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Ecological optimization of a non-isentropic Brayton power cycle with heat leakage

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Abstract: This paper reports the finite-time thermodynamic analysis based on the ecological optimization criterion for an irreversible and finite heat capacity rate Joule-Brayton heat engine model which includes three types of irreversibilities: finite rate heat transfer, heat leakage, and internal irreversibility. The optimal temperatures of the working fluid, the optimum power output, the optimum thermal efficiency, and the optimum exergetic efficiency are determined in terms of technical parameters. Results are reported for the effect of internal irreversibility, heat leakage, hot-cold temperature ratio, and the number of heat transfer units in hot and cold exchangers on the optimal performance parameters. The power and efficiency at maximum ecological function are found to be less than the maximum power and Curzon Ahlborn efficiency. Power output increases significantly with increasing hot cold side temperature ratio and number of heat transfer units. The thermal efficiency at maximum ecological function decreases almost linearly as the heat leak increases. The power output, thermal efficiency, and the exergetic efficiency increases as the turbine irreversibility parameter increased.

Keywords: ecological; finite-time; Brayton power cycle; variable temperature reservoir; non-isentropic.

Notation

A	=	Heat transfer area (m^2)
C	=	Heat capacitance rate (mass flow rate-specific heat product (kW/K)
E	=	Ecological function (kW)
N	=	Number of heat transfer units ($UA / \dot{C}_w$)
Q	=	Rate of heat transfer (kW)
$\dot{S}_{gen}$	=	Entropy generation rate (kW/K)
T	=	Temperature (K)
U	=	Overall heat transfer coefficient (kW/m^2-K)
$\dot{W}_C$	=	Compressor power input (kW)
$\dot{W}_T$	=	Turbine power output (kW)
$\dot{W}$	=	Net power output (kW)
x	=	Degree of irreversibility of compressor
y	=	Degree of irreversibility of turbine

Greek Letters

$\in$	=	Heat exchanger effectiveness
η	=	Thermal efficiency
η_{ex}	=	Exergetic or second-law efficiency
α	=	Hot-cold temperature ratio
ω	=	Dimensionless power output $\left(\dot{W}/\dot{C}_w T_{H1}\right)$

Subscripts

H	=	High-temperature heat source, hot side heat exchanger
L	=	Low temperature heat sink, cold-side heat exchanger
R	=	Regenerative heat exchanger
m	=	Maximum
max	=	Maximum
opt	=	Optimum
W	=	Working fluid
O	=	Environment

1. INTRODUCTION

The general theme of renewed analysis of heat engines under maximum power output has been applied to all the standard thermodynamic cycles. The Joule-Brayton cycle has received its fair share of attention. Joule-Brayton cycles have been used extensively in gas turbine power plants and aircraft propulsion systems. A closed cycle Brayton heat engine can be built to explore its suitability for special purposes, such as space power generation or nuclear power generation. Moreover, closed cycle Brayton heat engines can use various energy sources instead of high quality fuels. Bejan [1] showed that the power output of a Brayton cycle is maximized when the total thermal conductance is evenly distributed to the two heat exchangers. Wu and Kiang [2] examined the performance of work and power maximized Brayton cycle. Wu and Kiang [3] also studied the efficiency under maximum power condition by incorporating the nonisentropic expansion and compression processes into the cycle. Cheng and Chen [4] describe a study for the effect of regeneration on power output and thermal efficiency of an endoreversible gas turbine power cycle using finite time thermodynamic theory and evaluated the maximum power output, the corresponding thermal efficiency and the second law efficiency for the constant temperature thermal reservoir power cycle. Sahin et al. [5] introduced a new optimization criterion called the maximum power density, which allows one to consider heat engine sizes. They investigated the Carnot heat engine and non-regenerative Joule-Brayton engine. Yavuz [6] investigated the maximum power density performance for an internally irreversible Joule-Brayton power cycle free of heat transfer irreversibility. Medina et al. [7] applied the maximum power density method to a regenerative Joule–Brayton cycle and investigated optimal performance conditions in terms of isentropic efficiencies of the compressors and turbines. Sahin et al. [8] also extended the analysis for an irreversible Joule-Brayton heat engine by inclusion of reheating and regeneration into the model under maximum power density and maximum power condition. They obtained the optimal design parameters under maximum power density and maximum power condition in terms of the cycle temperature ratio, isentropic efficiencies of the compressors and turbines, and regenerator's efficiency. They also investigated the effects of the application of reheating and the component irreversibilities on the engine performance.

