Mathematical Model of Reliability Assessment for Traffic Control System

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Abstract: Traffic Control and management poses a major problem in urban cities, especially in rapidly growing & motorizing cities. In urban cities at road intersection traffic lights (Red, Green and Yellow) is used for controlling the congestion. For this there a set up of complete system called traffic control system at the road intersection. But there is chance of failure of individual unit of traffic control system. Reliability of traffic control system is therefore the most important feature to maintained above some acceptable threshold value. The maintenance of individual units can also be planned and implemented once the level of reliability for given instance of time is known. The paper therefore aims at determining the threshold reliability of traffic control system. In this paper mathematical model for calculating the reliability of traffic control system is designed.

Keywords- Traffic Control System, Mathematical Model, Reliability.

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Introduction

Optimization of functioning of traffic signal is considered one of the most effective measures to address road congestion problem. Traffic management is one of the measure term of road security. However the optimization of signal timing is complex due to randomness complexity and nonlinearity of the transportation system. Congestion problem on road can be avoided by using effective and reliable traffic control system. It is therefore drew the attention of researcher to determine the condition when reliability of traffic control system would be at stake. Therefore, it needed a model to deal with the reliability of traffic control system. Moved by these ideas the research for accessing the reliability of traffic system has been undertaken. The reliability is based on incremental reliability of its parts like error detector element, feedback element, traffic light controller and lights (Red, Yellow and Green). This paper therefore demonstrates a mathematical model for determining the equivalent reliability by taking each of its components all together.

Proposed Problem

It is aimed to assess the reliability of traffic control system consisting of various units like sensor, error detector element, feedback element, controller and lights. As shown in fig.1, obtain the mathematical model of equivalent reliability, Req.

Description of Traffic Control System

The main components of traffic control system are sensor, traffic controller and traffic lights. The block diagram for Traffic Control System can be shown as:
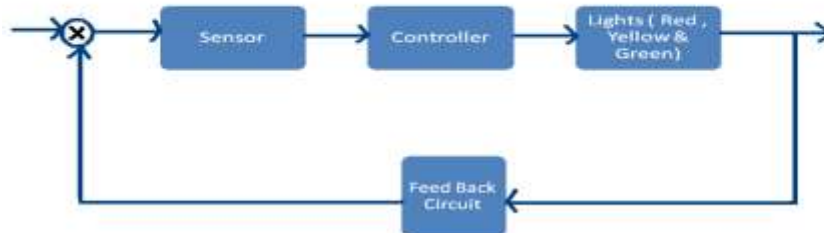
Fig.:- Block Diagram for Traffic Control System

It is required that the traffic control system should work without fail and uninterruptedly but such an operation could only be possible when every component of system works with acceptable reliability. Whole system for traffic control is mainly divided in to two parts. First part consists of sensor and error detector element and failure of controller and lights. The sensor is used to sense the inputs and gives output to controller. The error detector element compares the two inputs and gives the error output to sensor. The controller will give the required information to traffic lights. With the consideration of output of controller the traffic signal (Red, green and yellow) will glow .In order to get effective control it is required that system and its components should maintain a reliability $\geq R_{th}$.

It has been aimed to develop mathematical model for reliability assessment of traffic control system. The reliability of generation system basically depends on each component of the system. It is therefore a mathematical model has been developed in this dissertation by taking into account the reliability of individual component as partial reliability and there upon deriving of expression for equivalent reliability of generation system as a whole.

Mathematical Approach

The mathematical model for reliability assessment has been developed by way of equivalent reliability approach. It represents the overall reliability of traffic control system. Also it consists of resultant effect of reliability due to individual components.

1.   Expected Failure In Traffic Control System

In traffic Control System failures which affect overall system reliability are:

