A Review of Kinematics analysis, Workspace, Design and Control of 3-RPS parallel robots

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Abstract: Parallel robots find many applications in human-systems interaction, medical robots, rehabilitation, exoskeletons, to name a few. These applications are characterized by many imperatives, with robust precision and dynamic workspace computation as the two ultimate ones. This paper presents kinematic analysis, workspace, design and control to 3 degrees of freedom (DOF) parallel robots. Parallel robots have received considerable attention from both researchers and manufacturers over the past years because of their potential for high stiffness, low inertia and high speed capability. Therefore, the 3 DOF translation parallel robots provide high potential and good prospects for their practical implementation in human-systems interaction.

Keywords: kinematics, workspace, design, parallel robot, 3-RPS, 3 degrees of freedom.

I. INTRODUCTION

Parallel manipulators have become popular in recent years due to the advantages in terms of high stiffness, high force-toweight ratio, high load carrying capacity, and high precision control over the prescribed path of end-effecter. A parallel manipulator typically consists of a moving platform that is connected to a fixed base by several legs in parallel. Gough and Whitehall [1] devised a six-linear jack system for use as Universal tire testing machine. The six degrees of freedom (DOF) Stewart platform has designed for use as an aircraft simulator by Stewart [2]. A systematic study of possible kinematic structures of parallel manipulators is carried out by K.H. Hunt [3].

The six DOF parallel manipulators have many advantages mentioned above; however six DOF is not always required for many practical applications. Spatial parallel manipulators with less than six DOF gained prominence because of reduction in manufacturing cost and easiness in control. Many 3-DOF parallel manipulators have been designed and investigated for specific applications, such as the famous DELTA robot with three translational DOF [4, 5], the CaPaMan and HANA parallel manipulators with three spatial DOF [6,7]. A spatial 3-RPS parallel manipulator consists of three identical legs have been studied by Lee and Shah [8], Tsai [9] which allow the platform to rotate and translate. Joshi and Tsai [10] determined the singular positions of the 3-RPS manipulator through the Jacobian matrix. C.H. Liu. [11] applied a procedure to obtain direct singularity positions of a 3-RPS parallel manipulate or, in which the heights of the three spherical joints were used as coordinate axes and the workspace of the moving platform represented as an inclined solid cylinder in this coordinate system [12]. Yi Lu Bo Hu [13] proposed a unified and simple approach for solving inverse/forward velocity and acceleration of the limited-DOF PKMs with linear active legs. Jaime Gallardo [14] have carried out the forward position analysis of parallel manipulators with identical limb type revolute-prismatic-spherical (RP-S) leg by applying recursively the Sylvester dialytic elimination method. Rad [15] addressed the analysis of forward kinematics of a 3-DOF medical parallel robot with R-P-S joint structure using Newton- Kantorovich (N-K) method, in which workspace was determined through forward kinematics equation.

Stiffness is one of the most important aspects in the design of parallel kinematic machines because of higher stiffness allows higher feed rates with high accuracy in positioning of end-effecter. Dan Zhang [16] presented the kinetostatic modeling of the Tripod-based parallel kinematic machine and investigated the compliance over the workspace. Huy-Hoang Pham [17] presented the stiffness matrices of a double linear spring and a three DOF translational flexure parallel mechanism. Joo-Woo Kim et al. [18] performed the stiffness analysis of a 3 DOF parallel Robot with one constraining leg, which takes into account of elastic deformation of joints and links which is made up of three SPS (spherical-prismatic-spherical) legs and one UP (Universal-prismatic) leg at the center. Qingsong Xu and Yangmin Li [19] analyzed the stiffness of a three prismatic-revolute-cylindrical (3PRC) translational parallel manipulator, in which stiffness matrix is derived based upon screw theory with consideration of compliance of both actuators and legs. Yi Lu and Bo Hu [20] explained the stiffness analysis of over-constrained parallel manipulators.

Many scholars have performed optimum design of robot manipulators; Boeig [22] proposed numerical integration and sequential quadratic programming method for optimization of parallel manipulators.