In the finite time thermodynamic researches, an ecological optimization criterion has been proposed by Angulo-Brown [9] and improved by Yan [10] for the best mode of operation of an endo-reversible Carnot engine, i.e. Curzon-Ahlborn heat engine. It consists of maximizing function $E=\dot{W}-T_o\dot{S}_{gen}$, representing the best compromise between power $\dot{W}$ and the product of entropy production $\dot{S}_{gen}$ and the cold reservoir temperature T_L, and $T_L \dot{S}_{gen}$ is called "lost power". He made calculations at maximum ecological function, compared them with the results obtained for minimum entropy production, and found a good agreement between the two. He also remarked that efficiency at maximum ecological function is almost equal to the average of Carnot and Curzon-Ahlborn efficiencies. Yan [10] showed that it may be more reasonable to use $E=\dot{W}-T_o\dot{S}_{gen}$, if the cold reservoir temperature is not equal to the environment temperature T_o. This criterion function is more extended and more generalized than that presented by Angulo-Brown. In the definition of ecological function, the lost power is $T_o\dot{S}_{gen}$ which is produced because of entropy generation in the system and surroundings. Various studies on the subject argue that entropy generation in the system and surrounding is reduced significantly if the ecological function for a given heat engine is maximized, as it reduces the entropy generation in the surroundings (environment) significantly, and also the determination of environment state, i.e. temperature T_o, is a necessary condition to calculate the power loss. Hence, this criterion can be reported as having a long-range goal in the sense that it is compatible with the ecological objectives.

Cheng and Chen [11,12] applied the concept of finite time thermodynamics to ecologically optimize the power output of endoreversible and irreversible Brayton heat engine for constant temperature external reservoirs and reported that the optimization of ecological function represents the compromise between power output $\dot{W}$ and loss of power $T_o\dot{S}_{gen}$, which is produced by entropy generation in the system and its surroundings. Recently, Khaliq and Rajesh [13] carried out a study for the ecological optimization of an endoreversible and regenerative gas turbine power cycle coupled with the constant temperature thermal reservoirs using finite-time heat transfer theory. They presented optimum values of power output, thermal efficiency, and exergetic efficiency under a state of maximum ecological function and reported that both the power output and the entropy generation rate are increased significantly by the use of regenerators, but thermal efficiency and exergetic efficiency are decreased with the same.

More recently, Khaliq [14] studied the ecological optimization of an endoreversible and regenerative Joule-Brayton power cycle coupled with variable temperature thermal reservoirs using finite-time thermodynamic approach. He observed the effects of regeneration, hot-cold temperature ratio and the number of heat transfer units in hot and cold exchangers on the optimal performance parameters. He reported that, the optimization of ecological function leads to the improvement in exergetic efficiency and thermal efficiency, especially for low hot-cold side temperature ratios.

Thus an extension to Khaliq's [14] work to increase the degree of realism further, in this paper, a specific model of irreversible Joule-Brayton heat engine coupled with variable temperature thermal reservoirs is presented, where the irreversibilities come from finite thermal conductance between the working fluid and the reservoirs, heat leaks between the reservoirs, and internal irreversibility inside the Joule-Brayton heat engine. A steady flow approach for finite time thermodynamics is applied to calculate the maximum ecological function as well its corresponding thermal efficiency, exergetic efficiency and power output. The optimal performance and design parameters that maximize the ecological function are also investigated.

2. System description

The model of the regenerative and irreversible Joule-Brayton power cycle coupled to finite heat source and heat sink capacitance rates of hot / cold side working fluid is shown in Fig. 1, and its T-s diagram is also sketched in Fig. 2. The irreversible model is a modified Carnot cycle in which the finite-time heat transfer irreversibilities are taken into account. The internal working fluid (gas/air) enters the compressor at state 4 and compressed upto state 1' and enters the regenerator where it is partially heated up to state 1R by the turbine exhaust. In an ideal/real regenerator the working fluid (gas/air) leaves the regenerator at the temperature equal to/ less than the turbine exhaust $T_{3'}$ i.e.$T_{1R} \leq T_{3'}$. The primary heat addition takes place between state 1 and 1R. The working fluid (gas/air) leaving the regenerator enters the hot side heat exchanger (with finite heat capacitance rates of external working fluid) and heated up to state 2 while the external hot side working fluid cooled from T_{H1} to T_{H2}. The working fluid (gas/air) then enters the turbine and expands isentropically up to state 3'. The turbine exhaust enters the regenerator where it transfers heat partly to the compressor outlet and then enters the cold side heat exchanger of variable temperature reservoir where temperature increases from T_{L1} to T_{L2} and working fluid cooled up to state 4. The process 4-1' is the compression process in the compressor. Similarly 2-3' is the expansion process in the turbine. Thus the closed irreversible Joule-Brayton cycle 4-1'-1R-2-3'-3R-4, with real capacitance rate of external (hot/cold side) working fluid, is considered. The heat leak is also considered between the hot and cold temperature reservoirs T_{H1} and T_{L1}.