Error Detector element, Sensor, Light Controller and Signal Lights (Red, Yellow and Green).Each of these constitute small components and it is therefore the reliability of these parts are function of partial reliability of internal parts. For example: The generator is globally reliable only when all of its elements like generator winding, Excitation, Rotor etc. works reliability. Thus we can say that reliability of above system depends upon so many factors. The equivalent reliability model has been shown in Fig.2.
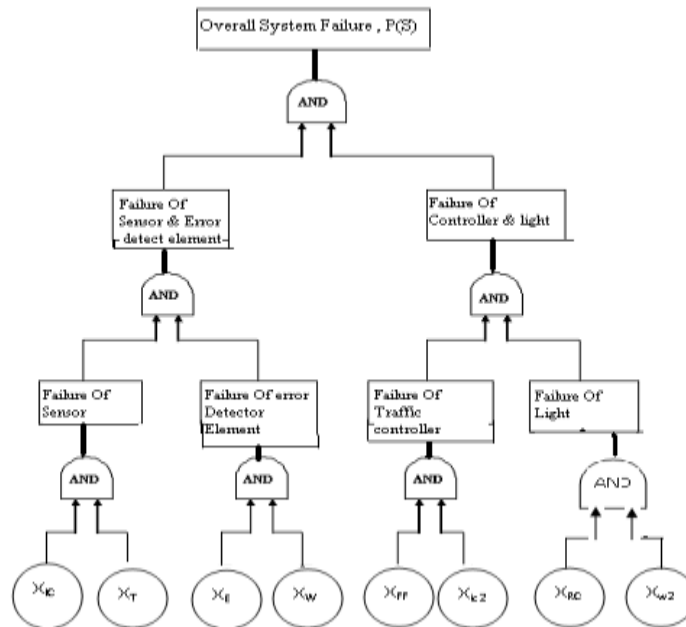
Fig.2. Equivalent Reliability Model For Traffic Control System

Reliability Rules:

a)   If the failure of an element depends on many factors, AND gate is a logical choice.

b)   If failure of an element is independent of many factors. OR gate is logical choice.

Reliability of sensor depends on temperature and connecting wire. Sensor is temperature dependant device. If temperature of sensor exceeds a certain limit than it would not work effectively. Hence it will also affect the overall reliability. Since the objective is dependent on both the factors, the AND gate has been used. In case of error detector element the objective is that it should give the correct difference between the input and feedback output, which is possible only when feedback element is reliable and connecting wires are working properly. Since the objective depends on both the factors, therefore AND gate is used. In part 2, the failure of system is dependent on working of controller and Signal lights. Thus logically AND gate is used to represent the failure of pat 2. For overall system reliability logical AND operation is used between part1 and part2.

2.   Proposed Mathematical Model Of Equivalent Reliability

As discussed above, the reliability of overall traffic control system depends upon so many factors individually and collectively, which is function of time. After appreciating the equivalent reliability model, it is required to find the

mathematical model for individual unit of traffic control system as well as equivalent reliability of overall traffic control system. In this process some terms has been used frequently. These have been abbreviated as below:

3.   Mathematical Model For Part 1

It is obvious from Fig 2.that the failure of part1 occurs when failure of sensor and error detector element occurs. Failure have been indicated by the bar above the letter e.g. S, the probability of failure of sensor shall be P(S),then the probability of failure of sensor is given by:

$$P(S) \quad = \quad P(\overline{X_{IC}} \text{ AND } \overline{X_T})$$

$$= \quad P(\overline{X_{ic}}).P(\bar{X}_T) \qquad\qquad (1)$$

If the reliability of sensor is indicated by R_S, then the reliability of sensor is given by

$$R_S = \quad 1 - P(\overline{X_{IC}}) \cdot P(\overline{X_T})$$

$$= \quad 1 - (1 - R_{IC})(1 - R_T)$$

$$= \quad R_{IC} + R_T - R_{IC}R_T \qquad\qquad (2)$$

Since the reliability of every component decreases with time, hence negative exponential function will be used. Therefore

$$R_{IC} = \exp(-\lambda_1 t) \quad \text{and}$$

$$R_T = \exp(-\lambda_2 t)$$

where, λ_1 represents the failure rate of integrated circuit and λ_2 represents failure rate due to rise in temperature.

Thus, applying the usual probability laws, we have

$$R_S = \exp(-\lambda_1 t) + \exp(-\lambda_2 t) - \exp(-\lambda_1 t) . \exp(-\lambda_2 t) \qquad (3)$$

a)   Mathematical model for error detector element:

Similarly, for error detector, if the probability of failure of error detector is indicated by P(E) , then

$$P(E) = \quad P(X_{FE} \text{ AND } \overline{X_W})$$

$$= \quad P(\overline{X_{FE}}).P(\overline{X_W}) \qquad\qquad (4)$$

Where, $P(\overline{X_{FE}})$ represents the probability of failure of feedback circuit and $P(\overline{X_W})$ represents the probability of failure of connecting wire.

