

# Synergy of Filtering with Delayed States and Missing Data in Measurement Level Fusion

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**Abstract:** In certain engineering situations, and especially in wireless sensor networks, there are events wherein some delays occur in data transmission and at times some measurements might be randomly missing. Processing such data for tracking and prediction of targets of interest would cause inaccuracies. Kalman filter or its equivalent algorithms are mainly used for estimation of the states of the dynamic systems. In this paper the problem of modelling and filtering such delayed states and missing data is handled in a synergy of Kalman filter-like optimal filtering algorithms in the measurement data level fusion process. This combined approach of filtering of delayed states, missing observations and data fusion is relatively novel. The sensor data fusion is becoming increasingly matured soft technology and has found numerous applications in science and engineering and in image fusion also. Performance evaluation is carried out using numerical simulation.

**Keywords:** Delayed states, missing data, Kalman like filtering algorithms, data level fusion.

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## I. INTRODUCTION

The process of multisensory data fusion (MSDF) is very important for many civilian and military applications: a) target tracking, and b) automatic target recognition (ATR), and c) WSN. The idea is that we want to ascertain the status and an identity of the object under investigation and observation wherefore the measurements are available from more than one sensor. Then more complete information about the status of the object is obtained by fusing the information from these sensors in some appropriate way [1]. Sometimes, in a data-communications channel a few or many measurements might be missing (from one or more sensors). Then, it becomes very important to study and evaluate the performance of the data processing algorithms in presence (and despite) of missing measurements (missing data, MD), i.e. in the absence of certain signal-data during certain times; we might have only the random (measurement) noises present for these channels.

There are two basic methods of fusion of information from two (or more) sets of data: a) measurement data level, and b) state vector fusion (SVF). In the data level fusion, the data sets from the two (or more) sensor channels are directly combined and used in the Kalman filter (KF, or in any other optimal filter) and after processing (by the filter) we obtain one automatically fused state of the target. In the SVF, the data sets coming from each sensor are initially processed by individual KFs and then by a well-known formula of SVF [1] these individual estimates of the states are fused. This formula obtains a weighted average of the individual state estimates, and the weights are derived from the covariance matrices of the individual KFs which have been used for the processing the original signal-data sets. If some measurements are missing in the processing of the data by KF, then the performance of the filter would not be as good, since the KF does not have so much of inherent robustness to handle the situations when the data are missing for a long time, or/and there are delayed states; sometimes the states themselves are the measurements and hence, the delayed states would result into delayed measurements. What would happen is that the filter would continue to predict the state estimates based on the time propagation equations which do not take any measurements into account anyway, and the measurement data update would not be very effective, since some data are missing anyway. Hence, it is very important to study the problem of missing measurements in the filter processing cycle along with the state delays. This joint/combined aspect has not gained much attention in the context of multisensory data fusion for target tracking, though some work has been done in certain special

cases [2-10]. In some studies [2-4] certain special aspects have been considered: a) intermittent observations, b) missing data in online condition monitoring, and c) packet dropouts. In [5] a system with multiple sensor delay is considered, but the algorithm seems quite involved. The problem of outliers and missing data is considered in [6], however, the illustration is only for simple time series cases. Refs. [7,8,10] deal with only missing observations, and aspect of state delay is not treated. In [9] only the problem with system delay is considered. In the present paper we consider the state delay and the randomly missing measurement data in filtering algorithms in a synergistic manner and as well as in the measurement data level fusion. The data may be missing due to: a) a failure of a sensor, and/or b) there is a problem in a communication channel, and the received data might be only the noise and the signal is missing. So, it is very important to handle the situation of missing data in KF in some formal and optimal way. Time delay is encountered in many real time systems, due to latency time of certain channels. Time delay is a key factor that influences the overall system stability and performance. If these aspects of system's state delay and missing data are not handled appropriately in the tracking filter, then we might lose the track, and the performance of the tracking filter would also be very poor.

We discuss and derive certain algorithms based on KF and evaluate the performance of these algorithms when some measurement data are randomly missing and are not available for further processing in the filters, and also there are delayed states, in the measurement level data fusion scenarios. The detailed derivations are not presented for the brevity of the paper. We present, first two models for representing the delayed states, and then corresponding filtering algorithms, then we consider the problem of missing measurement data, the related algorithms, and combined filtering algorithms for data level fusion.