Finite-time thermodynamic analysis:

When heat transfer obeys a linear law, then thermodynamics equations of interest are

$$\dot{Q}_{HC} = \dot{C}_W \left(T_2 - T_{1R}\right) = \in_H \dot{C}_W \left(T_{HI} - T_{IR}\right) \tag{1}$$

$$\dot{Q}_{LC} = \dot{C}_w \left(T_{3R} - T_4\right) = \in_L \dot{C}_W \left(T_{3R} - T_{L1}\right) \tag{2}$$

The heat transfer rate in the regenerator is given by

$$\dot{Q}_R = \dot{C}_W \left(T_{3'} - T_{3R}\right) = \in_R \dot{C}_W \left(T_{3'} - T_{1'}\right) = \dot{C}_W \left(T_{1R} - T_{1'}\right) \tag{3}$$

The heat leak in the system is given by

$$\dot{Q}_I = C_I \left(T_{H1} - T_{L1}\right) \tag{4}$$

The net heat rates transferred from the hot reservoir and to the cold reservoir are

$$\dot{Q}_H = \dot{Q}_{HC} + \dot{Q}_I \tag{5}$$

$$\dot{Q}_L = \dot{Q}_{LC} + \dot{Q}_I \tag{6}$$

The first-law of thermodynamics requires the power output of the heat engine to be

$$\dot{W} = \dot{Q}_{HC} - \dot{Q}_{LC} = \dot{Q}_H - \dot{Q}_L = \dot{C}_W \in_H (T_{HI} - T_{IR}) - \dot{C}_W \in_L (T_{3R} - T_{L1}) \tag{7}$$

and the effectiveness of the hot side, cold side and regenerative counter flow heat exchangers $\in_H$, $\in_L$, $\in_R$ may be given as

$$\in_H = \frac{1 - e^{-NTU_H}\left(1 - \frac{\dot{C}_{H,min}}{\dot{C}_{H,max}}\right)}{1 - \dot{C}_{L,min} / \dot{C}_{L,max} e^{-NTU_L}\left(1 - \dot{C}_{L,min} / \dot{C}_{L,max}\right)}$$

$$\in_L = \frac{1 - e^{-NTU_L\left(1 - \frac{\dot{C}_{L,min}}{\dot{C}_{L,max}}\right)}}{1 - \dot{C}_{L,min}/\dot{C}_{L,max}\, e^{-NTU_L\left(1 - \dot{C}_{L,min}/\dot{C}_{L,max}\right)}}$$

and $$\in_R = \frac{NTU_R}{1 + NTU_R}$$

where $\dot{C}_{H,min}$ and $\dot{C}_{H,max}$ are, respectively, the smaller and the larger of the two capacitance rates $\dot{C}_H$ and $C_{wf.}$ $\dot{C}_{L,min}$ and $\dot{C}_{L,max}$ are, respectively, the smaller and the larger of $\dot{C}_L$ and $\dot{C}_{wf}$. The number of heat transfer units are based on the minimum thermal capacitance rates :

$$N_H = \frac{U_H A_H}{\dot{C}_W}, N_L = \frac{U_L A_L}{\dot{C}_W}, N_R = \frac{U_R A_R}{\dot{C}_W}$$

Equations (1) to (3) respectively, gives

$$T_2 = \in_H T_{H1} + (1-\in_H) T_{1R} \quad (8)$$

$$T_4 = \in_L T_{LI} + (1-\in_L) T_{3R} \quad (9)$$

$$T_{3'} = \frac{T_{1R}}{\in_R} + \left(1 - \frac{1}{\in_R}\right) T_{1'} \quad (10)$$

Equations (7) and (9) give

$$T_4 = \frac{\in_H (1-\in_L)}{\in_L} T_{HI} + T_{LI} - \frac{\in_H (1-\in_L)}{\in_L} T_{1R} - \frac{(1-\in_L)\dot{W}}{\dot{C}_W \in_L} \quad (11)$$

Using equations (10) and (3), equation (12) may be obtained as

$$T_{1'} = \frac{(\in_R - 1) T_{1R} + \in_R T_{3R}}{(2\in_R - 1)} \quad (12)$$

$T_{3'}$ can be obtained after using equations (3) and (12) as

$$T_{3'} = \frac{\in_R T_{1R} + (\in_R - 1) T_{3R}}{(2\in_R - 1)} \quad (13)$$

Denoting the degree of irreversibility for the real compressor and turbine which takes into account their nonisentropic nature in terms of temperatures respectively as

$$\frac{T_1}{T_{1'}} = x \quad (14)$$

$$\frac{T_3}{T_{3'}} = y \quad (15)$$

Since processes 2-3 and 4-1 are isentropic, after putting this constraint on the four temperatures, equation (16) may be obtained as

$$T_1 T_3 = T_2 T_4 \quad (16)$$

This equation can readily be obtained after applying the isentropic relations between pressures and temperatures.
Using equations (14), (15) and (16), equation (17) is obtained

