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Design of Piezoelectric Smart Structures with Single Input Single Output (SISO) System for Active Vibration Control

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Abstract: This paper deals with the Active Vibration control of beam like structures with distributed piezoelectric actuator and sensor layers bonded on top and bottom surfaces of the beam. The patches are located at the different positions to determine the better control effect. The piezoelectric material is placed on the free, middle and fixed end alternately. The study is demonstrated through simulation in MATLAB for various controllers like Proportional Controller by Output Feedback, Proportional Integral Derivative controller (PID) and Pole Placement technique. A smart cantilever beam is modeled with SISO system. The entire structure is modeled using the concept of piezoelectric theory, Euler-Bernoulli beam theory, Finite Element Method (FEM) and the State Space techniques. The numerical simulation shows that the sufficient vibration control can be achieved by the proposed method.

Keywords: Smart Structure, Finite Element model, State Space model, Proportional Output feedback, PID, Pole Placement, Vibration Control.

1. Introduction

The Piezoelectric materials are used because if the forces are applied on that material it produces voltage and this voltage goes to active devices and control the vibration. Their reliability, nearly linear response with applied voltage and their low cost make piezoelectric materials the most widely preferred one as collocated sensor and actuator pair. Active vibration control is the active application of force in an equal and opposite fashion to the forces imposed by external vibration.

The smart structures can be defined as:
"The structure that can sense external disturbance and respond to that with active control in real time to maintain the mission requirements."

The present work considers the application of piezoelectric patches to smart beam-like structures for the purpose of active vibration control with Single Input Single Output (SISO) system. The finite element method is powerful tool for designing and analyzing smart structures. Both structural dynamics and control engineering need to be dealt to demonstrate smart structures. A design method is proposed by incorporating control laws such as Propotional Output Feedback (POF) and Proprotional Integral Derivative (PID) and Pole Placement technique to suppress the vibration. **Baz and Poh (1988) (2)** investigated methods to optimize the location of piezoelectric actuators on beams to minimize the vibration amplitudes. **Suleman (1998) (8)** proposed the effectiveness of the piezo-ceramic sensor and actuators on the attentuation of vibrations on an experimental wing due to the gust loading. **Brij N Agrawal and Kirk E Treanor (1999) (3)** presented the analytical and experimental results on optimal placement of Piezoceramics actuators for shape control of beam structures. **Manning, Plummer & Levesley (2000) (5)** presented a smart structure vibration control scheme using system identification and pole placement technique. **Raja, S, prathap G & Sihna (2002) (7)** studied active vibration control of a composite sandwich beam with two kinds of piezoelectric actuator or such as extension-bending and shear. **Xu & Koko (2004) (10)** proposed results by using the commercial FE-package and ANSYS. **Baillargeon & Vel (2005) (1)** presented vibration suppression of adaptive sandwich cantilever beam using PZT shear actuators by experiments and numerical simulations. **T. C. Manjunath & B. Bandyopadhyay (2007) (9)** presented the modeling and design of a multiple output feedback based discrete sliding mode control scheme application for the vibration control of a smart cantilver beam of three four and five elements. **N.S. Viliani1, S.M.R. Khalili (2009) (6)** studied the active buckling control of smart functionally graded (FG) plates using piezoelectric sensor/actuator patches. **M. Yaqoob Yasin, Nazeer Ahmad (2010) (4)** presented the active vibration control of smart plate equipped with patched piezoelectric sensors and actuators.

In most of present researches, FEM formulation of smart cantilever beam is usually done by ANSYS and by design of control laws are carried out in Mat LAB toolbox. The objective of this work is to design and analysis of piezoelectric smart structures with control laws. Proportional Output Feedback (POF) Controller, Proportional Integral Derivative (PID) control law and Pole Placement Technique is used to suppress the vibrations. The eigenvalues of the closed loop system are also controlled with Pole Placement Technique. Numerical examples are presented to demonstrate the validity of the proposed design scheme. This paper has organized in to three parts, FEM formulation of piezoelectric smart structure with control laws, Numerical simulation and Conclusion.

2. Modeling of Smart Cantilever Beam

2.1 Finite Element Formulation of Beam Element

A beam element is considered with two nodes at its end. Each node is having two degree of freedom (DOF) i.e. translation and rotation is considered. The shape functions of the element are derived by applying boundary conditions. The mass and stiffness matrix is derived using shape functions for the beam element. To obtain the mass and stiffness matrix of smart beam element which consists of two piezoelectric materials and a beam element, are added. The global mass and stiffness matrix is formed. The boundary conditions are applied on the global matrices for the cantilever beam. The first two rows and two columns should be deleted as one end of the cantilever beam is fixed. The actual response of the system i.e. the tip displacement is obtained for all the various models of the cantilever beam with and without the controllers.

