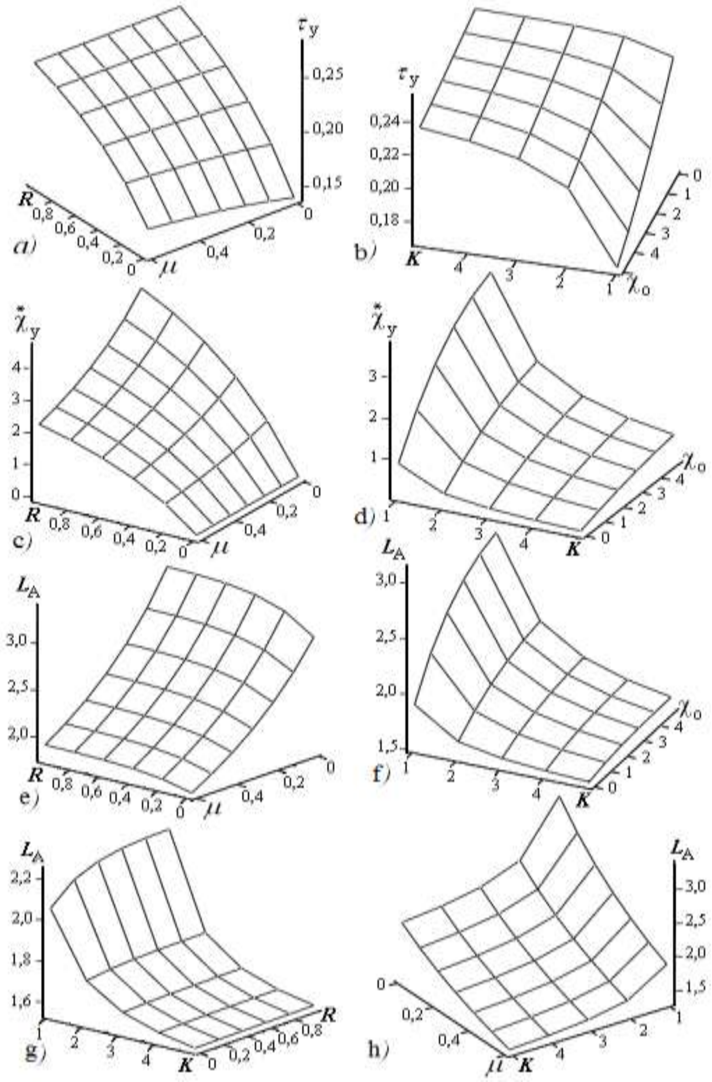

Figure 3: Dependence of the impact time (τ_y), rebounding velocity ($\overset{*}{\chi}_y$) and amplitude of vibration the particle(L_A) on parameters μ, R, χ_0, K.

Figure (3) shows the graphs of the calculated change in the impact time τ_y, the rebounding velocity of the particle after the impact $\overset{*}{\chi}$ and the amplitude of oscillation of the particle along the deck $L_A = L/A$ in relation to the indicatorof inertia of the particle rotation μ; coefficient of speed restitution upon impact R; initial deflection of the particle $\chi_0 = x_0/A$, determined by the position of the reflective plate (Fig. 1) and the coefficient of kinematic mode K.

Rotational inertia (μ) inhibits movement of the particles, reducing the scope L_A (Fig. 3d). In this case, the rebounding speed decreases (Fig. 3c), and the collision time increases (Fig. 3a).

Particle elasticity (R) increases the amplitude of oscillation L_A (Fig. 3d) and rebounding velocity (Fig. 3 c), which is to be understandable. With the increase of elasticity the particle receives more energy upon impact, which is counterbalanced by an increase at the moment of collision τ_y (Fig. 3 a) and, thereby, decrease in the rate of the reflective plate at the moment of impact, which is proportional to $\cos 2\pi\tau_y$. With $R=1$ (regardless of the parameter μ value) $\tau_y = 0.25$, which indicates that given absolute elasticity the periodic motion mode is also possible even at a fixed deck.

The increase of the initial deviation χ_0 results in the increase of rebounding speed $\overset{*}{\chi}$ while reducing the moment of impact (Fig.3 b, d). This isdue to the need to overcome the gravity component, which increases with the increase of the χ_0 value. With the increase in intensity of vibrations (K) this component is mainly overcome due to the increase of energy transferred to the particle at the moment of impact, and the role of the χ_0 parameter is much lowered.

An increase in vibration intensity (K) leads to reduction of the oscillation amplitude of the particle movement along the deck (Fig. 3e), which is consistent with the results of the particle movement along a rectilinear profile [2-5]. But, in case of sinusoidal profile the increase of intensity leads to violation of non-separated movement. A change in the initial bias χ_0 affects the amplitude L_A (Fig. 3 e, f, g), depending on the values of other parameters.

6. Conclusions

On the basis of the resulting differential equation in the environment of Mathcadwe have developed a program that describes non-separated periodic vibro-impact mode of movement of a rounded particle along the vibrating gutter-shaped deck with a profile of arbitrary shape, provided with a reflective plate.It has been demonstrated that in case of a sinusoidal profile deck,The violation of the given mode most probably occurs in the area of maximum particle removal from reflective plate, where the normal reaction has a minimum value.

The amplitude of the vibrational motion of the particles along the deck has proved decisive for separation of mixtures in small size deck. Elastic particles have greater amplitude than those inelastic, suggesting possibility of particle separation on small size decks according to their physical and mechanical properties with sinusoidal profile.

The best separation effect of the mixture for the considered mode of particle movement in a sinusoidal profile can be achieved once the values of the coefficient of the kinematic regime are closest to the lower limit. At high values of the coefficient, the difference in movement trajectories of the particles having different properties, comes to nought, thus making mixture separation almost impossible.

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