Multi Regression Analysis to Predict the Gas In-Place and Aquifer Constant of the X Field in Sirte Basin

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ABSTRACT

Traditionally, the p/z plot has been used to describe the behavior of volumetric depletion gas reservoirs. Several authors [1,2,3,] have pointed out, that this technique can lead to serious misinterpretations of the drive mechanism and overestimation of the original gas in place (OGIP) if the reservoir produces under the influence of an encroaching aquifer; since the p/z plot may be straight line during the early life , but a non- linear flattening of the plot may ultimately develop. Extrapolations are of little value. In this article the gas in-place, aquifer constant, aquifer to reservoir ratio and the constant of the dimensionless time were determined. To determine the above mentioned quantities; unsteady state calculation using the method developed in 1949 by Van Everdingen & Hurst[4] and then modified by Jawad[5] and the material balance equation as a straight line developed by Havlena & Odeh[6] were employed. Different Values were assumed for aquifer size to reservoir size ratio and the dimensionless time constant; then multi regression analysis and the method of least square[7] were used to determine the gas in-place and the aquifer constant. To confirm the results and the conclusions of this method, an oil field example was taken from reference (8).

INTRODUCTION

Material balance analysis is a standard reservoir engineering tool for the analysis of the performance of oil and gas reservoirs. Applied to single , tank-type gas reservoirs, the material balance yields a characteristic relationship between the ratio of pressure to z factor (p/z) and cumulative gas production. In the ideal case of volumetric depletion, i.e no changes in the hydrocarbon pore volume during depletion, this relation simplifies to a straight line. For reservoir under water drive or abnormal pressure there is no straight line relationship between p/z and cumulative gas production In this study It is required to determine the gas in-place of the **X** field which is located in Sirte basin in Libya using the production data and the PVT properties of the fluid.

The reservoir lithology is complex and having low porosity and permeability, the reservoir is associated with moderate aquifer strength and the aquifer properties are not well known. This is merely because one rarely drills wells into an aquifer to acquire the essential information about porosity, permeability, thickness and fluid properties. The basic objectives in applying the material balance equation to the production-pressure history of a gas reservoir are to define the drive mechanism (natural water drive or volumetric depletion) and estimate the OGIP.

Traditionally, two techniques have been used: the p/z [1] plot and the method of Havlena-Odeh[6]. The former, though most popular method in the industry, can be extremely insensitive in practice, leading to misjudgment of the drive mechanism and a serious overestimation of the OGIP. Because of this, the method of Havlena-Odeh[7,8] is recommended as a means of checking the validity of the p/z plot Nonetheless, the simplicity of the p/z technique has resulted in its almost universal application.

Should an encroaching aquifer support the reservoir pressure as production advances, an independent mathematical model is required to describe the behavior of the aquifer-reservoir system

The unsteady –state equation developed by Van Everdingen & Hurst[4] and then modified by Jawad[5] and the material balance equation as a straight line developed by Havlena & Odeh[6] , multi regression analysis and least square method[7] were used to determine the gas in-place, aquifer constant, the constant of the dimensionless time and ratio of aquifer size to

reservoir size. To confirm the results and conclusion an example (Oil reservoir) from reference (8) solved using this procedure.

PROCEDURE

To perform the required calculations the following tables were prepared.
Table (1) shows the production data of the oil reservoir.
Table (2) shows the production data of the gas reservoir.
Table (3) shows the fluid and petrophysical properties for both reservoirs.

Table(1) Production Data For The Example

Time	P	Bo	Np	Rp	Rs	Bg
Year	psia	Bbl/Stb	MM-Stb	Scf/Stb	Scf/Stb	Bbl/Scf
0.00	2740	1.404	0.00	650	650	0.00093
1.00	2500	1.374	7.88	760	592	0.00098
2.00	2290	1.349	18.42	845	545	0.00107
3.00	2109	1.329	29.15	920	507	0.00117
4.00	1949	1.316	40.69	975	471	0.00128
5.00	1818	1.303	50.14	1025	442	0.00139
6.00	1702	1.294	58.42	1065	418	0.00150
7.00	1608	1.287	65.39	1095	398	0.00160
8.00	1535	1.280	70.74	1120	383	0.00170
9.00	1480	1.276	74.54	1145	371	0.00176
10.00	1440	1.273	77.43	1160	364	0.00182