However, the traditional optimization methods suffer from local optima and lack of convergence of the optimization algorithm. Holland [23] has described how genetic algorithms may be applied as powerful and broadly applicable stochastic search methods and optimization techniques, since they can escape from local optima. Liu X-J [24] introduces an approach to do optimum design of 3-DOF spherical parallel manipulators in order to optimize the performance indices GCI and GSI. Shiakolas [25] used three methods (simple genetic algorithm, genetic algorithm with elitism and differential evolution) to get optimum design of 2-link and 3-link serial manipulators. Study of parallel manipulators with the optimization criteria as the manipulability or dexterity of the manipulator was done by Alice [26]. Marco Ceccarelli [27] formulated a multi-objective optimization problem for 3R serial link manipulators by taking the workspace volume and robot dimensions as objective functions for the given workspace limits as constraints. Liu [28] presented a method for optimal kinematic design of a 3-dof parallel manipulator; the dimensional synthesis being carried out by introducing a tilt angle to achieve a nearly axial symmetry of kinematic performance with respect to configuration. N.M. Rao and K.M. Rao [29] have performed the dimensional synthesis of a 3-RPS parallel manipulator using a hybrid optimization method called GA-simplex method. Manuel R.

Barbosa [30] presented the kinematic design of a 6-DOF parallel manipulator for maximum dexterity, using the GA and neuro-GA in order to explore the advantages of neural networks and GA's. Raza Ur- Rehman [31] proposes a methodology to deal with the multi-objective design optimization of 3-PRR planar parallel manipulator with the size of the regular shaped workspace and the mass in motion of the mechanism are as the objective functions [32]. F.A. Lara-Molina [33] performed the optimal design of a spatial Stewart-Gough platform based on multi-objective optimization; the objective functions are Global Condition Index (GCI), Global Payload Index (GPI) and Global Gradient Index (GGI) using multi-objective Evolutionary Algorithm (MOEA). Chun-Ta Chen [34] presented a constrained multi-objective genetic algorithm for a general motor-driven parallel kinematic manipulator 3UPS for the optimal trajectory of a PKM with linear actuators, in which travelling time and energy expended in driving the platform from one pose to another are minimized. Antonio M. Lopes and E.J. Solteiro Pires [35] formulated an optimization problem in order to minimize power consumption and maximize the stiffness of the manipulator using a multi-objective genetic algorithm in order to find the work piece location in a machining Robotic cell. Ridha Kelaiaia [36] presented an approach for dimensional synthesis for parallel manipulators.



Fig. 1: The 3-RPS parallel robot

II. KINEMATICS ANALYSIS

Robot kinematics deals with the study of the robot motion as constrained by the geometry of the links [37]. Typically, the study of the robot kinematics is divided into two parts, inverse kinematics and forward (or direct) kinematics [38]. The inverse kinematics problem involves a known pose (position and orientation) of the output platform of the robot to a set of input joint variables that will achieve that pose [39]. The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the moving platform that results from those given inputs [40]. Fig. 2 shows a spatial parallel robot, 3-DOF, and 3-RPS of parallel robot. It consists of three identical links that connect the moving platform at points B_i by spherical joints to the fixed base at points Ai, by revolute joints. Each link consists of an upper and a lower member connected by a prismatic joint.

These three prismatic joints are used as inputs for the parallel robot.

Overall, there are eight links, three revolute joints, three prismatic joints and three spherical joints. Thus, the degree of freedom of the parallel robot can be computed with [41]:

$$\mathbf{F} = \lambda(\mathbf{n} - \mathbf{j} - \mathbf{1}) + \sum_{i} \mathbf{f}_{i} = \mathbf{6}(\mathbf{8} - \mathbf{9} - \mathbf{1}) + (\mathbf{3} + \mathbf{3} + \mathbf{9}) = \mathbf{3}$$
(1)

For the kinematic analysis, two Cartesian coordinate systems A(x, y, z) and B(u, v, w) are attached to the fixed base and moving platform, respectively, as shown in Fig. 2. The following assumptions are made. Points A1, A2, A3 lie on the xyplane and B1, B2 and B3 lie on the uv-plane. As shown in Fig. 2, the origin O of the fixed coordinate system is located at the centroid of Δ A1A2A3 and the x axis points in the direction of $\overline{OA_1}$.



Fig. 2: 3-RPS parallel robot with linear actuators



Fig. 3: Top views of the 3-RPS parallel robot

Similarly, the origin **P** of the moving coordinate system is located at the centroid of **AB1B2B3** and the **u**-axis in the direction of $\overline{PB_1}$. Both **AA1A2A3** and **AB1B2B3** are equilateral triangles with $|OA_1| = |OA_2| = |OA_3| = g$ and $|PB_1| = |PB_2| = |PB_3| = h$. Furthermore, the axis of each revolute joint, **Ji**, lies on the **x-y** plane and is perpendicular to the vector $\overline{OA_1}$.

The transformation from the moving platform to the fixed base can be described by a position vector

 $\mathbf{P} = \overline{\mathbf{OP}}$ and a $\mathbf{3} \times \mathbf{3}$ rotation matrix $\mathbf{R}_{\mathbf{A}}^{\mathbf{B}}$.