$$x T_{1'}\, y T_{3'} = T_2 T_4 \quad (17)$$

Substituting equations (8), (11) to (13) into equation (17), we may have

$$x\left\{\frac{\in'_R T_{1R} + \in_R T_{3R}}{(2\in_R - 1)}\right\} y\left\{\frac{\in_R T_{1R} + \in_R' T_{3R}}{(2\in_R - 1)}\right\} = \{\in_H T_{H1} + \in'_H T_{1R}\}\left\{\frac{\in_H \in'_H}{\in_L} T_{H1} + T_{L1} - \frac{\in_H}{\in_L}\in'_H T_{1R} - \frac{\in'_L \dot{W}}{\dot{C}_W \in_L}\right\} \quad (18)$$

where $\in'_R = \in_R - 1$

$$\epsilon'_H = 1 - \epsilon_H$$
$$\epsilon'_L = 1 - \epsilon_L$$

Substituting equation (7) for T_{3R} in equation (18), equation (19) may be obtained:

$$a_1 T_{1R}^2 + \left(a_2 + \dot{W} a_3\right) T_{1R} + a_4 + \dot{W} a_5 + \dot{W} a_6^2 = 0 \tag{19}$$

This is quadratic in T_{1R}, and it gives

$$T_{1R} = \frac{-\left(a_2 + \dot{W} a_3\right) \pm \left[\left(a_2 + \dot{W} a_3\right)^2 - 4a_1\left(a_4 + \dot{W} a_5 + \dot{W}^2 a_6\right)\right]^{1/2}}{2a_1} \tag{20}$$

The constants a_1, a_2, a_3, a_4, a_5 and a_6 are defined in the Appendix 2.

The objective function for ecological optimization, which was proposed by Angulo-Brown [23] and modified by Yan [24] is given by

$$E = \dot{W} - T_o \dot{S}_{gen} \tag{21}$$

where E is the objective function, $\dot{W}$ is the power output, $\dot{S}_{gen}$ is the entropy generation rate, and the term $T_o \dot{S}_{gen}$ is the lost power.

The optimization of the ecological function represents the best compromise between power output and lost power which is produced by entropy generation in the system and its surroundings.

Applying the second law to the heat engine , equation (22) may be obtained:

$$E = \dot{W} - T_o \left(\frac{\dot{Q}_L}{T_{L1}} - \frac{\dot{Q}_H}{T_{H1}} \right) \tag{22}$$

Substituting equations (5) and (6), and the value of T_{3R} in terms of T_{1R} from equation (7) into equation (22), it may be reported as

$$E = b_1 \dot{W} + b_2 T_{1R} + b_3 \tag{23}$$

Maximizing the ecological function E with respect to $\dot{W}$ by setting

$$\frac{\partial E}{\partial \dot{W}} = 0, \text{ yields}$$

$$\frac{b_1}{b_2} = \frac{\partial T_{1R}}{\partial \dot{W}} \tag{24}$$

Differentiating equation (20) with respect to $\dot{W}$ for T_{1R} and then substituting it in equation (24), it may be observed that

$$b_4 \dot{W}^2 + b_5 \dot{W} + b_6 = 0 \tag{25}$$

This is quadratic in power $\dot{W}$, hence the optimum power output may be given as

$$\dot{W}_{opt} = \frac{-b_5 \pm \left(b_5^2 - 4b_4 b_6\right)^{1/2}}{2b_4} \tag{26}$$

Constants b_1 , b_2, b_3, b_4, b_5 and b_6 are defined in the Appendix.

Substituting equation (26) into equation (20) yields the expression for $T_{1R,opt}$

$$T_{1R,opt} = \frac{-(a_2 + \dot{W}_{opt} a_3) \pm \left[(a_2 + \dot{W}_{opt} a_3)^2 - 4a_1(a_4 + \dot{W}_{opt} a_5 + \dot{W}^2{}_{opt} a_6) \right]^{1/2}}{2a_1} \tag{27}$$

The optimum cycle temperatures $T_{2,opt}$, $T_{3,opt}$, $T_{4,opt}$, $T_{1,opt}$ and $T_{3R,opt}$ can be obtained using equation (8) to (15) and equation (27).

The optimum power output $\dot{W}_{opt}$ and the corresponding heat transfer rates $\dot{Q}_{H,opt}$, $\dot{Q}_{L,opt}$ and $\dot{Q}_{R,opt}$ can also be calculated after using equation (1) to (13) and equation (27)