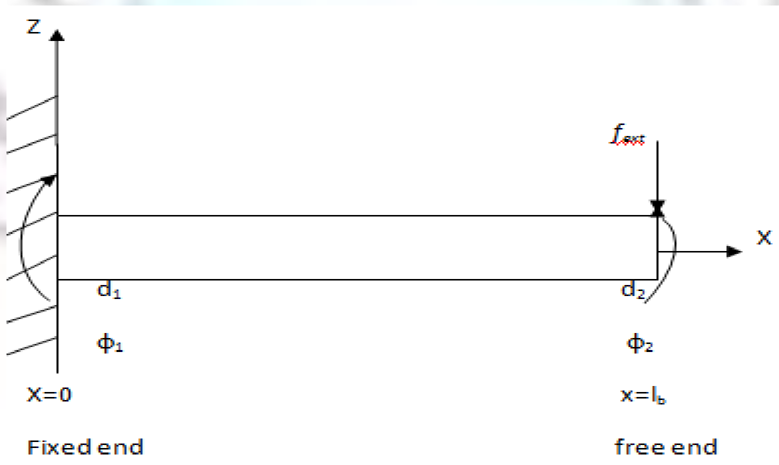

Fig. 1 Cantilever beam subjected to force

The displacement d is given by

$$
[n] = \begin{array}{l} [f_1(x)] = [1 - 3x^2/l_b^2 + 2x^3/l_b^3] \\ [f_2(x)] = [x - 2x^2/l_b + x^3/l_b^2] \\ [f_3(x)] = [3x^2/l_b^2 - 2x^3/l_b^3] \\ [f_4(x)] = [-x^2/l_b + x^3/l_b^2] \end{array} \tag{1}
$$

Where [n] gives the shape functions and [q] is the vector of displacements and slopes (nodal displacement vector).
The equation of motion of the regular beam element is obtained by the lagrangian equation

$$
\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_i}\right] + \left[\frac{\partial U}{\partial \dot{q}_i}\right] = [F_i]
$$

As $M^b\ddot{q} + K^b q = f^b(t)$ (2)

Where M^b, K^b and F^b are the mass, stiffness and force co-efficient vector matrices respectively of the regular beam element. The mass and stiffness matrices are obtained as

$$\left[M^b\right] = \rho_b A_b \int_{l_b} [n_3]^T [n_3] dx$$

$$= \rho_b A_b \int_{l_b} \begin{bmatrix} 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3} \\ x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} \\ \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} \\ -\frac{x^2}{l_b} + \frac{x^3}{l_b^2} \end{bmatrix} \begin{bmatrix} \left(1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3}\right) & \left(x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2}\right) & \left(\frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3}\right) & \left(-\frac{x^2}{l_b} + \frac{x^3}{l_b^2}\right) \end{bmatrix}$$

Combining all the elements the mass matrix becomes

$$\left[M^b\right] = \int \frac{\rho_b A_b l_b}{420} \begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ 13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix} \quad (3)$$

$$\left[K^b\right] = E_b I_b \int_{l_b} [n_3]^T [n_3] dx$$

$$= \begin{bmatrix} -\frac{6}{l_b^2} + \frac{12x}{l_b^3} \\ -\frac{4}{l_b} + \frac{6x}{l_b^2} \\ \frac{6}{l_b^2} - \frac{12x}{l_b^3} \\ -\frac{2}{l_b} + \frac{6x}{l_b^2} \end{bmatrix} \begin{bmatrix} -\frac{6}{l_b^2} + \frac{12x}{l_b^3} & -\frac{4}{l_b} + \frac{6x}{l_b^2} & \frac{6}{l_b^2} - \frac{12x}{l_b^3} & -\frac{2}{l_b} + \frac{6x}{l_b^2} \end{bmatrix}$$

$$K_b = \frac{E_b I_b}{l_b} \begin{bmatrix} \frac{12}{l_b^2} & \frac{6}{l_b} & -\frac{12}{l_b^2} & \frac{6}{l_b} \\ \frac{6}{l_b} & 4 & -\frac{6}{l_b} & 2 \\ -\frac{12}{l_b^2} & -\frac{6}{l_b} & \frac{12}{l_b^2} & -\frac{6}{l_b} \\ \frac{6}{l_b} & 2 & -\frac{6}{l_b} & 4 \end{bmatrix} \quad (4)$$

$$EI = E_b I_b + 2E_P . I_P \quad (5)$$

$$I_P = \frac{1}{12} b t_a^3 + b t_a \left(\frac{t_a + t_b}{2}\right)^2 \quad (6)$$

$$\rho A = b(l_b t_b + 2\rho_P t_a) \quad (7)$$

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Table no. 1 Properties of flexible cantilever beam and piezoelectric