Table(2) Production Data For The X- Field

Time	P	Z	Bg	Gp	Wp,	Bw
Year	psia		Cf/Scf	BScf	MBbl	Bbl/Bbl
0.00	4265	1.0150	0.013431	0.000	0.000	1.080
5.58	3997	1.0019	0.014147	230.517	1357.400	1.080
6.75	3923	0.9987	0.014368	295.940	1742.200	1.080
7.33	3886	0.9970	0.014480	337.519	1986.900	1.080
8.83	3837	0.9951	0.014637	421.695	2482.800	1.080
9.50	3795	0.9934	0.014774	459.810	2707.500	1.080
10.41	3762	0.9920	0.014882	508.420	2993.600	1.080
11.41	3709	0.9900	0.015064	568.138	3345.200	1.080
12.24	3672	0.9888	0.015198	602.276	3545.800	1.090
13.08	3646	0.9879	0.015292	650.988	3832.000	1.090
15.17	3461	0.9812	0.016000	798.654	4702.300	1.090
17.00	3333	0.9763	0.016532	929.707	5473.700	1.090
19.00	3215	0.9725	0.017072	1071.868	6310.700	1.090
19.75	3170	0.9712	0.017291	1125.064	6642.000	1.090
20.58	3113	0.9699	0.017584	1185.288	7020.900	1.090
21.75	3045	0.9683	0.017947	1267.995	7449.500	1.090
22.66	2987	0.9667	0.018266	1330.540	8044.000	1.090
23.83	2887	0.9640	0.018845	1406.141	8844.100	1.090
24.75	2824	0.9625	0.019236	1454.759	9492.700	1.090
25.75	2772	0.9614	0.019574	1512.137	10275.400	1.090
27.00	2685	0.9596	0.020171	1584.244	11430.300	1.090

Table (3) Fluid and Petrophysical Prperties

Parameter	Oil Field	Gas Field
k,md	200	0.7
h,ft	100	---
μw,cp	0.55	0.21
ro,ft	9200	20000
ø, fraction	0.25	0.05
Cw,psi^-1	3.00E-06	3.50E-06
cf, psi^-1	4.00E-06	6.50E-06
Sw, fraction	0.05	0.44
α,degree	140	360
T,degree F		318
Ψ	0.39	1.00

The following steps used:

1. Write the material balance equation in the form of :

Production Term =Expansion Term + Water Influx.

For the oil reservoir:

$N_P[\beta_o +(R_P - R_S) \beta_g] +W_P \beta_W = N[\beta_{o-} \beta_{oi} +(R_{Si} - R_S) \beta_g +(C_W S_{Wc} +C_f) \quad \beta_{oi} \Delta P/(1- S_{Wc})] + W_e$
....(1)

Or :

$Z = N X + B \sum \Delta P Q_{tD}$ ....(1a)

$Z = N X + B S(\Delta p, t_D, r_D)$ or   ....(1b)

$Z = N X + B Y$ ....(1c)

For the gas reservoir:

$G_P \beta_g +W_P \beta_W = G[\beta_{g-} \beta_{gi} + (C_W S_{Wc} +C_f) \beta_{gi} \Delta P/(1- S_{Wc})] + W_e$ . ...(2)
Or:

$Z = G X + B \sum \Delta P Q_{tD}$ or   ....(2a)

$Z = G X + B S(\Delta p, t_D, r_D)$ or   ....(2b)

$Z = G X + B Y$ ....(2c)

In equations (1c) & (2c) , Z is the production term and can be called $Z_{Observed}$ and the only unknown are N & B or G & B

2. Since Q_{tD} is a function of t_D and r_D ,where :

$t_D = 2.309 \, kt/ (ø\mu_w c_t r_o^2) = C_{tD} t$ (time in years)
$r_D = r_e/r_o$

Both C_{tD} & r_D are unknown or there are some uncertainties in their values.
Start with equation (1c) :

A - Assume a value to r_D, say =2

B - Assume a value to C_{tD} (less   than what do you expect).

C - Call the subroutine to calculate Q_{tD} and then calculate Y.

D - Calculate N & B using multi regression analysis and least square Method.

E - With values of N & B, calculate Z_{Cal}.

F - Calculate the square difference between ($Z_{observed}$ - Z_{Ca})^2 = Δ

G – Increase the value C_{tD}

H - Go to step C and repeat the calculations (50 times)

I - Find the standard deviation σ: $\sigma = \sqrt{\sum \Delta / n_d}$

J - At the minimum value of standard deviation & maximum value of correlation coefficient chose N & B.