## II. Filter with delayed state - Model 1

For this case the linear delayed state model is given by

$$\begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = \begin{bmatrix} \phi_0 & \phi_1 \\ I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} + G \begin{bmatrix} w(k) \\ 0 \end{bmatrix} \quad (1)$$

Here,  $\phi(\cdot)$  are the respective transition matrices for the given system for the present and delayed states,  $w(k)$  is the white Gaussian process noise (WGN) sequence with zero mean and known covariance matrix Q. We have the following equivalence

$$\begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} = x_k; \quad \begin{bmatrix} \phi_0 & \phi_1 \\ I & 0 \end{bmatrix} = \phi; \quad \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = x_{k+1}; \quad \begin{bmatrix} w(k) \\ 0 \end{bmatrix} = w_k$$

With the above equivalence we rewrite (1) as

$$x_{k+1} = \phi x_k + G w_k \quad (2)$$

The measurement equation is given as

$$z_{k+1} = H \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} + v_{k+1}$$

Here,  $v_{k+1}$  is the zero mean white Gaussian measurement noise (WGN) with covariance matrix R, and we have

$$z_{k+1} = H x_{k+1} + v_{k+1} \quad (3)$$

Let  $\tilde{x}_{k+1}$  represent the state estimate at time k+1 and  $\hat{x}_k$  the (filtered) updated or posterior (after the measurement data are incorporated) estimate at time k. Then the time propagation of the state estimate is

$$\tilde{x}_{k+1} = \phi \hat{x}_k \quad (4)$$

The measurement prediction is given by,

$$\tilde{z}_{k+1} = H\tilde{x}_{k+1} \quad (5)$$

The filtering equations for this model 1 of the delayed state, as can be easily seen from the equivalence in equations (1) to (3), are the same as that of the conventional KF

$$\text{(covariance propagation)} \quad \tilde{P}_{k+1} = \phi \hat{P}_k \phi^T + GQG^T \quad (6)$$

$$\text{(Kalman gain)} \quad K = \tilde{P}_{k+1} H^T (H\tilde{P}_{k+1} H^T + R)^{-1} \quad (7)$$

$$\text{(measurement update)} \quad \hat{x}_{k+1} = \tilde{x}_{k+1} + K(z_{k+1} - \tilde{z}_{k+1}) \quad (8)$$

$$\text{(covariance update)} \quad \hat{P}_{k+1} = (I - KH) \tilde{P}_{k+1} (I - KH)^T + KRK^T \quad (9)$$

Thus, in Model 1, the state delay is incorporated in a composite way.

### III. Filter with delayed state-Model 2

In this model 2 the delayed state is accommodated as

$$x_{k+1} = \phi_0 x_k + \phi_1 x_{k-1} + Gw_k \quad (10)$$

The measurement equation is given by

$$z_{k+1} = Hx_{k+1} + v_{k+1} \quad (11)$$

Then, the time propagation of the state is given as

$$\tilde{x}_{k+1} = \phi_0 \hat{x}_k + \phi_1 \hat{x}_{k-1} \quad (12)$$

The measurement prediction equation is given as

$$\tilde{z}_{k+1} = H\tilde{x}_{k+1} \quad (13)$$

The KF equations for this model 2 of the delayed state, by comparing equations (2), (6) and (10), are given as

$$\text{(covariance propagation)} \quad \tilde{P}_{k+1} = \phi_0 \hat{P}_k \phi_0^T + \phi_1 \hat{P}_{k-1} \phi_1^T + GQG^T \quad (14)$$

$$\text{(Kalman gain)} \quad K = \tilde{P}_{k+1} H^T (H\tilde{P}_{k+1} H^T + R)^{-1} \quad (15)$$

$$\text{(measurement update)} \quad \hat{x}_{k+1} = \tilde{x}_{k+1} + K(z_{k+1} - H\tilde{x}_{k+1}) \quad (16)$$

$$\text{(covariance update)} \quad \hat{P}_{k+1} = (I - KH) \tilde{P}_{k+1} (I - KH)^T + KRK^T \quad (17)$$

Thus, in this case the state delay is incorporated directly in the state equation.

#### IV. Combined filter with delayed states-Model 2 and missing data

The equation (10) for the delayed state of model-2 is considered here :

$$x_{k+1} = \phi_0 x_k + \phi_1 x_{k-1} + Gw_k \quad (18)$$

The measurement equation is given as

$$z_{k+1} = \gamma(k+1)Hx_{k+1} + v_{k+1} \quad (19)$$

The scalar quantity  $\gamma$  is a Bernoulli sequence which takes values 0 and 1 randomly; thus, we have  $E\{\gamma(k) = 1\} = \beta(k)$  and  $E\{\gamma(k) = 0\} = 1 - \beta(k)$  with  $\beta$  as the percentage of measurements that arrive truthfully to the sensor fusion node (if there are two or more sensors) [2]. This also means that some measurement data are randomly missing. The constant  $\beta$  is assumed to be known and pre-specified. Then, the KF-like filtering algorithm to handle and incorporate the missing measurements along with delayed state is given as

$$\text{(state propagation)} \quad \tilde{x}_{k+1} = \phi_0 \hat{x}_k + \phi_1 \hat{x}_{k-1} \quad (20)$$