Parameter for beam (with units)	Symbol	Numerical value	Parameter for piezoelectric (with units)	Symbol	Numerical value
Length (m)	l_b	0.075	Length (m)	l_p	0.075
Width (m)	b	0.03	Width (m)	b	0.03
Thickness (mm)	t_b	0.5	Thickness (mm)	t_a	0.35
Young modulus (GPa)	E_b	193.06	Young modulus (GPa)	E_p	68
Density (kg/m^3)	ρ_b	8030	Density (kg/m^3)	ρ_p	7700
Damping constant	α, β	0.001, 0.0001	Pierostrain constant	d_{31}	125 x 10^{-12}
			Piezostress constant	g_{31}	10.5 x 10^{-3}

Beam with Piezoelectric at Different Positions

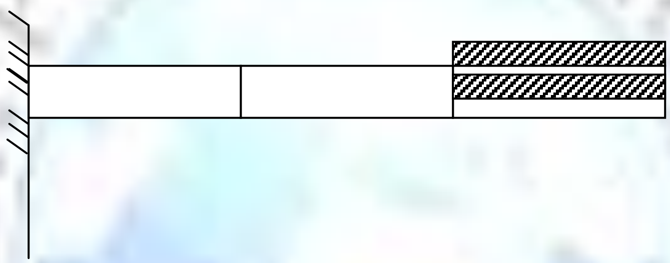

Fig. 2 Piezoelectric placed on free end

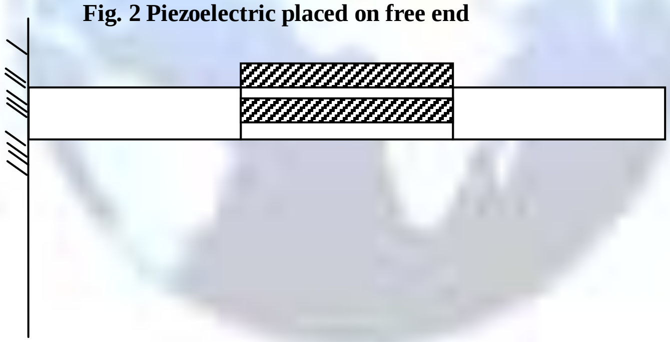

Fig. 3 Piezoelectric placed on middle end

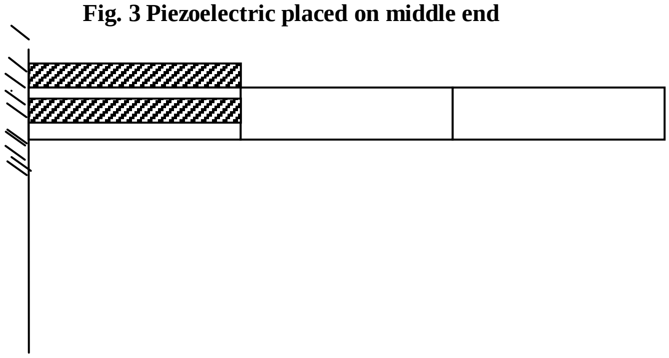

Fig. 4 Piezoelectric placed on fixed end

The equation of motion of the smart structure is finally given by

$$M\ddot{q} + Kq = f_{ent} + f_{cntrl} = f_t \tag{8}$$

Where $M, K, q, f_{ent}, f_{cntrl}, f^t$ is the global mass matrix, global stiffness matrix of the smart beam, the vector of displacements and slopes, and the external force applied to the beam, the controlling force from the actuator and the total force vector respectively. The generalized coordinates are introduced into the (4.16) equation using a transformation q = Tg in order to reduce it.

The equation (4.16) now becomes

$$MT\ddot{g} + KTg = f_{ent} + f_{cntrl} = f_t \tag{9}$$

Premultiplying (2) by T^T, we get

$$T^T MT\ddot{g} + T^T KTg = T^T f^t \tag{10}$$

Which can be written as $M^*\ddot{g} + K^*g = f^*_{ent} + f^*_{cntrl} = f^*_t$ (11)

The above equation (4.19) can be written as

$$M^*\ddot{g} + C^*\dot{g} + K^*g = f_t^* \tag{12}$$

By introducing the generalized structural model damping matrix $C^* = aM^* + bK^*$ where α and β are frictional damping constants and the structural damping constant used in C*.

2.2 Sensor equation

The total charge Q(t) developed on the sensor surface is the spatial summation of all the point charges developed on the sensor layer. Thus, the expression for the current generated is obtained as

$$i(t) = \frac{dQ(t)}{dt} = \frac{d}{dt}\int_A e_{31}e_x dA = ze_{31}b\int_0^{l_p} n_1^T \dot{q}dx, \tag{13}$$

Where $z = \frac{t_b}{2} + t_a$

This current is converted into the open circuit sensor voltage V^s using a signal-conditioning device with the gain G_c. The sensor output voltage is obtained as

$$V^s(t) = G_c e_{31} zb\int_0^{l_p} n_1^T \dot{q}dx, \tag{14}$$

Where d_{31} is the piezoelectric constant, e_{31} is the piezoelectric stress / charge constant, E_p is the young's modulus and ε_x is the strain that is produced.