K – Assume another initial value for C_{tD} in step B and repeat the   calculations.

L - Go to step A with a new value   of r_D and find another set of N & B.

For the oil reservoir the assumed values for r_D are :   2,4,5,5.1,5.2,5.25,5.3,5.5,6.
    For the gas reservoir the assumed values for r_D are :   2,2.25,2.5,2.75,3,4,6,8,10

RESULTS & INTERPRETATIONS

Table (4) & (5) show the values of C_{tD} , σ_{min} , N or G, & B for each value of r_D of the oil and gas reservoir respectively.

Table: 4

CtD	St.Dev.	N,MM STB	B,Bbl/psi	CtD	St.Dev.	N,MM STB	B,Bbl/psi	CtD	St.Dev.	N,MM STB	B,Bbl/psi
RD=2				**RD=4**				**RD=5**			
0.225	0.1909	332.88	44803.26	2.250	0.7285	284.40	11814.38	4.500	0.5042	286.06	7432.54
0.225	0.1909	332.87	44805.17	2.565	0.1023	315.18	9974.09	4.995	0.1617	317.34	6270.33
0.203	0.3237	289.27	56141.40	2.565	0.1023	315.17	9974.42	4.950	0.1616	313.73	6393.54
0.270	0.6529	401.36	28288.74	2.610	0.1252	322.49	9592.00	4.950	0.1616	313.73	6393.54
0.225	0.1909	332.87	44805.17	2.587	0.1091	318.84	9782.21	4.995	0.1617	317.31	6271.38
0.270	0.6529	401.36	28288.74	2.565	0.1023	315.19	9973.77	4.882	0.1779	308.37	6578.37
0.315	0.8608	435.48	20385.41	2.520	0.1261	308.06	10353.08	4.860	0.1875	306.62	6639.55
0.360	0.9770	454.14	16164.55	2.520	0.1261	308.06	10353.08	4.860	0.1875	306.62	6639.55
0.405	0.3237	465.60	13619.05	2.633	0.1467	326.10	9404.93	4.950	0.1616	313.71	6394.19
4.275	0.1607	316.14	6056.33	4.500	0.1635	312.51	5931.14	4.500	0.3436	292.45	6341.04
4.253	0.1612	314.03	6125.48	4.500	0.1635	312.51	5931.14	4.500	0.2281	301.40	6166.04
4.230	0.1648	311.90	6195.39	4.523	0.1609	314.49	5868.59	4.658	0.1610	314.37	5756.91
4.275	0.1607	316.11	6057.11	4.500	0.1635	312.47	5932.17	4.680	0.1608	316.34	5696.33
4.320	0.1679	320.39	5918.44	4.500	0.1635	312.47	5932.17	4.725	0.1674	320.18	5578.68
4.253	0.1612	314.04	6125.12	4.590	0.1682	320.47	5681.71	4.725	0.1674	320.19	5578.47
4.320	0.1679	320.39	5918.44	4.567	0.1634	318.46	5744.15	4.725	0.1674	320.18	5578.68
4.253	0.1612	314.04	6125.21	4.500	0.1635	312.51	5931.19	4.680	0.1608	316.31	5696.99
4.275	0.1607	316.11	6057.11	4.455	0.1767	308.59	6055.95	4.658	0.1610	314.37	5756.90
RD=5.3				**RD=5.5**				**RD=6**			
4.500	0.3436	292.45	6341.04	4.500	0.9370	297.45	5992.38	0.248	1.4771	558.15	-5122.73
4.793	0.1612	314.09	5653.05	5.535	0.2067	306.07	5418.11	0.248	1.4771	558.15	-5122.73
4.770	0.1640	312.21	5710.10	5.490	0.2081	302.23	5523.79	0.495	1.4778	559.32	-2815.31
4.838	0.1624	317.88	5539.01	5.513	0.2066	304.15	5470.86	6.615	0.4655	274.84	5294.07
4.860	0.1662	319.73	5483.57	5.535	0.2067	306.00	5419.71	7.380	0.1967	285.98	4923.97
4.882	0.1718	321.63	5427.01	5.512	0.2066	304.15	5470.76	7.425	0.1969	289.38	4847.34
4.860	0.1662	319.77	5482.51	5.580	0.2114	309.83	5314.85	7.425	0.1969	289.40	4846.91
4.860	0.1662	319.76	5482.82	5.468	0.2113	300.33	5576.42	7.402	0.1963	287.71	4885.08
4.725	0.1769	308.53	5822.92	5.625	0.2210	313.69	5210.10	7.380	0.1967	285.94	4924.80