The optimum first-law efficiency or the thermal efficiency at maximum ecological function is equal to the optimum power divided by the heat input rate, namely;

$$\eta_{th,opt} = \frac{\dot{W}_{opt}}{\dot{Q}_{H,opt}} \tag{28}$$

The exergetic or second-law efficiency which has been a measure of how close the heat power cycle operates to the maximum first-law efficiency, namely the ideal Carnot efficiency. The second-law efficiency at maximum ecological function is given by,

$$\eta_{ex,opt} = \frac{\eta_{th,opt} T_{H1}}{T_{H1} - T_{L1}} \tag{29}$$

The second law of thermodynamics applied to the heat power cycle gives the entropy generation rate and it is found to be

$$\dot{S}_{gen} = \frac{\dot{Q}_L}{T_{L1}} - \frac{\dot{Q}_H}{T_{H1}} \tag{30}$$

Substitution of equations (5) and (6) into equation (30), results in

$$\dot{S}_{gen} = \frac{\dot{C}_W \in_L}{T_{L1}} \left(\frac{\in_H}{\in_L} T_{H1} - \frac{\in_H}{\in_L} T_{1R} - \frac{\dot{W}}{\dot{C}_W \in_L} \right) - \frac{\in_H \dot{C}_W}{T_{H1}} (T_{H1} - T_{1R}) + C_I (T_{H1} - T_{L1}) \left(\frac{1}{T_{L1}} - \frac{1}{T_{H1}} \right) \tag{31}$$

Dividing equation (31) by $\dot{C}_w$ on both sides, the dimensionless entropy generation rate may be obtained as

$$\frac{\dot{S}_{gen}}{\dot{C}_w} = \in_H (\alpha - 1)\left(1 - \frac{T_{1R}}{T_{H1}}\right) - \alpha\omega + \frac{C_I}{\dot{C}_w}\left(\alpha - 2 + \frac{1}{\alpha}\right) \tag{32}$$

Where $\alpha = \frac{T_{H1}}{T_{L1}}$ and $\omega = \frac{\dot{W}}{\dot{C}_W T_{H1}}$

The optimum values of dimensionless entropy generation rate can be calculated substituting equations (26) and (27) into (32).

The finite-time efficiency or efficiency at maximum power is given by

$$\eta_m = 1 - \sqrt{T_{L1}/T_{H1}} \tag{33}$$

3. Results and discussion

The variation of the dimensionless entropy generation rate at maximum ecological function with respect to the hot-cold side temperature ratio T_{H1}/T_{L1} for different values of N_H & N_L, and for the fixed values of N_R and the irreversibility parameters x and y are plotted in Fig. 3. From the figure, it is shown that the dimensionless optimum entropy generation rate $(\dot{S}_{gen,opt}/\dot{C}_w)$ increases as the temperature ratio T_{H1}/T_{L1} increases at constant N_H & N_L values. It is further shown in Fig. 3 that the optimum entropy generation rate increases with the increase in N_H & N_L values. Fig. 3 also shows that the thermal efficiency increases significantly as T_{H1}/T_{L1} increases. Fig.3 further shows the variation of the second law or exegetic efficiency at maximum ecological function given by equation 29 with respect to the temperature ratio for different values of N_H & N_L and fixed N_R. It is observed that the optimum exegetic efficiency increases significantly as temperature ratio increases. It is further shown that the optimum exegetic efficiency increased as values of N_H & N_L increased at constant N_R. The optimum exegetic efficiency is higher at $N_H=N_L=4$ than at $N_H=N_L=3$. The maximization of ecological function leads to the improvement in exegetic efficiency and in thermal efficiency, especially for low hot-cold side temperature ratios. It is seen from the figure that large difference exists between η_m and $\eta_{th,opt}$

The maximum ecological function $\dot{E}_{max}/\dot{C}_W T_{H1}$ and optimum power output $\dot{W}_{opt}/\dot{C}_W T_{H1}$ are plotted as the function of heat reservoir temperature ratios T_{H1}/T_{L1} for different values of heat transfer units N_H & N_L and for the fixed values of N_R and the irreversibility parameters x and y in Fig. 4. The maximum ecological function increases sharply with an increase in temperature ratio. The maximum ecological function at $N_H=N_L= 4$ is significantly higher than at $N_H=N_L=3$. When the heat reservoir temperature ratio is increased, the power output at maximum ecological function increases significantly. Moreover, the power output at maximum ecological function is higher at the larger values of N_H & N_L than at lower values.

The maximum ecological function $\dot{E}_{max}/\dot{C}_W T_{H1}$, the power output at maximum ecological function $\dot{W}_{opt}/\dot{C}_W T_{H1}$, the thermal efficiency and exegetic efficiency at maximum ecological function are plotted as a function of turbine irreversibility parameter for the fixed values of reservoir temperature ratio T_{H1}/T_{L1}, number of heat transfer units N_H, N_L& N_R, heat transfer rate between the two reservoirs per unit temperature difference $C_I/\dot{C}_W$ and the compressor irreversibility parameters x in Fig. 5. It is observed that the maximum ecological function increases significantly as the values of turbine irreversibility parameter y increased. Similarly the power output, thermal efficiency and the exergetic efficiency increases as the turbine irreversibility parameter increased. Moreover, increasing the turbine irreversibility y would lead to an increase in the maximum ecological function as well as its corresponding thermal efficiency. This points the direction; the engineers should concentrate on the technology for decreasing the turbine irreversibility and thus could obtain higher values of ecological function for an irreversible Joule-Brayton power cycle.