2.3 Actuator equation:

The actuator strain is derived from the converse piezoelectric equation. The strain developed e_a on the actuator layer is given by

$$e_a = d_{31}E_f \tag{15}$$

Where d_{31} and E_f are the piezo strain constant and the electric field respectively. When the input to the piezoelectric actuator $V^a(t)$ is applied in the thickness direction t_a, the electric field, E_f which is the voltage applied $V^a(t)$ divided by the thickness of the actuator t_a and the stress, s_a which is the actuator strain multiplied by the young's modulus E_p of the piezo actuator layer are given by

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$$E_f = \frac{V^a(t)}{t_a} \tag{16}$$

Finally, the control force applied by the actuator is obtained as

$$f_{ctrl} = E_p d_{31} b \bar{z} \int_{l_p} n_2 dx V^a(t) \tag{17}$$

Where $\bar{z} = \frac{(t_a + t_b)}{2}$ is the distance between the neutral axis of the beam and the piezoelectric layer or can be expressed as a scalar vector product as

$$f_{ctrl} = hV^a(t) = hu(t) \tag{18}$$

where n_2^T is the first spatial derivative of the shape function of the flexible beam, h^T is a constant vector which depends on the type of actuator and its location on the beam, given by $h = \begin{bmatrix} E_p d_{31} b\bar{z} & 0 & E_p d_{31} b\bar{z} & 0 \end{bmatrix}$ and u(t) is nothing but the control input to the actuator, i.e. $V^a(t)$ from the controller. If any external forces are acting on the beam, then the total force vector becomes

$$f_t = f_{ext} + f_{ctrl}. \tag{19}$$

2.4 State space model of the smart cantilever beam:
The following equation can be written in state space from as follows:

$$M^* \ddot{g} + C^* \dot{g} + K^* g = f_{ent}^* + f_{ctrl}^* = f_t^*$$

Let the states of the system be defined as

$$\dot{g} = x = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \text{and } \ddot{g} = \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \tag{20}$$

Now equation becomes

$$M^* \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} + c^* \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + K^* \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = r_{ent}^* + r_{ctrl}^* \tag{21}$$

This can be further simplified as

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = -M^{*-1} K^* \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - M^{*-1} C^* \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + M^{*-1} r_{ent}^* + M^{*-1} r_{ctrl}^* \tag{22}$$

and finally written in state equation form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ M^{*-1} K^* & -M^{*-1} C^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{*-1} T^T n \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ M^{*-1} T^T r \end{bmatrix} r(t)$$

i.e. $\dot{x} = Ax(t) + Bu(t) + Er(t)$ (23)

The output equation for sensor for a SISO case is given by

$$y(t) = v^s(t) = p^T \dot{q} = p^T T \dot{g} = P^T T \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \tag{24}$$

$$= \begin{bmatrix} 0 & P^T T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

and written in output equation form as

$$y(t) = c^T x(t) + Du(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ M^{*-1}K^* & -M^{*-1}C^* \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ M^{*-1}T^T h \end{bmatrix}$$ symbol h

$$C^T = \begin{bmatrix} 0 & P^T T \end{bmatrix}$$

D=null matrix

$$E = \begin{bmatrix} 0 \\ M^{*-1}T^T r \end{bmatrix} r(t)$$

3. Control Laws

The various control laws such as one control law, which is based on Proportional Output Feedback by assuming arbitrary value and one classical control law Proportional Integral Derivative (PID) based on state feedback and one control law which is based on Pole Placement by state feedback has been explained as:-

3.1 Control with POF controller

In the first case, the responses are taken by giving impulse input. The Proportional Output Feedback controller is designed by taking the arbitrary value of gain. Output Feedback control provides a more consequential design. The responses are also plotted by changing the position of sensor and actuator on the beam i.e. free end, middle end and fixed end alternately.