(covariance propagation)

$$\tilde{P}_{k+1} = \phi_0 \hat{P}_k F_0^T + \phi_1 \hat{P}_{k-1} F_1^T + GQG^T \quad (21)$$

(Kalman gain)

$$K = \beta \tilde{P}_{k+1} H^T (\beta^2 H \tilde{P}_{k+1} H^T + R)^{-1} \quad (22)$$

(filter equation)

$$\hat{x}_{k+1} = \tilde{x}_{k+1} + K(z_{k+1} - \beta H \tilde{x}_{k+1}) \quad (23)$$

(covariance update)

$$\hat{P}_{k+1} = (I - \beta KH) \tilde{P}_{k+1} (I - \beta KH)^T + KRK^T \quad (24)$$

This filter and its variants have been studied in [2] for randomly missing data, but without state delay.

#### V. Filter with delayed state-Model 1 and measurement level fusion

Here, we consider the delayed state model 1 and the measurement data level fusion together. The state model is the same as in equations (1) and (2). The measurement models for the two sensors are given as

$$z_{1k+1} = H_1 \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} + v_{1k+1} \quad (25)$$

$$z_{2k+1} = H_2 \begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} + v_{2k+1} \quad (26)$$

In the compact form the measurement models are, considering the equivalent form as in section 2

$$z_{1k+1} = H_1 x_{k+1} + v_{1k+1} \quad (27)$$

$$z_{2k+1} = H_2 x_{k+1} + v_{2k+1} \quad (28)$$

We see that since the state model is the same as in (1) and (2), the time propagation parts of the filter are given by equations (4) and (6). However, since we have the two channels of the measurement data, i.e two sensors, we have the following equations for the measurement data update part of the filter

(measurement update)

$$\hat{x}_{k+1} = (I - K_2 H_1 - K_3 H_2) \tilde{x}_{k+1} + K_2 z_{1k+1} + K_3 z_{2k+1} \quad (29)$$

(covariance update)

$$\hat{P}_{k+1} = (I - K_2 H_1 - K_3 H_2) \tilde{P}_{k+1} (I - K_2 H_1 - K_3 H_2)^T + K_2 R_1 K_2^T + K_3 R_2 K_3^T \quad (30)$$

The Kalman gains can be obtained from the following set of equations (derivations avoided)

$$K_2 (H_1 \tilde{P}_{k+1} H_1^T + R_1) + K_3 H_2 \tilde{P}_{k+1} H_1^T = \tilde{P}_{k+1} H_1^T \quad (31)$$

$$K_3 (H_2 \tilde{P}_{k+1} H_2^T + R_2) + K_2 H_1 \tilde{P}_{k+1} H_2^T = \tilde{P}_{k+1} H_2^T \quad (32)$$

$$K_2 s_{11} + K_3 s_{12} = c_1 \quad (33)$$

$$K_2 s_{21} + K_3 s_{22} = c_2 \quad (34)$$

$$\begin{bmatrix} K_2 & K_3 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}^{-1} \quad (35)$$

With  $s_{11} = (H_1 \tilde{P}_{k+1} H_1^T + R_1)$ ,  $s_{12} = H_2 \tilde{P}_{k+1} H_1^T$ ,  $s_{21} = H_1 \tilde{P}_{k+1} H_2^T$  and  $s_{22} = (H_2 \tilde{P}_{k+1} H_2^T + R_2)$ .

The residuals are computed as a part of equation (29). Thus, this filter takes into account the state delay in measurement level data fusion.

## VI. Filter with delayed state-Model 2-and measurement level fusion

For this case the state model is the same as in equation (10), and the measurement models are given by equations (27) and (28). Hence, combining the appropriate filtering equations, we have the following filter for this case :

$$\tilde{x}_{k+1} = \phi_0 \hat{x}_k + \phi_1 \hat{x}_{k-1} \quad (36)$$

$$\tilde{P}_{k+1} = \phi_0 \hat{P}_k \phi_0^T + \phi_1 \hat{P}_{k-1} \phi_1^T + G Q G^T \quad (37)$$

$$\hat{x}_{k+1} = (I - K_2 H_1 - K_3 H_2) \tilde{x}_{k+1} + K_2 z_{1k+1} + K_3 z_{2k+1} \quad (38)$$

$$\hat{P}_{k+1} = (I - K_2 H_1 - K_3 H_2) \tilde{P}_{k+1} (I - K_2 H_1 - K_3 H_2)^T$$

$$+K_2R_1K_2^T + K_3R_2K_3^T \quad (39)$$

$$\begin{bmatrix} K_2 & K_3 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}^{-1} \quad (40)$$

with  $c(\cdot)$  and  $s(\cdot)$  defined as in equations (33)-(35). Thus, this obtains the filter for state delay of Model 2 in measurement level data fusion.

