

Trilokesh Method of Triangles

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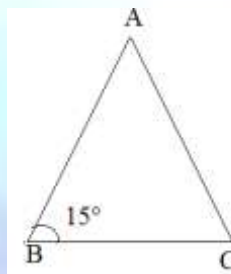
Abstract: In this manuscript the authors have evaluated a method to find the third side of triangles very simply by simple calculations.

Keywords: perpendicular, vertex, expanding, numerator, evaluate, constant.

INTRODUCTION

This method evaluates the third side of a triangle. The author is aware of large calculations and use of trigonometric tables. So he derived a new method which will help in determining the third side of a triangle without doing such a large calculations. Other methods such as Pythagoras theorem are applicable only on right triangles but this method is applicable on all triangles but author has given some constants (only for common angles). In fact Pythagoras theorem is also a special case of this formula.

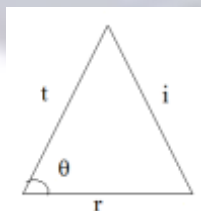
Given a triangle with side AB=4 cm, BC=3 cm and $\angle B = 15^\circ$. Can you calculate the side AC?



You will find it difficult to calculate but by using Trilokesh method you can calculate it easily.

Let you know how to calculate this

Consider a triangle with side as t, r and i. the angle between t and r is θ . side t and r are given. We have to calculate the third side i.e. i.



Here is a table which will help you in calculations.

Angle (θ)	15°	22.5°	30°	45°	60°	67.5°	75°	90°
Constant (Original) (K)	$\sqrt{2 + \sqrt{3}}$	$\sqrt{2 + \sqrt{2}}$	$\sqrt{2 + \sqrt{1}}$	$\sqrt{2 + \sqrt{0}}$	$\sqrt{2 - \sqrt{1}}$	$\sqrt{2 - \sqrt{2}}$	$\sqrt{2 - \sqrt{3}}$	$\sqrt{2 - \sqrt{4}}$
(K) in decimal	1.93	1.84	1.73	1.41	1	0.76	0.51	0

Angle (θ)	105°	112.5°	120°	135°	150°	157.5°	165°	180°
Constant original (K)	$\sqrt{2-\sqrt{3}}$	$\sqrt{2-\sqrt{2}}$	$\sqrt{2-\sqrt{1}}$	$\sqrt{2+\sqrt{0}}$	$\sqrt{2+\sqrt{1}}$	$\sqrt{2+\sqrt{2}}$	$\sqrt{2+\sqrt{3}}$	$\sqrt{2+\sqrt{4}}$
Constant decimal	0.51	0.76	1	1.41	1.73	1.84	1.93	2

The formula is (0° to 90°)

$$t^2 + r^2 - K(\theta) * t * r = i^2 \quad (\text{method 1})$$

The formula is (90 to 180)

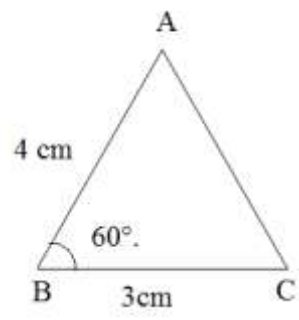
$$t^2 + r^2 + K(\theta) * t * r = i^2 \quad (\text{method 2})$$

where
 t = side of triangle (given),
 r = side of triangle (given),
 I = side of triangle (to find),
 K = constant at particular angle (θ)
 θ = angle between side t and r

Note: using values of K in points is more convenient than using them in roots. Constant on all other angles also exist but they will make formula more complex and they also do not have a series like these. These constants are sufficient to evaluate third side of common angles.

Let us take some examples to understand more clearly.

➤ Consider a triangle ABC with $AB=4\text{cm}$, $BC=3\text{cm}$ and $\angle B=60^\circ$. Find AC



Solution:

Since angle is below 90° so we will apply method 1 i.e.

$$(t^2 + r^2 - k_{\theta} * t * r = i^2)$$

Here $AB=4\text{ cm}$ (t)
 $BC=3\text{ cm}$ (r)
 $\theta = 60^\circ$

Since $\theta = 60^\circ$ so value of $K=1$
 By applying Trilokesh method 1

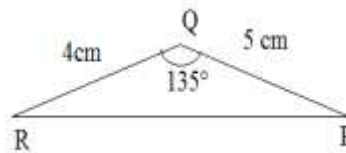
$$(t^2 + r^2 - k_{\theta} * t * r = i^2)$$

On putting values we get,

$$\begin{aligned} 4^2 + 3^2 - 1 * 4 * 3 &= i^2 \\ 16 + 9 - 12 &= i^2 \\ 25 - 12 &= i^2 \\ 13 &= i^2 \\ &= i \\ 3.60 &= i \end{aligned}$$

Hence, the third side is 3.60 cm.