The maximum ecological function, the power output at maximum ecological function, the thermal efficiency, entropy generation and exegetic efficiency at maximum ecological function are plotted as a function of heat transfer rate between the two reservoirs per unit temperature difference $C_I/\dot{C}_W$ for fixed values of turbine and compressor irreversibility parameter y and x, reservoir temperature ratios T_{H1}/T_{L1}, number of heat transfer units (N_H, N_L& N_R) in Fig. 6. It is noted that the thermal efficiency, exegetic efficiency and maximum ecological function decreases almost linearly as the heat leak increase. On the other hand the entropy generation at maximum ecological function increases sharply as the heat leak increased. The optimum entropy generation rate increases with the increase in N_H & N_L values at constant N_R, and thus values is higher at N_H=N_L= 4 than at N_H=N_L=3. From the above discussion, a decrease in the heat leak would result in an increase in the maximum ecological function and its corresponding thermal efficiency. Therefore, using a better design in the thermal insulation between the two hot reservoirs to decrease the heat leaks, we could increase the maximum ecological function and its corresponding thermal efficiency for an irreversible Joule-Brayton power cycle.

4. Conclusion

Using finite-time ecological optimization criterion, a non-isentropic closed regenerative Joule-Brayton power cycle operating between finite heat source and finite heat sink with heat leakage has been studied. The non-isentropic nature of the compressor and expansion processes is taken into account by the introduction of compressor and turbine irreversibility parameters. The ecological function is defined as the power output minus the loss power, which is equal to the product of environment temperature and entropy generation rate. The maximum ecological function is an increasing function of reservoir temperature ratios, irreversibility parameters, and total number of transfer units of the heat exchangers; however, it is found to be a decreasing function of heat leaks. The power and efficiency at the maximum ecological function are found to be less than the maximum power and corresponding efficiency. Optimization of ecological function leads to the improvement in exergetic and thermal efficiencies especially for low hot-cold side temperature ratios. The optimum values of temperature ratios and number of heat transfer units can be used as important criterion in the design of an irreversible Joule-Brayton power cycle.

5. Appendix

$$a_1=\left[\frac{(\epsilon_R-1)\epsilon_R-(\epsilon_R-1)^2\epsilon_H/\epsilon_L-\epsilon_R^2\epsilon_H/\epsilon_L+\dfrac{\epsilon_R\epsilon_H^2(\epsilon_R-1)}{\epsilon_L^2}}{(2\epsilon_R-1)^2}\right]xy+\frac{\epsilon_H}{\epsilon_L}(1-\epsilon_H)(1-\epsilon_L)$$

$$a_2=\left[\begin{array}{l}\dfrac{(\epsilon_R-1)^2\epsilon_H}{\epsilon_L(\epsilon_R-1)^2}T_{H1}+\dfrac{(\epsilon_R-1)^2T_{L1}}{(2\epsilon_R-1)^2}+\dfrac{\epsilon_R^2\epsilon_H T_{H1}}{\epsilon_L(2\epsilon_R-1)^2}-\dfrac{(\epsilon_R-1)\epsilon_R\epsilon_H^2}{\epsilon_L^2(2\epsilon_R-1)^2}T_{H1}\\+\dfrac{\epsilon_R^2T_{L1}}{(2\epsilon_R-1)^2}-\dfrac{\epsilon_H\epsilon_R(\epsilon_R-1)T_{L1}}{\epsilon_L(2\epsilon_R-1)^2}-\dfrac{\epsilon_R\epsilon_H^2(\epsilon_R-1)T_{H1}}{\epsilon_L^2(2\epsilon_R-1)^2}-\dfrac{\epsilon_R\epsilon_H(\epsilon_R-1)}{\epsilon_L(2\epsilon_R-1)^2}T_{L1}\end{array}\right]xy$$
$$+\frac{\epsilon_H^2(1-\epsilon_L)}{\epsilon_L}T_{H1}-\frac{\epsilon_H(1-\epsilon_H)(1-\epsilon_L)}{\epsilon_L}T_{H1}-(1-\epsilon_H)T_{L1}$$

$$a_3=\left[\frac{-(\epsilon_R-1)^2}{C_W\epsilon_L(2\epsilon_R-1)^2}+\frac{(\epsilon_R-1)\epsilon_R\epsilon_H}{\epsilon_L^2\dot{C}_W(2\epsilon_R-1)^2}-\frac{\epsilon_R^2}{\dot{C}_W\epsilon_L(2\epsilon_R-1)^2}+\frac{\epsilon_R(\epsilon_R-1)\epsilon_H}{\dot{C}_W\epsilon_L^2(2\epsilon_R-1)^2}\right]xy+\frac{(1-\epsilon_L)(1-\epsilon_H)}{\dot{C}_W\epsilon_L}$$