## VII. Combined filter with delayed state-Model 2 and missing data in measurement level fusion

In this case the delayed state model 2 is the same as in (10)

$$x_{k+1} = \phi_0 x_k + \phi_1 x_{k-1} + Gw_k \quad (41)$$

The measurements are obtained from two sensors

$$z_{1k+1} = \beta_1 H_1 x_{k+1} + v_{1k+1} \quad (42)$$

$$z_{2k+1} = \beta_2 H_2 x_{k+1} + v_{2k+1} \quad (43)$$

We assume that the measurement data can miss in any one of the sensors or both the sensors simultaneously. The combined filter to handle the delayed state (in model 2) and the randomly missing measurement data from the sensors, in the measurement level data fusion, is given as

$$\tilde{x}_{k+1} = \phi_0 \hat{x}_k + \phi_1 \hat{x}_{k-1} \quad (44)$$

$$\tilde{P}_{k+1} = \phi_0 \hat{P}_k \phi_0^T + \phi_1 \hat{P}_{k-1} \phi_1^T + GQG^T \quad (45)$$

$$\tilde{z}_{1,k+1} = \beta_1 H_1 \tilde{x}_{k+1} \quad (46)$$

$$\tilde{z}_{2,k+1} = \beta_2 H_2 \tilde{x}_{k+1} \quad (47)$$

$$\hat{x}_{k+1} = (I - \beta_1 K_2 H_1 - \beta_2 K_3 H_2) \tilde{x}_{k+1} + K_2 z_{1,k+1} + K_3 z_{2,k+1} \quad (48)$$

The Kalman gains can be computed from the following equations

$$K_2 (\beta_1^2 H_1 \tilde{P}_{k+1} H_1^T + R_1) + \beta_1 \beta_2 K_3 H_2 \tilde{P}_{k+1} H_1^T = \beta_1 \tilde{P}_{k+1} H_1^T \quad (49)$$

$$K_3 (\beta_2^2 H_2 \tilde{P}_{k+1} H_2^T + R_2) + \beta_1 \beta_2 K_2 H_1 \tilde{P}_{k+1} H_2^T = \beta_2 \tilde{P}_{k+1} H_2^T \quad (50)$$

$$K_2 s_{11} + K_3 s_{12} = c_1 \quad (51)$$

$$K_2 s_{21} + K_3 s_{22} = c_2 \quad (52)$$

$$\begin{bmatrix} K_2 & K_3 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}^{-1} \quad (53)$$

Here,  $s_{11} = (\beta_1^2 H_1 \tilde{P}_{k+1} H_1^T + R_1)$ ,  $s_{12} = \beta_1 \beta_2 H_2 \tilde{P}_{k+1} H_1^T$ ,  $s_{21} = \beta_1 \beta_2 H_1 \tilde{P}_{k+1} H_2^T$  and  $s_{22} = \beta_2^2 (H_2 \tilde{P}_{k+1} H_2^T + R_2)$

Thus, we have now the KF-like, optimal filters that can handle delayed state (in model 2) and randomly missing data in measurement level data fusion scenario. It must be mentioned here that the fusion filters for the delayed state Model 1, Model 2 and the missing data in the combined measurement level data fusion have been obtained based on the same principle as the derivation of the conventional KF.

### VIII. Evaluation of the filters

The performance of the presented filters is validated using numerical simulations carried out in MATLAB. The simulations are done for a period of 200 seconds with a sampling interval of 1 second. The data of the target under consideration are generated using the appropriate state and measurement equations corresponding to each filter by adding zero mean WGN process and measurement noise with  $\sigma = 0.001$  (for Q) and  $\sigma = 0.5$  (for R) respectively. However, for the data fusion exercises involving two sensors, the covariance for the second sensor is assumed to be three times higher than the first sensor. The state vector  $x$  consists of position, velocity and acceleration of a moving object, however, only position measurements are used for filtering. The state-model 1 of the system is chosen with the state transition matrices as

$$\phi_0 = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}; \quad \phi_1 = \text{constant} \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad (54)$$