Let **u**, **v**, and **w** be three unit vectors defined along **u**, **v**, and **w** axes of the moving coordinate system B, respectively; then the rotation matrix can be expressed in terms of the direction cosines of **u**, **v** and **w** as:

$$R_{B}^{A} = \begin{bmatrix} u_{x} & v_{x} & w_{x} \\ u_{y} & v_{y} & w_{y} \\ u_{z} & v_{z} & u_{z} \end{bmatrix}$$
(2)

We note that the elements of R_B^A must satisfy the following orthogonal conditions:

$$u_{x}^{2} + u_{y}^{2} + u_{z}^{2} = 1$$

$$v_{x}^{2} + v_{y}^{2} + v_{z}^{2} = 1$$

$$w_{x}^{2} + w_{y}^{2} + w_{z}^{2} = 1$$

$$u_{x}v_{x} + u_{y}v_{y} + u_{z}v_{z} = 0$$

$$u_{x}w_{x} + u_{y}w_{y} + u_{z}w_{z} = 0$$

$$v_{x}w_{x} + v_{y}w_{y} + v_{z}w_{z} = 0$$
(3)

Let ai and b_i^B be the position vectors of points Ai and Bi, respectively. Then the coordinates of Ai and Bi are given by:

$$a_1 = [g, 0, 0]^T$$

 $a_2 = \left[-\frac{1}{2}g, \frac{\sqrt{3}}{2}g, 0 \right]^T$

$$a_{3} = \left[-\frac{1}{2}g, -\frac{\sqrt{3}}{2}g, 0 \right]^{t}$$

$$B_{b_{1}} = [h, 0, 0]^{T}$$

$$B_{b_{2}} = \left[-\frac{1}{2}h, \frac{\sqrt{3}}{2}h, 0 \right]^{T}$$

$$B_{b_{3}} = \left[-\frac{1}{2}h, -\frac{\sqrt{3}}{2}h, 0 \right]^{T}$$
(4)

The position vector qi and Bi with respect to the fixed coordinate system is obtained by the following transformation:

$$q_{i} = P + A_{R_{B}}B_{b_{i}}$$
(5)

$$q_{1} = \begin{bmatrix} px + hu_{x} \\ px + hu_{y} \\ px + hu_{z} \end{bmatrix}$$

$$q_{2} = \begin{bmatrix} px - \frac{1}{2}hu_{x} + \frac{\sqrt{3}}{2}hv_{x} \\ py - \frac{1}{2}hu_{y} + \frac{\sqrt{3}}{2}hv_{y} \\ pz - \frac{1}{2}hu_{z} + \frac{\sqrt{3}}{2}hv_{z} \end{bmatrix}$$

$$q_{3} = \begin{bmatrix} px - \frac{1}{2}hu_{x} - \frac{\sqrt{3}}{2}hv_{x} \\ py - \frac{1}{2}hu_{y} - \frac{\sqrt{3}}{2}hv_{x} \\ py - \frac{1}{2}hu_{z} - \frac{\sqrt{3}}{2}hv_{z} \end{bmatrix}$$
(6)

For the implementation and resolution of forward and inverse kinematic problems of a parallel robot, a MATLAB environment was chosen. This is where a user friendly graphical user interface was developed, as well.

III. WORKSPACE EVALUATION

Calculation of the workspace and its boundaries with perfect precision is crucial, because they influence the dimensional design, the manipulator's positioning in the work environment, and its dexterity to execute tasks [42]. In this section, the workspace of the proposed robots will be discussed in details. For a robot in the context of industrial application and given parameters, it is very important to analyze the area and the shape of its workspace. The workspace is limited by several conditions [43]. The prime limitation is the boundary obtained through solving inverse kinematics. Further, the workspace is limited by the reachable extent of drives and joints [44], then by the occurrence of singularities, and finally by the link and platform collisions. The parallel robot 3-RPS realizes a wide workspace, as presented in Fig. 4.In order to generate a reachable workspace of parallel manipulators; a numerical algorithm was introduced [45]. Analysis, i.e. visualization of the workspace is an important aspect of performance analysis.

For the sake of simplicity, other design specific factors such as the end-effector size, drive volumes have been ignored.



Fig. 4: Workspace volume of 3-RPS parallel manipulator

IV. CONTROL

In this section discusses the kinematic characteristics of the 3-RPS parallel mechanism. The mechanism has three degrees of freedom, i.e., one translation and two rotations.