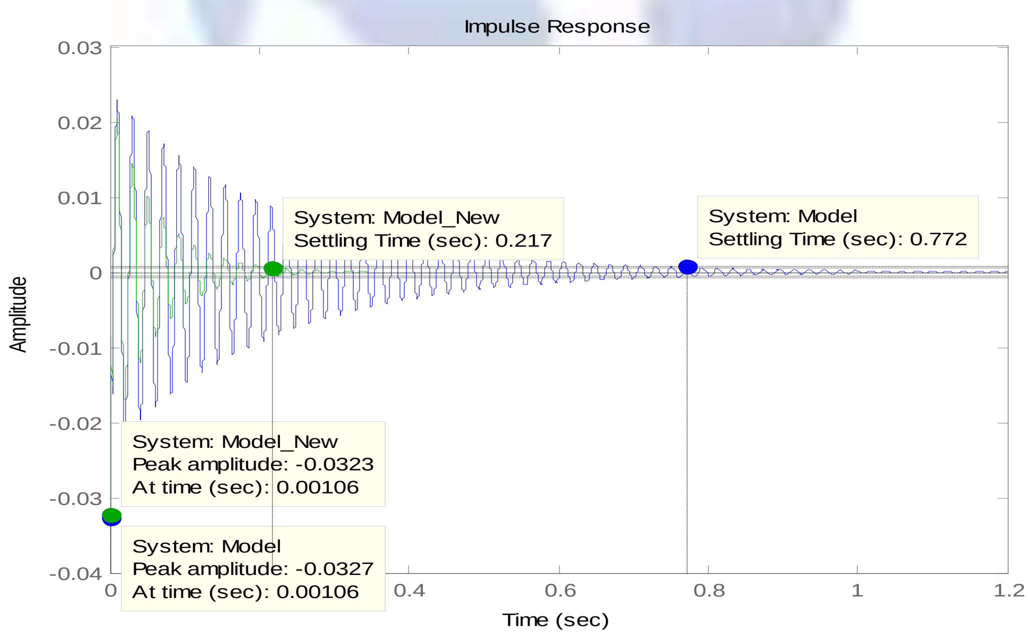

Fig. 5 Tip displacement of cantilever beam when piezoelectric materials at free end position

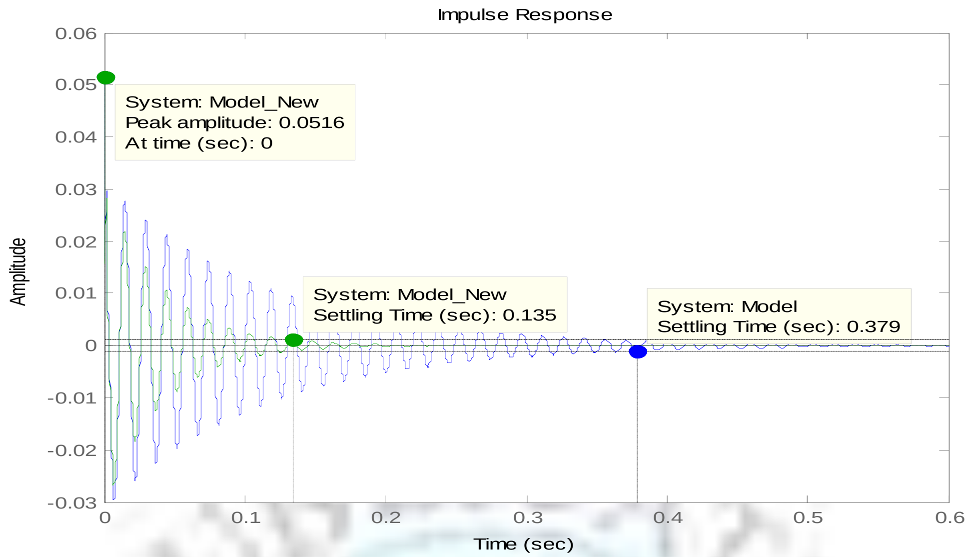

Fig. 6 Tip displacement of cantilever beam when piezoelectric materials at middle end position