In state delay-model 2 also these matrices are the same, but are handled in a different way as studied earlier, and a small constant controls the effect of the delayed state. The process noise coefficient matrix and the measurement model are given as

$$G = \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix}; \quad H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad (55)$$

The state initial conditions used for the simulation and for the filter are chosen appropriately. As a check it was established that when there are some measurement data missing, but if this missing aspect is not taken into account in the optimal filtering algorithm, then the performance of the filter was poor; also, it was established that when there is a state delay present in the data, but when this aspect was not incorporated in the filter, the performance of the tracking filter was found to be degraded; and hence, these two aspects are not further studied here. So, we evaluate the performance of the filters presented under the conditions that either there is a state-delay or/and some measurements/data are randomly missing, in either case: i) without fusion, and ii) with the measurement data level fusion. Table 1 (with legends) shows the performance metrics (% states-errors and % measurement fit errors) for these filters; reasonably small numerical values of these metric

indicate that the performance of the filter is relatively good. Also, this is corroborated by performance plots. Figure 1 depicts the basic filter performance with the three states and residuals from the filter DS2. Figure 2 shows the state-errors and position measurement autocorrelations (ACR) with bounds from the same filter with delayed state DS2 (with one sensor and no missing data). Also, similar performance plots were obtained for the filter DS1, but are not reproduced here. Figure 3 depicts state errors and position measurement autocorrelations (ACR) with bounds from the filter with delayed state DS2M, and for one sensor and missing data with the threshold value at 0.5. The measurement data are missed randomly when the absolute value of the random number exceeds this threshold value, this means that if the threshold value is smaller, more data are missing. Some more guidelines are needed in order to select the realistic thresholds and their relations to the tuning parameter to be used in the algorithm, this requires further research. Here, for the sake of illustration of the algorithms some adhoc approach has been taken. With all this, we see that the performance of the filters is satisfactory. Thus, the performance of the basic filters with delayed state and/or missing data has been found to be satisfactory, because the residuals and their autocorrelations are found to be within their respective theoretical bounds. Also, numerical values of the performance metrics are small (Table 1). Next, the performance of filters for measurement data level fusion with delayed states and/or missing data is evaluated. Figure 4 shows the state errors with bounds from the filter with delayed state DS2F, two sensors and no missing data. Figure 5 shows the position measurements and residuals from filter with delayed state, DS2MF and data level fusion with two sensors and missing data with the thresholds 0.5 for sensor 1 and 0.95 for sensor 2. Figure 6 depicts the state errors with bounds from the filter with delayed state DS2MF, and data level fusion with two sensors and missing data thresholds 0.5, 0.95. Figure 7 shows the position measurements and residuals from the filter with delayed state DS2MF and data level fusion with the two sensors and missing data thresholds 0.95 for sensor 1 and 0.5 for sensor 2.

**Table 1: Performance results for various filters**

Filter Type(**)	Filters with delayed states and/or missing data with only one sensor						
	% state errors			% position (meas.) errors		Missing levels of 'no' measurement data(+)	
	position	velocity	acceleration	Sensor 1	Sensor 2	Sensor 1	Sensor 2
DS1	0.0188	0.1718	5.893	0.019	-	-	-
DS2	0.015	0.140	5.707	0.0189	-	-	-
DS2M	0.015	0.1412	5.67	0.0193	-	0.95 (*)	-
	0.0198	0.2125	6.446	0.03	-	0.5	-
	0.0295	0.308	6.594	0.0507	-	0.3	
	0.0743	0.5562	7.264	0.1833	-	0.1 (#)	
	Filters with delayed states and/or missing data with two sensors-Data level fusion						
DS1F S1	0.0152	0.136	5.607	0.0188	-	-	-
S2	0.017	0.148	5.605	-	0.0207	-	-
F	0.0149	0.134	5.635	-	-	-	-



DS2F	S1	0.0152	0.133	5.510	0.0188	-	-	-
	S2	0.0170	0.145	5.529	-	0.0207	-	-
	F	0.0149	0.1301	5.541	-	-	-	-
DS2MF	S1	0.019	0.169	5.916	0.0298	-	0.5	0.95
	S2	0.021	0.166	5.877	-	0.0214		
	F	0.019	0.158	5.890	-	-		
	S1	0.0150	0.1361	5.531	0.0193	-	0.95	0.5
	S2	0.0186	0.157	5.531	-	0.024		
	F	0.0150	0.137	5.685	-	-		

(+) The threshold used for generating the missing data; (\*)→Less data missing; (#)→More data missing; S1 sensor 1, S2 sensor 2, F fused. (\*\*): **DS1**-Filter with delayed state model 1 and one sensor; **DS2**-Filter with delayed state model 2 and one sensor; **DS2M**-Filter with delayed state model 2 & missing measurement data with one sensor; **DS1F**-Filter with delayed state model 1 & fusion; **DS2F**-Filter with delayed state model 2 & data level fusion; **DS2MF**-Filter with delayed state model 2, missing measurement data & data level fusion.