$$a_4=\left[\frac{\epsilon_R\epsilon_H^2(\epsilon_R-1)}{\epsilon_L^2(2\epsilon_R-1)^2}T_{H1}^2+\frac{\epsilon_R\epsilon_H(\epsilon_R-1)}{(2\epsilon_R-1)^2\epsilon_L}T_{H1}T_{L1}+\frac{\epsilon_R\epsilon_H(\epsilon_R-1)T_{H1}T_{L1}}{\epsilon_L(2\epsilon_R-1)^2}+\frac{\epsilon_R(\epsilon_R-1)}{(2\epsilon_R-1)^2}T_{L1}^2\right]xy$$

$$-\frac{\in_H^2 (1-\in_L)T_{H1}^2}{\in_L} - \in_H T_{H1} T_{L1}$$

$$a_5 = \left[-\frac{\in_R \in_H (\in_R - 1)T_{H1}}{\dot{C}_W \in_L^2 (2\in_R - 1)^2} - \frac{\in_R (\in_R - 1)T_{L1}}{\dot{C}_W \in_L (2\in_R - 1)^2} - \frac{\in_R (\in_R - 1)\in_H T_{H1}}{\dot{C}_W \in_L^2 (2\in_R - 1)^2} - \frac{\in_R (\in_R - 1)T_{L1}}{\dot{C}_W \in_L (2\in_R - 1)^2} \right] xy + \frac{\in_H (1-\in_L)T_{H1}}{\dot{C}_W \in_L}$$

$$a_6 = \left[\frac{\in_R (\in_R - 1)}{\dot{C}_W^2 \in_L^2 (2\in_R - 1)^2} \right] xy$$

$$b_1 = \left(1 + \frac{T_0}{T_{L1}}\right)$$

$$b_2 = \left(\frac{1}{T_{L1}} - \frac{1}{T_{H1}}\right) \in_H \dot{C}_W T_0$$

$$b_3 = \in_H \dot{C}_W T_0 \left[\left(1 - \frac{T_{H1}}{T_{L1}}\right) - \frac{C_I}{\in_H \dot{C}_W T_{L1}} (T_{H1} - T_{L1}) \left(1 - \frac{T_{L1}}{T_{H1}}\right) \right]$$

$$b_4 = \left[-4a_1 a_6 \left(a_3 - \frac{2a_1 b_1}{b_2} \right)^2 - a_3^4 - 16a_1^2 a_6^2 + 8a_1 a_3^2 a_6 \right]$$

$$b_5 = \left(a_3 - \frac{2a_1 b_1}{b_2} \right)^2 (a_3 - 4a_1 a_5) - 2a_2 a_3^3 + 16a_1^2 a_5 a_6 + 8a_1 a_3 a_2 a_6 + 4a_1 a_3^2 a_5$$

$$b_6 = \left(a_3 - \frac{2a_1 b_1}{b_2} \right)^2 (a_2 - 4a_1 a_4) - a_2^2 a_3^2 - 4a_1^2 a_5 + 4a_1 a_2 a_3 a_5$$

References

[1]. Bejan A..Theory of heat transfer-irreversible power plants. International Journal of Heat Mass Transfer 1988; 31(6):1211-1219.

[2]. Wu C, Kiang RL. Work and power optimization of a finite time Brayton cycle. Int. Journal of ambient Energy 1990;11(3):129-136.

[3]. Wu C, Kiang RL. Power Performance of a non-isentropic Brayton cycle. Trans ASME – Journal of Engineering for Gas Turbines and Power 1991; 113: 501-504.

[4]. Cheng CY, Chen CK..Power optimization of an endoreversible regenerative Brayton cycle. Energy-The International Journal 1996; 21 (4): 241-247.

[5]. Sahin B, Kodal A, Yavuz H.. Maximum power density for an endoreversible Carnot heat engines. Energy- The Int. Journal 1996; 21 :1219-1231.

[6]. Yavuz H.. Maximum power density analysis of an irreversible Joule- Brayton engine. J Physics. D: Appl. Physics 1996; 29 (8): 1162-1167.

[7]. Medina A, Roco JMM, Calvo Hernandez A. Regenerative gas turbines at maximum power density conditions. J. Phys., D: Appl. Phys 1996; 29 :2802-2811.

[8]. Sahin, B, Kodal A, Kaya SS. A comparative performance analysis of irreversible regenerative reheating Joule- Brayton engines under maximum power density and maximum power conditions. J. Phys., D: Appl. Phys 1998; 31: 2125-2131.

[9]. Angulo-Brown F. An ecological optimization criterion for finite-time heat engines. Journal Applied Physics 1991; 69 (11): 7465-7469.