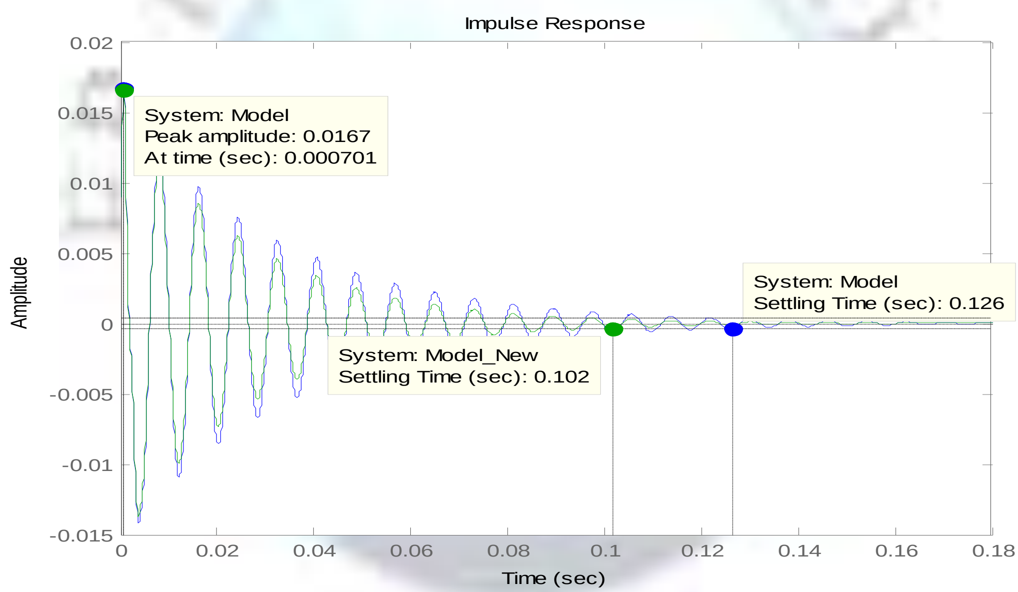

Fig. 7 Tip displacement of cantilever beam when piezoelectric materials at fixed end position

3.2 Control with PID controller:

In a PID controller the control action **s** generated as a sum of three terms. It is given by $G_1(s) = k_p + \frac{k_i}{s} + k_d s$ (23)

- K_p = Proportional gain
- K_I = Integral gain
- K_d = Derivative gain

We use $k_d = 10, k_p = 100, k_i = 40$

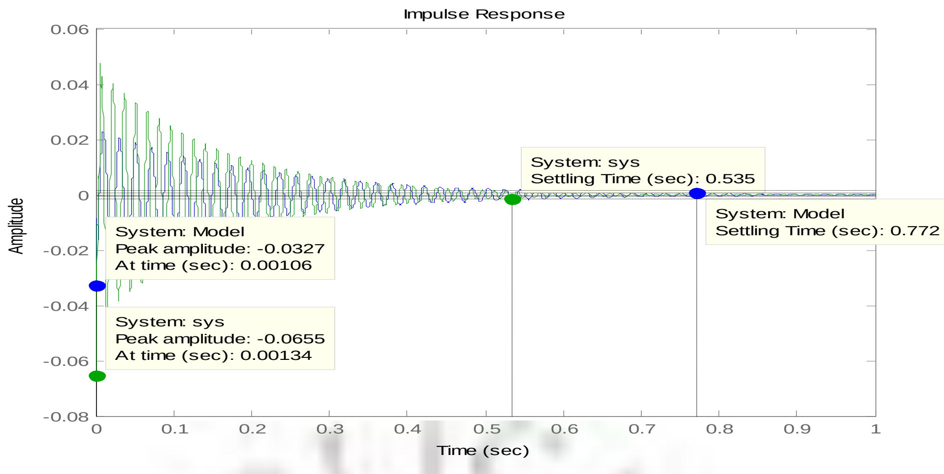

Fig. 8 Tip displacement of cantilever beam with or without PID controller when Piezoelectric materials at free end position

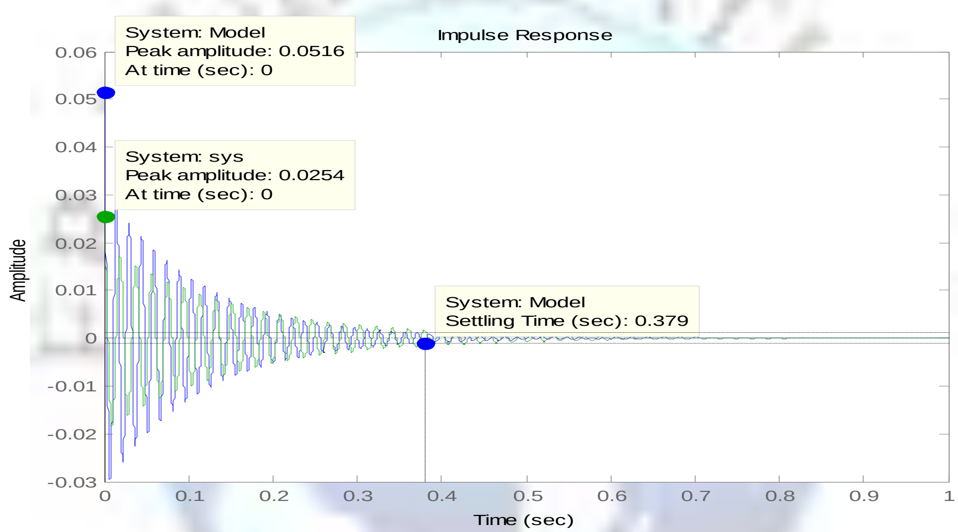

Fig. 9 Tip displacement of cantilever beam with or without PID controller when piezoelectric materials at middle end position