Finally, Figure 8 depicts the state errors with bounds from the filter with delayed state DS2MF and data level fusion with two sensors and missing data thresholds 0.95, 0.5. We observe that all these filters handle the delayed states and missing data very satisfactorily in the measurement level fusion, since various errors have been found to be within their respective theoretical bounds. From the results of Table 1 we infer: i) when more data are missing these errors increase, still the performance of the corresponding filter is good and acceptable, ii) percentage fit errors (state errors and the position measurement errors, where applicable) are less for measurement level fusion compared to the use of one sensor data, iii) in the measurement data level fusion with missing data when more and more data are missing, the performance metric values increase, but the performance is still good and acceptable, iv) in case of DS1F and DS2F, the position states-fit errors are nearly the same because the mathematical models are equivalent and that we use only the position state as the only measurement, and v) the acceleration-state errors' metrics are relatively large, since we do not use velocity and/accelerations as the observable. These inferences are qualitatively supported by Figures 1 to 8.

Thus, it is seen, from the numerical values of various performance metrics and the plots of various errors within their bounds (with a few or more missing data), that the performance of these filters with delayed states and/or missing data in one sensor or two sensor configurations is found to be good and satisfactory. Thus, the KF type filters for delayed states and/or missing data and for measurement level fusion have been obtained in a synergistic way and validated using numerically simulated data. When more data are missing, the increasing, but slight degradation of the metrics values has been seen. Even if there are more measurement data missing, the performance of the corresponding filters with one sensor or two sensors has been bound to be very satisfactory. Hence, in the overall sense, these filters have shown satisfactory and acceptable performance.

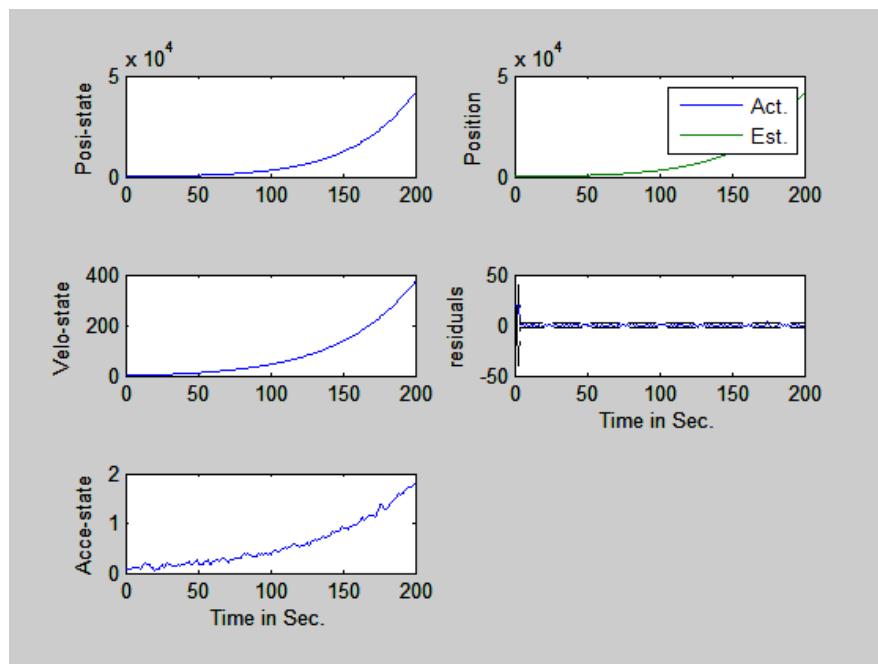
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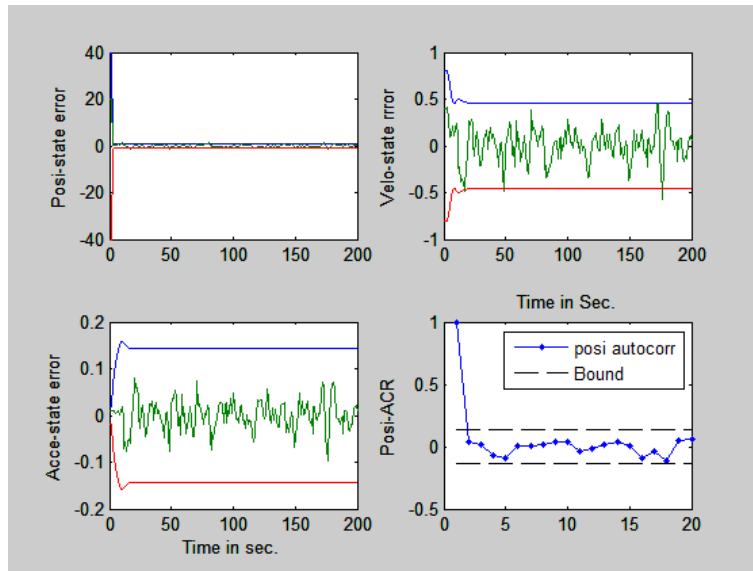
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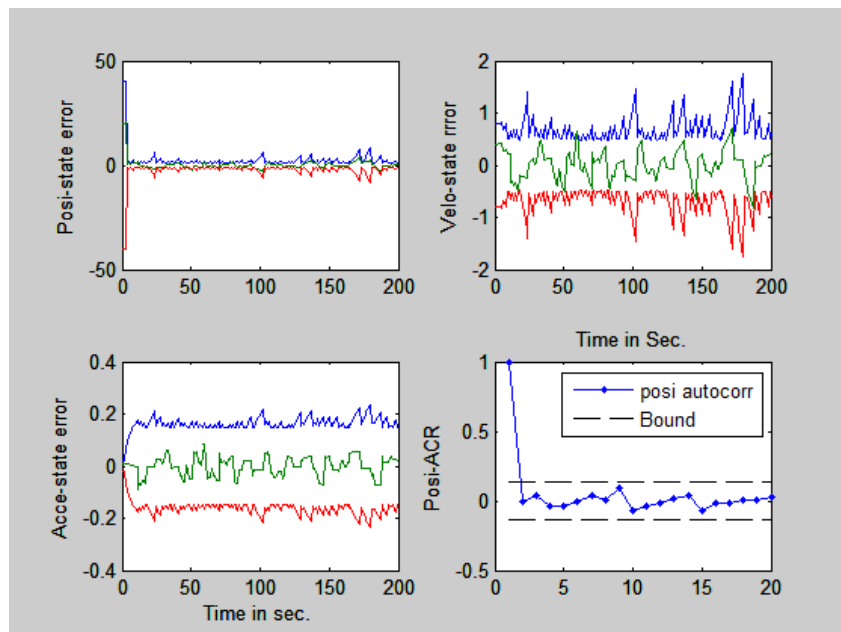
### FIGURES