Table: 5

$R_D=2$				$R_D=2.25$				$R_D=2.5$			
CtD	St.Dev.	G, BSCF	B,bbl/psi	CtD	St.Dev.	G, BSCF	B,bbl/psi	CtD	St.Dev.	G, BSCF	B,bbl/psi
0.080	0.0876	3097.92	354210.10	0.120	0.0856	2987.36	286487.40	0.160	0.0847	2832.359	251056.00
0.160	0.0876	3662.42	204076.40	0.160	0.0939	3322.21	214672.30	0.160	0.0847	2832.359	251056.00
0.240	0.1004	3779.82	176798.20	0.240	0.0856	3621.24	158018.50	0.240	0.0937	3318.547	166681.50
0.320	0.1078	3813.46	169293.10	0.320	0.0939	3733.56	138361.00	0.320	0.0847	3551.68	132045.40
0.400	0.1119	3824.13	166953.60	0.400	0.1010	3782.84	130056.20	0.400	0.088	3668.513	115785.10
0.480	0.1142	3827.80	166159.10	0.480	0.1059	3806.19	126203.80	0.480	0.0937	3732.51	107204.20
0.560	0.1156	3828.96	165905.30	0.560	0.1092	3817.66	124334.80	0.560	0.0988	3769.656	102341.00
0.640	0.1164	3829.39	165813.20	0.640	0.1116	3823.47	123396.10	0.640	0.1027	3792.045	99457.80
0.720	0.1169	3829.55	165779.90	0.720	0.1132	3826.40	122924.40	0.720	0.1058	3805.879	97697.51
0.800	0.1171	3829.59	165769.90	0.800	0.1144	3827.96	122675.00	0.800	0.1081	3814.457	96614.04
$R_D=2.75$				$R_D=3$				$R_D=4$			
0.240	0.0848	2902.71	189843.70	0.320	0.0847	2880.10	158572.60	0.840	0.0825	2949.59	81025.16
0.240	0.0848	2902.71	189843.70	0.320	0.0847	2880.04	158578.00	0.880	0.0835	3000.97	77554.74
0.240	0.0848	2902.71	189843.70	0.360	0.0863	3031.42	140351.90	0.840	0.0825	2949.63	81023.74
0.320	0.0917	3254.22	141421.40	0.320	0.0847	2880.04	158578.00	0.800	0.0837	2882.40	85322.13
0.400	0.0878	3462.16	116006.90	0.400	0.0890	3161.93	125831.90	0.800	0.0837	2882.40	85322.13
0.480	0.0848	3585.76	101789.40	0.480	0.0950	3356.98	105549.70	0.720	0.0868	2747.88	94928.33
0.560	0.0876	3663.01	93232.91	0.560	0.0866	3486.82	92821.34	0.840	0.0825	2949.59	81025.16
0.640	0.0917	3713.48	87786.84	0.640	0.0847	3575.64	84446.85	0.960	0.0874	3119.65	70423.42
0.720	0.0956	3747.50	84185.34	0.720	0.0863	3638.60	78684.27	0.720	0.0868	2747.88	94928.33
0.800	0.0990	3771.05	81730.56	0.800	0.0890	3684.21	74602.12	0.800	0.0837	2882.40	85322.13
$R_D=6$				$R_D=8$				$R_D=10$			
1.920	0.0863	2704.19	43586.92	2.400	0.1515	3339.93	22939.01	0.600	0.1635	5008.83	14904.13
1.920	0.0863	2703.98	43591.59	4.800	0.0914	2642.33	24721.22	4.800	0.1534	3555.96	11840.54
1.920	0.0863	2704.03	43590.34	6.000	0.0847	2870.39	20293.01	7.200	0.1107	2665.67	16978.23
1.920	0.0863	2704.02	43590.61	5.920	0.0847	2853.49	20569.44	9.600	0.0871	2709.70	14737.28
2.000	0.0877	2707.37	42976.05	6.000	0.0847	2870.43	20292.51	11.000	0.0847	2866.56	12953.69
1.920	0.0863	2704.03	43590.34	6.000	0.0847	2870.50	20291.81	11.040	0.0847	2871.02	12907.14
2.240	0.0880	2796.89	39450.69	5.880	0.0847	2845.11	20708.62	10.920	0.0847	2857.37	13049.39
1.920	0.0863	2703.91	43593.03	6.080	0.0847	2887.34	20020.25	10.880	0.0847	2852.71	13098.00
2.160	0.0882	2746.90	41041.82	6.120	0.0847	2895.62	19887.58	11.160	0.0847	2884.86	12765.32
2.000	0.0877	2707.44	42974.53	6.000	0.0847	2870.61	20290.55	10.800	0.0847	2843.60	13194.37