Here R(E) indicates the reliability of error detector and is given by:

$$R(E) = 1 - P(\overline{X_{FE}}).P(\overline{X_W})$$

$$= 1 - (1 - R_{FE}).(1 - R_W)$$

$$= R_{FE} + R_W - R_{FE}.R_W \qquad (5)$$

where, R_{FE} and R_W are the reliabilities of the feedback circuit and the connecting wires respectively. As the reliability of both these elements decreases with time, therefore

$$R_{FE} = \exp(-\lambda_3 t) \quad \text{and}$$

$$R_W = \exp(-\lambda_4 t)$$

where λ_3 is failure rate due to feedback circuit and λ_4 is the failure rate of connecting wires. Both these function decreases with time. Applying usual probability laws we have

$$R_E = \exp(-\lambda_3 t) + \exp(-\lambda_4 t) - \exp(-\lambda_3 t).\exp(-\lambda_4 t) \qquad (6)$$

Equivalent reliability for part 1 can be calculated as:

$$P(P1) = P(\overline{X_S} \text{ AND } \overline{X_E})$$

$$= P(\overline{X_S}).P(\overline{X_E})$$

$$R(P1) = 1 - P(\overline{X_S}).P(\overline{X_E})$$

$$= 1 - [(1 - R_S).(1 - R_E)]$$

$$= 1 - [(1 - (\exp(-\lambda_1 t) + \exp(-\lambda_2 t) - \exp(-\lambda_1 t).\exp(-\lambda_2 t)))(1 - (\exp(-\lambda_3 t) + \exp(-\lambda_4 t) - \exp(-\lambda_3 t).\exp(-\lambda_4 t)))] \qquad (7)$$

b)   Mathematical model for Controller:

The reliability of controller also depends upon the integrated circuit used in it and the feedback element. Hence the reliability for the controller used in traffic control system is

$$R(C) = R_{IC} + R_{FE} - R_{IC}.R_{FE}$$

By using value for R_{IC} and R_{FE} from previous expression, we get

$$= \exp(-\lambda_1 t) + \exp(-\lambda_3 t) - \exp(-\lambda_1 t).\exp(-\lambda_3 t) \qquad (8)$$

c)   Mathematical model for Signal Lights

The reliability of traffic control system also depends on the light source used for providing signals. The failure of light source depends upon the components like reflector cup and diode used in light source. Hence probability of failure of light source is

$P(LS)$ = $P(\overline{X_{RC}} \text{ AND } \overline{X_D})$

= $P(\overline{X_{RC}}).P(\overline{X_D})$ (9)

Reliability of light source R(LS) is given by

$R(LS)$ = $1 - P(\overline{X_{RC}}).P(\overline{X_D})$

= $1 - (1 - R_{RC})(1 - R_D)$

= $R_{RC} + R_D - R_{RC}.R_D$ (10)

Where, $R_{RC} = \exp(-\lambda_5 t)$ and $R_{D} = \exp(-\lambda_6 t)$

where, λ_4 = failure rate or reflector cup and λ_5 = failure rate of diode.

Applying usual probability law we have

$R(LS) = \exp(-\lambda_5 t) + \exp(-\lambda_6 t) - \exp(-\lambda_5 t).\exp(-\lambda_6 t)$ (11)

Equivalent reliability for part 2 is given by

$R(P2)$ $= 1 - P(X_C).P(X_{LS})$

$= 1 - [(1 - R_C).(1 - R_{LS})]$

$= 1 - [(1 - (\exp(-\lambda_1 t) + \exp(-\lambda_3 t) - \exp(-\lambda_1 t).\exp(-\lambda_3 t)))(1 - (\exp(-\lambda_5 t) + \exp(-\lambda_6 t) - \exp(-\lambda_5 t).$

$\exp(-\lambda_6 t)))]$ (12)

Overall reliability of traffic control system is given by:

$R_{eq} = 1 - P(P1).P(P2)$

$= 1 - [(1 - R(P1)).(1 - R(P2))$

By putting the values of R(P1) and R(P2) from equation 7 and 11 respectively, we get the equivalent reliability as:

$R_{eq} = 1 - [(1 - 1 - [(1 - (\exp(-\lambda_1 t) + \exp(-\lambda_2 t) - \exp(-\lambda_1 t).\exp(-\lambda_2 t)))(1 - (\exp(-\lambda_3 t) + \exp(-\lambda_4 t) - \exp(-\lambda_3 t).\exp(-\lambda_4 t)))]*(1 - 1 - [(1 - (\exp(-\lambda_1 t) + \exp(-\lambda_3 t) - \exp(-\lambda_1 t).\exp(-\lambda_3 t)))(1 - (\exp(-\lambda_5 t) + \exp(-\lambda_6 t) - \exp(-\lambda_5 t).\exp(-\lambda_6 t)))]$
(13)

Equation (13) represents the equivalent model of reliability for overall traffic control system. It takes into account the reliability of individual model in a collective manner. Furthermore the model is a function of time.

Conclusion

The mathematical model is designed for calculating the reliability of the traffic control system. The threshold reliability is calculated from the various equations used in the paper and is also demonstrated through some required figures.

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