**Figure 1. The states of the model, actual & predicted measurements and residuals: Filter with delayed state DS2 (Model 2, One sensor-no missing data).**



**Figure 2: The state-errors and position measurement autocorrelations (ACR) with bounds- Filter with delayed state DS2 (Model 2, One sensor-no missing data).**



**Figure 3: State-errors and position measurement autocorrelations (ACR) with bounds-Filter with delayed state DS2M (Model 2, One sensor-missing data-Threshold 0.5).**

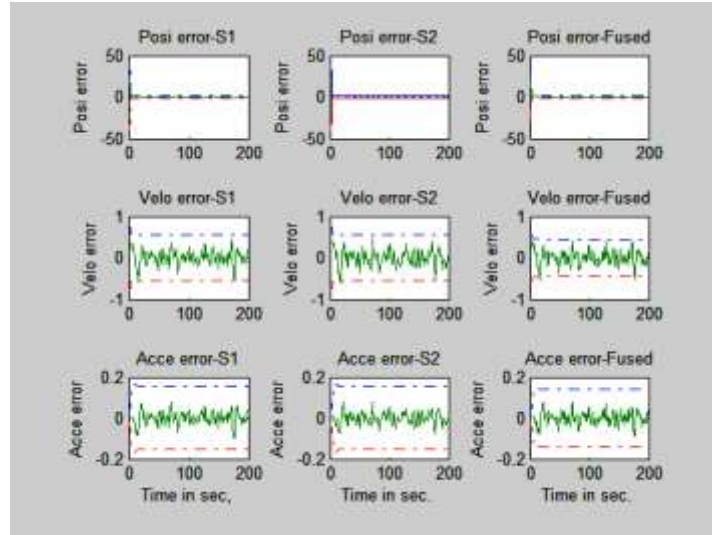


Figure 4. State-errors with bounds-Filter with delayed state DS2F and data level fusion (Model 2, Two sensors and no missing data).