Figures (1) through (9) shows the plot of σ_{min}, N & B vs. C_{tD}.

For the oil reservoir the values of C_{tD}, r_D, and B are 5.67, 5, 6446 bbl/psi and the calculated value for N is 312 MMSTB (as shown in reference 8).

Figures (1) through (3) show no trend for the minimum standard deviation and non of the N or B values become constant; so the values of r_D are not correct.

Figures (4) through (6) show that r_D value is between 5 & 5.25

Figures (7) through (9) show that r_D value is between 5.1 & 5.2.

Finally the value of r_D is between 5, 5.1 & 5.2 and the exact value corresponding to the minimum value of the standard deviation which is 5.1 and the values of C_{tD}, N, B are 4.275, 316.MMstb, 6057 Bbl/psia respectively.

Figures (10) through (18) shows the plot of σ_{min}, G & B vs. C_{tD}.

Figures (11) through (14) show no trend for the minimum standard deviation and non of the G or B values become constant; so the values of r_D are not correct.

Figure(10) shows an indication that r_D almost correct; so another values of r_D were assumed (2.25,2.5,2.75,3.0) to be close to the correct value. Figure (15) ($r_D=2.25$) shows a smooth trend like the trend of $r_D=2$., therefore r_D is between 2.0 &2.25; choose the one with minimum standard deviation, in this case $r_D=2.25$ and hence the values of C_{tD}, G, and B are 0.24, 3.62 TSCF,158.018 Mbbl/psia.

CONCLUSION

1. Results of this field and the example study show the modified approach by Jawad, unsteady –state equation to be a powerful tool in the prediction of the water influx especially in case of trial & error on value of r_D.

2. The accurate data on aquifer and reservoir geometry and fluid are not required to predict the values of N,G, and B.

3. It is necessary to get the minimum of the minimum values of the standard deviation to determine values of N,G, and B.
4. It is not enough to start with one initial value of C_{tD}.

Nomenclature :

B = Aquifer Constan, bbl/psia
C_f = aquifer compressibility ,psi^{-1}
C_w = Water compressibility, psi^{-1}
c_t = $c_f + c_w$
G = gas in-place, TSCF
G_p = Cumulative gas produced, Scf
h = aquifer thickness, ft
k = permeability, md
N = Oil in-place, MMstb
N_p = Cumulative oil Produced, Stb
R_S = Initial gas solubility, Scf/Stb
R_P = Cumulative Gas Oil ratio,Scf/Stb
r_o = reservoir radius ,ft
r_e = aquifer radius , fr
r_D = dimensionless radius = r_e / r_o
t = time , year
t_D = dimensionless time
W_e = Cumulative Water Influx, Bbl
ΔP = Pressure drop, $(P_o - P_{wf})$, psia
P_o = Initial Reservoir Pressure, psia
ΔP_i = $(P_o - P_i)/2$ for i=1,2
ΔP_i = $(P_i - P_{i+2})/2$ for i >=3
Φ = porosity ,fraction
μ_w = water viscosity , cp
β_o = Oil formation volume factor, Bbl/Stb
β_g = Gas formation volume factor Bbl/Scf
σ = Standard deviation
Ψ = Shape Factor.
i is initial.

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SI Metric Conversion Factors:

Bbl x 1.589873 E-01 = m^3
ft x 3.048 E-01 = m
ft^3 x 2.831685 E -02 = m^3
md x 9.869233 E-04 = m^2
psi x 6.894757 E+00 = kpa
scf/Bbl x 1.801175 E-01 = m^3/m^3(st)

Fig(1) $R_D = 2$

Fig(4) $R_D = 5$

Fig(2) $R_D = 4$

Fig(5) $R_D = 5.25$

Fig(3) $R_D = 6$

Fig(6) $R_D = 5.5$