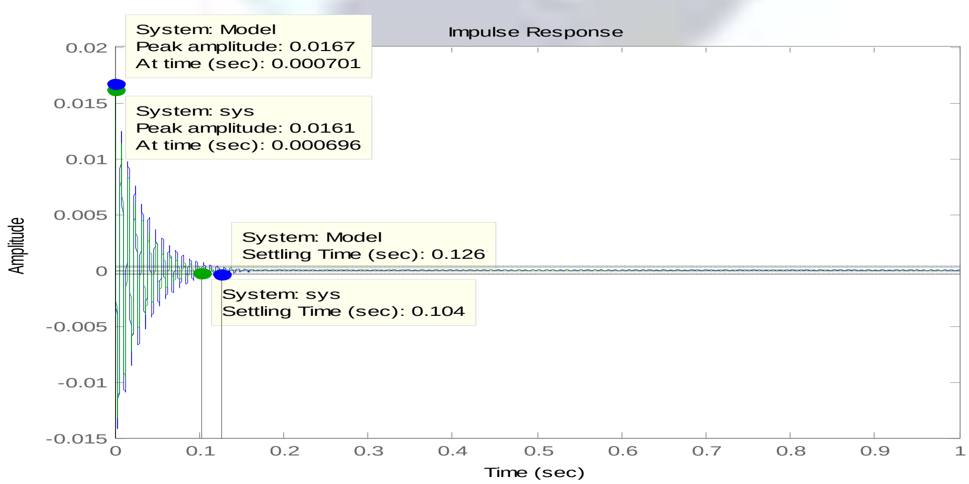

Fig. 10 Tip displacement of cantilever beam with or without PID controller when piezoelectric material fixed end position

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3.3 Control with Pole Placement technique:

Pole Placement Technique is used to determine the value of feedback Gain K for the required control in which we can control according to the desired Eigen vectors and frequency. The present design technique begins with a determination of the desired closed-loop poles based on the transient-response and/or frequency-response requirements such as Eigen vectors, damping ratio. By applying the Pole Placement Technique, we get

Free end

K =1.0e+006 *[-4.7904 -0.1697 -0.0002 0.0000]

Middle end

K = 1.0e+004 *[-4.3243 1.2622 -0.0002 -0.0000]

Fixed end

K = 1.0e+009 *[-0.5593 1.4531 0.0000 0.0001]

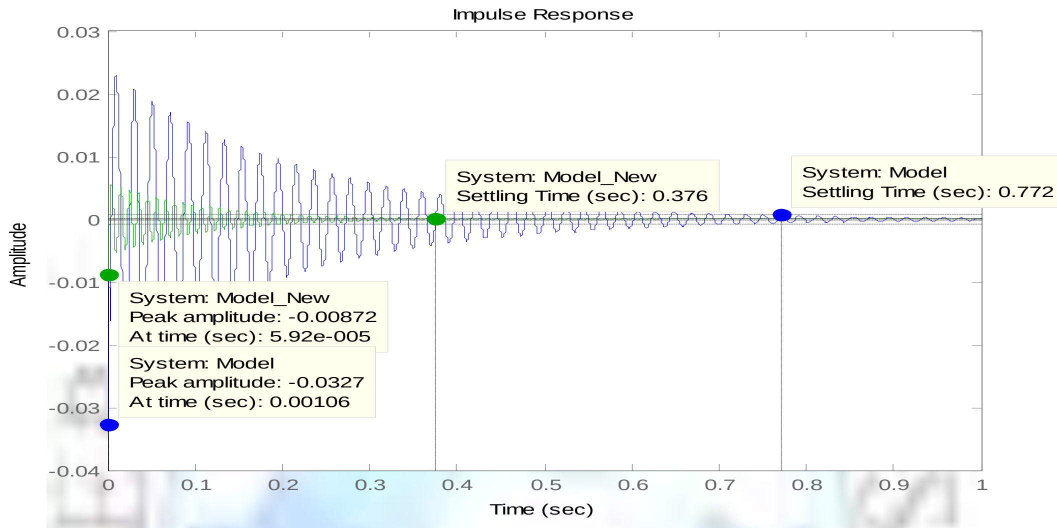

Fig. 11 Tip displacement of cantilever beam with or without Pole Placement technique when piezoelectric material on free end position