➤ Given a triangle PQR with PQ = 5 cm, QR = 4 cm, $\angle Q = 135^\circ$. Find PR.



Solution:

Since angle is above 90° so we will use method 2

$$(t^2 + r^2 + K(\theta) * t * r = i^2)$$

Here, PQ = 5 cm (t)
QR = 4 cm (r)
 $\theta = 135^\circ$

Since $\theta = 135^\circ$ So value of K = 1.41

By applying Trilokesh method 2,

$$(t^2 + r^2 + K(\theta) * t * r = i^2)$$

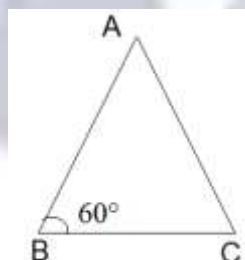
On putting values we get,

$$\begin{aligned} 5^2 + 4^2 + 1.41 * 5 * 4 &= i^2 \\ 25 + 16 + 28.2 &= i^2 \\ 69.2 &= i^2 \\ \sqrt{69.2} &= i \\ 8.31 &= i \end{aligned}$$

Hence the third side is 8.31 cm.

Here is one more interesting theorem

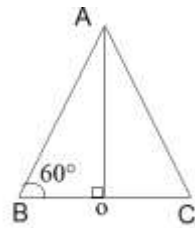
According to this theorem in a triangle having one of its angle 60° , the square of the opposite side to it is equal to the ratio of sum of cubes of other two sides to the sum of these two sides.



In this triangle $(AC)^2 = \frac{(AB)^3 + (BC)^3}{(AB) + (BC)}$

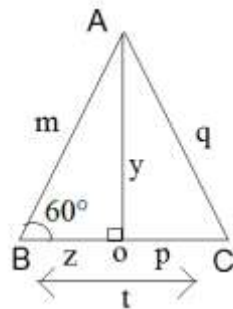
Proof:

Given: ABC is a triangle having $\angle B = 60^\circ$



To prove:

Construction : Construct a perpendicular from the vertex A on its opposite side BC. Let it be AO.



Let $AB = m$, $AC = q$, $BC = t$, $BO = z$ and $OC = p$

Now in ΔABO ,

$$BO = AB \cdot \cos 60^\circ \text{ or } z = m \cdot \cos 60^\circ$$

$$Z = m/2 [\cos 60^\circ = 1/2] \dots\dots\dots(1)$$

Also, by Pythagoras theorem,

$$m^2 - z^2 = y^2$$

From (1),

$$m^2 - [m/2]^2 = y^2$$

$$\frac{4m^2 - m^2}{4} = y^2$$

$$= \dots\dots\dots(2)$$

Now in ΔAOC ,

By Pythagoras theorem,

$$y^2 + p^2 = q^2$$

From (2),

$$\frac{3m^2}{4} + p^2 = q^2$$

or

.... (From fig.)

From (1),

$$\frac{3m^2}{4} + \left(t - \frac{m}{2}\right)^2 = q^2$$

$$\frac{3m^2}{4} + \frac{(2t - m)^2}{4} = q^2$$

On expanding we get,

$$\frac{3m^2 + 4t^2 + m^2 - 4mt}{4} = q^2$$

$$\frac{4m^2 + 4t^2 - 4mt}{4} = q^2$$

Now, multiply and divide by (m+t) on L.H.S.

$$\frac{(m+t)4m^2 + 4t^2 - 4mt}{(m+t)4} = q^2$$

Taking 4 common from numerator

$$\frac{(m+t)(4)(m^2 + t^2 - mt)}{(m+t)(4)} = q^2$$

$$\frac{(m+t)(4)(m^2 + t^2 - mt)}{(m+t)(4)} = q^2$$

Now we know that,

$$(a+b)(a^2+b^2-ab) = (a^3+b^3)$$

So,

$$(q)^2 =$$

Or

$$(AC)^2$$

Hence proved.

About authors

Trilok Kaushik is a student of X class in Greenland Public School, residing in Dattaur, Rohtak, Haryana. Mathematics is one of his favorite subjects.

Lokesh Kaushik is a student of XII class in Charan Singh memorial convent school, Residing in Dattaur, Rohtak, Haryana. Right now he is taking coaching classes from Suryodya institute.

They performed a lot of experiments in mathematics, resulted in very wonderful formulas. Their hard work of years proved to be fruitful when their first formula approved entitled as "Trilokesh Ultimate Sum Method". They hope that their formulas will help the students in solving many mathematical problems as well as in understanding the concepts more clearly.