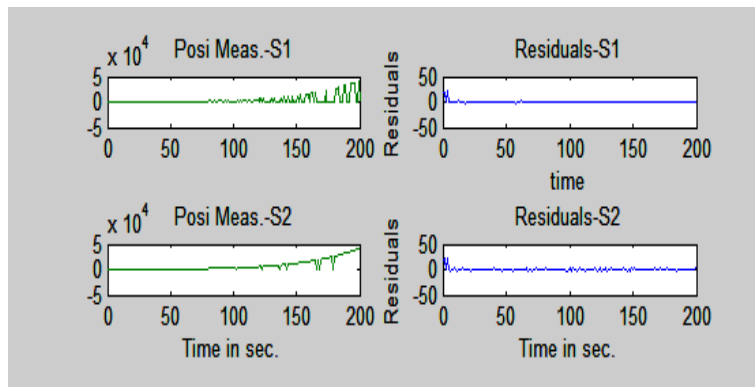


Figure 5. Position measurements and residuals-Filter with delayed state, missing data DS2MF and data level fusion (Model 2, Two sensors and missing data thresholds 0.5, 0.95).

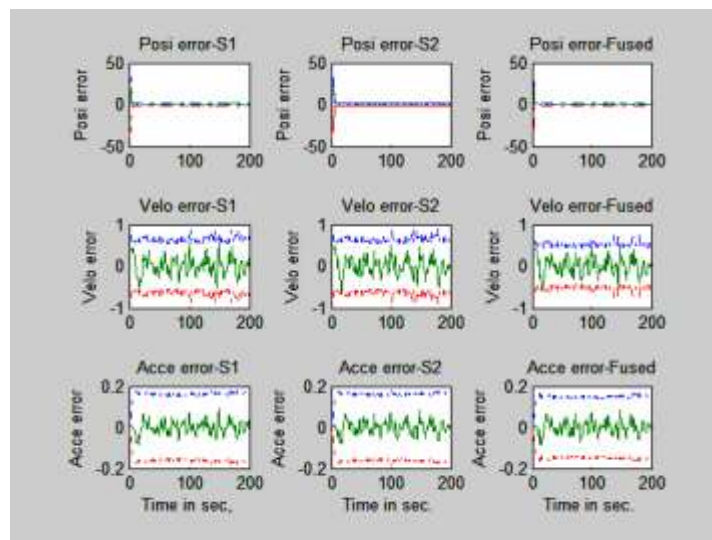
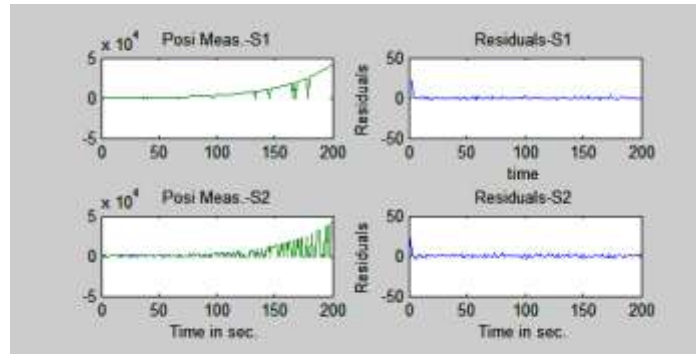
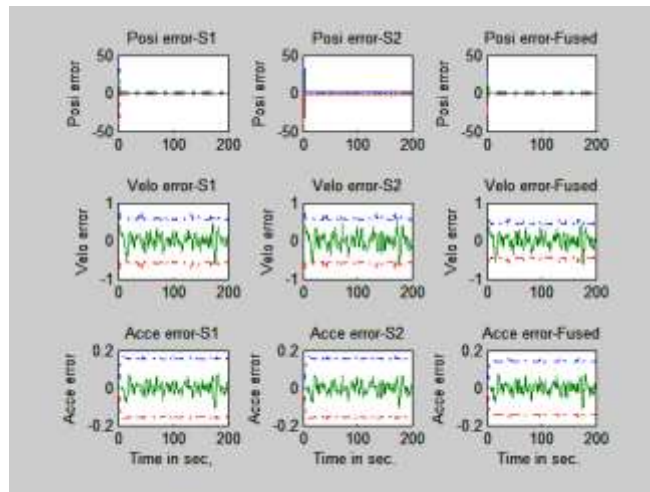


Figure 6. State-errors with bounds-Filter with delayed state DS2MF, missing data and data level fusion (Model 2, Two sensors and missing data thresholds: 0.5, 0.95).



**Figure 7. Position measurements and residuals-Filter with delayed state, missing data DS2MF and data level fusion (Model 2, Two sensors and missing data thresholds 0.95, 0.5).**



**Figure 8. State-errors with bounds-Filter with delayed state DS2MF, missing data and data level fusion (Model 2, Two sensors and missing data thresholds: 0.95, 0.5).**