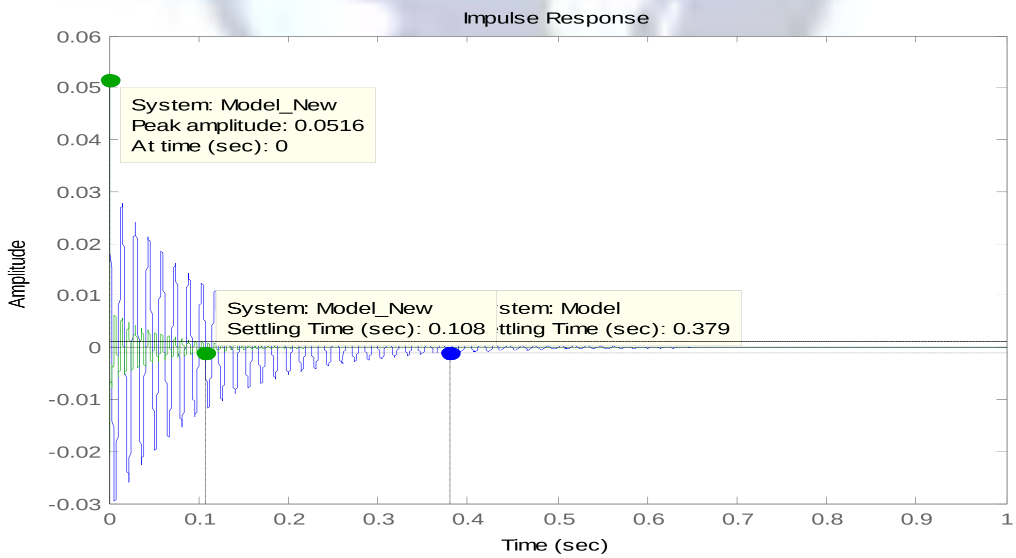

Fig. 12 Tip displacement of cantilever beam with or without Pole Placement technique when piezoelectric material on middle end position

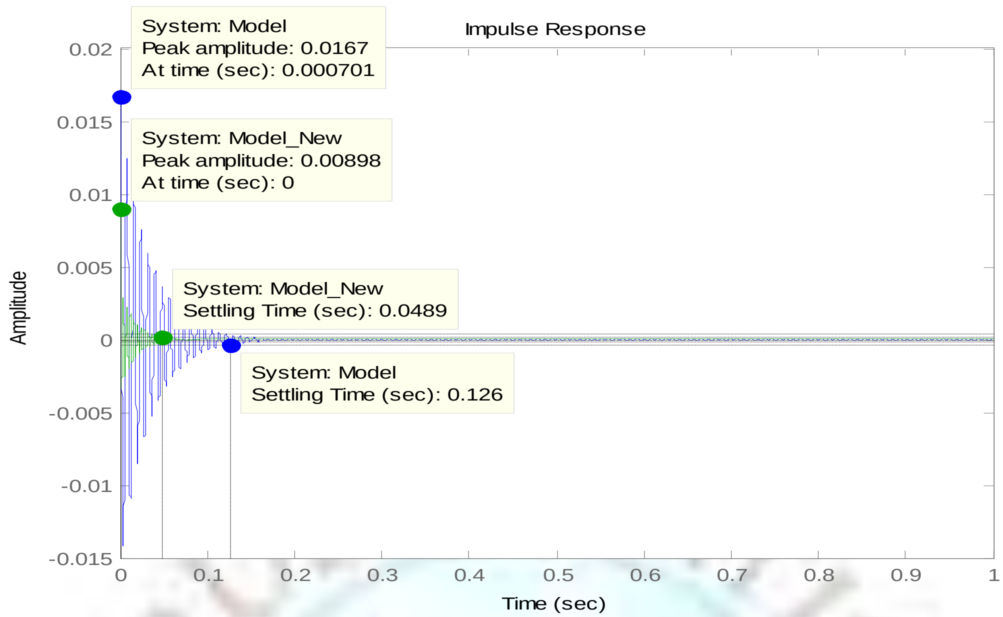

Fig. 13 Tip displacement of cantilever beam with or without Pole Placement technique when piezoelectric material on fixed end position

4. Results and Conclusion

Present work deals with the mathematical formulation and the computational model for the active vibration control of a beam with piezoelectric smart structure. A general scheme of analyzing and designing piezoelectric smart structures with control laws is successfully developed in this study. It has been observed that without control the transient response is predominant and with control laws, sufficient vibrations attenuation can be achieved. Numerical simulation showed that modeling a smart structure by including the sensor / actuator mass and stiffness and by varying its location on the beam from the free end to the fixed end introduced a considerable change in the system's structural vibration characteristics. From the responses of the various locations of sensor/actuator on beam, it has been observed that best performance of control is obtained, when the piezoelectric element is placed at fixed end position.

Active Vibration control for different types of controller

Different type of controller	Free end		Middle end		Fixed end	
	Settling time(in sec.)	peak response (in mm)	Settling time (in sec.)	Peak response	Settling time (in sec.)	Peak response
Uncontrolled loop	0.772	-0.0327	0.379	0.0516	0.126	0.0167
POF Controlled loop	0.217	-0.0323	0.135	0.0516	0.102	0.0167
PID Controlled loop	0.535	-0.0655	0.379	0.0254	0.104	0.0161
Pole Placement Controlled loop	0.378	-0.0872	0.108	0.0516	0.0489	0.00898

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