In them, it is difficult to identify the possible rotational axes by observation, and more difficult to identify the possible continuous rotational axes. The simulation of running the robot was based on the Simulink module from MATLAB. The control of the robot is implemented using a joint based control scheme. In such a scheme, the end-effector is positioned by finding the difference between the desired quantities and the actual ones expressed in the joint space [34]. The command of the robot is expressed in Cartesian coordinates of the end-effector. Using the inverse kinematic problem, these coordinates become displacements. These displacements further become the reference points for the control algorithm.

The inputs of the algorithm were the differences between the angles computed (via inverse kinematic problem equations), and the values from sensors. The control signal was applied on three hydraulic cylinders which were actuating the robot structure. The controller parameters k_p and k_i were optimized for a given trajectory and a maximum error, using the block Signal Constraint from the Simulink Response Optimization toolbox. The control of the parallel robot is implemented using a joint-based control scheme [46]. In such a scheme, the endeffector is positioned by finding the difference between the desired extent of movements and the actual one, expressed in the joint space. The command of the robot is expressed in Cartesian coordinates of the end-effector. Using the inverse kinematic problem, these coordinates become displacements [47]. These displacements will further become the reference points for the control algorithm. The control scheme of the robot is presented in Fig. 5.

MATLAB/Simulink was chosen as a tool that is a widely used for modeling, simulation and testing of dynamical systems. It indicates the rotational axes of the 3-RPS mechanism cannot be chosen arbitrarily in the path planning. Therefore, making sure the continuous rotational axes of 3-RPS manipulator has important significance in control. A model in Simulink is represented graphically by means of a number of interconnected blocks.



Fig. 5: Control block scheme for 3-RPS parallel robot

The simplest control strategy, which can be taken into account, is view on the robots-manipulators, powered by group of the independent systems (drives - actuators), controlled separately, as a set of single-input / single-output systems [48]. 3-RPS parallel robot is controlled by means of traditional PID schemes in position/velocity, considering only their kinematics: the reference trajectory of the end-effector is established a priori. It's planned in the future to apply the use of more sophisticated algorithms, such as hybrid position-force control (HPFC) and impedance control, which allows fulfilling the requirements of complex and critical tasks, sometimes still performed manually and in general to enhance the performance of the robot [49].

V. SIMULATION

There are several reasons for realizing a model of the platform. Firstly, it is possible to check the functionality of the construction and to determine the working area by simulation. Furthermore, the control program can be developed and tested before the real platform is available. The mechanical construction is performed with the CAD program Solid Works [50] and the data is exported to Sim-Mechanics [40], a simulation tool for mechanical systems. Using Sim-Mechanics the dynamic behavior of the platform can be tested with a real or simulated control before it is set up. The model of the manipulator respects geometrical constraints, joints and mass distribution. Friction is neglected in this model. In order to simulate the behavior of the robot, a dynamic model of the robot has to be built. This is a complex subject and different methods were developed in order to solve it. A classical approach to closed-chain dynamic

modeling is to first consider an equivalent tree-structure, and then to consider system constraints via Lagrange multipliers or d'Alembert's principle [51]. Other approaches include the use of virtual work, Lagrange formalism, Hamilton's principle, and Newton- Euler equations [52].

In this article the dynamic model of the robot is built using the SimMechanics toolbox from Simulink. The toolbox uses the standard Newtonian dynamics of forces and torques in order to solve both the direct and the inverse problem [53]. The model was built from Simulink blocks that represent the kinematic elements and joints of the robot.

These blocks allow modeling of mechanical systems consisting of any number of rigid bodies, connected by joints representing translational and rotational degrees of freedom. In order to build a Sim-Mechanics model, one has to specify the inertial properties of the body such are degrees of freedom and constraints, along with coordinate systems that is attached to each body of the structure. This procedure can be very difficult for bodies with complex geometric forms; however, the process can be simplified by use of a Solid Works tool.

All three kinematic chains are defined by body and joint blocks. The inertia properties and the coordinates of the joints for each body were determined automatically when the CAD model was imported in MATLAB/Simulink environment. The connection of the mechanical model of the robot to the rest of the robot model was realized via actors and n joint blocks. Inputs of the model can be one of the following [54]: the generalized force, the position, speed, or the acceleration of the motor joints. In this paper, the inputs chosen were the speed of all three motor joints of the robot. As outputs, the angles of each motor element were chosen. In addition, sensors were used to determine the position of the end-effector.

CONCLUSIONS

The adoption of standard hydraulic actuators can allowed designers to design a low cost parallel robot with high payload and Reasonable operating speed. This design based on a 3 degrees of freedom kinematic scheme, can perform basic tasks like Flight Simulator, medical applications and some tasks with enough precision for industrial applications. Future researches will be addressed either the improved control strategy, or a new joint design exploiting reduced backlash.

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