[10]. Yan Z. Comment on - an ecological optimization criterion for finite-time heat engines. Journal Applied Physics 1993; 73 (7): 3583.

[11]. Cheng CY, Chen CK. Ecological optimization of an endoreversible Brayton cycle. Energy Conversion and Management 1998; 39 (½): 33-44.

[12]. Cheng CY, Chen CK. Ecological optimization of an irreversible Brayton heat engine. J. Phys. D: Appl. Phys 1999; 32 :350-357.

[13]. Khaliq A, Rajesh K. Finite-time heat transfer analysis and ecological optimization of an endoreversible and regenerated gas turbine power cycle. Applied Energy – An International Journal 2005;81(1): 73-84.

[14]. Khaliq A. Finite- thermal reservoir effects on ecologically optimized closed regenerative Joule-Brayton power cycles. Proc. Instn. Mech. Engrs, Part A, Journal Power and Energy 2006; 220(4): 425-434.

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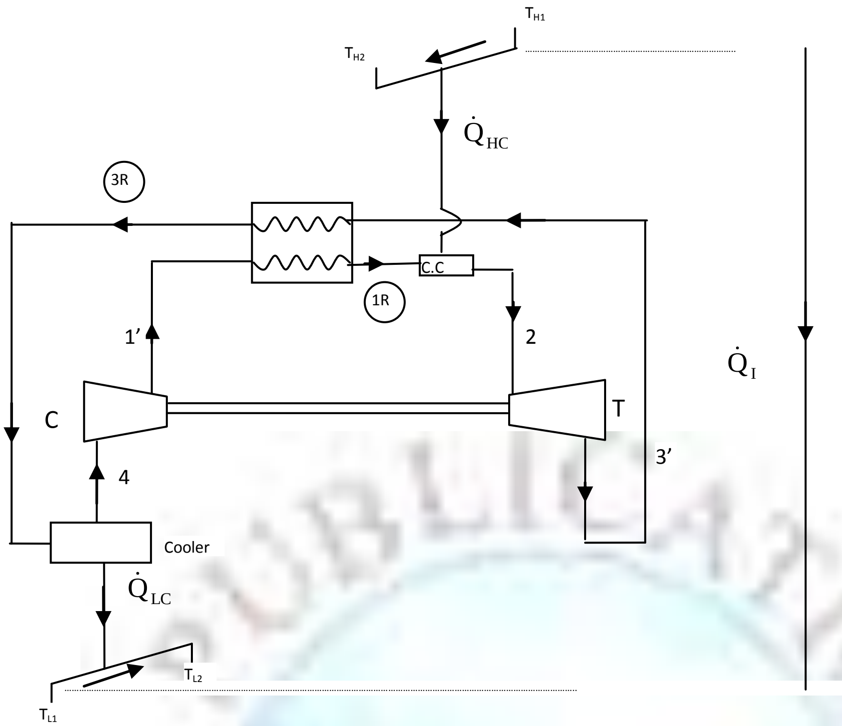

Fig. 1: Schematic diagram of a Joule-Brayton power cycle

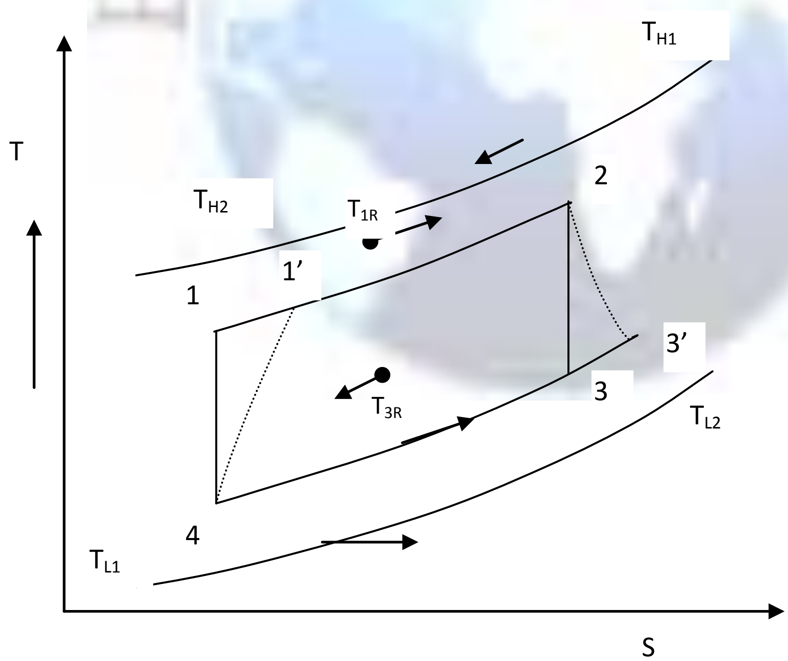

Fig. 2: Temperature-entropy representation of a Joule- Brayton power cycle

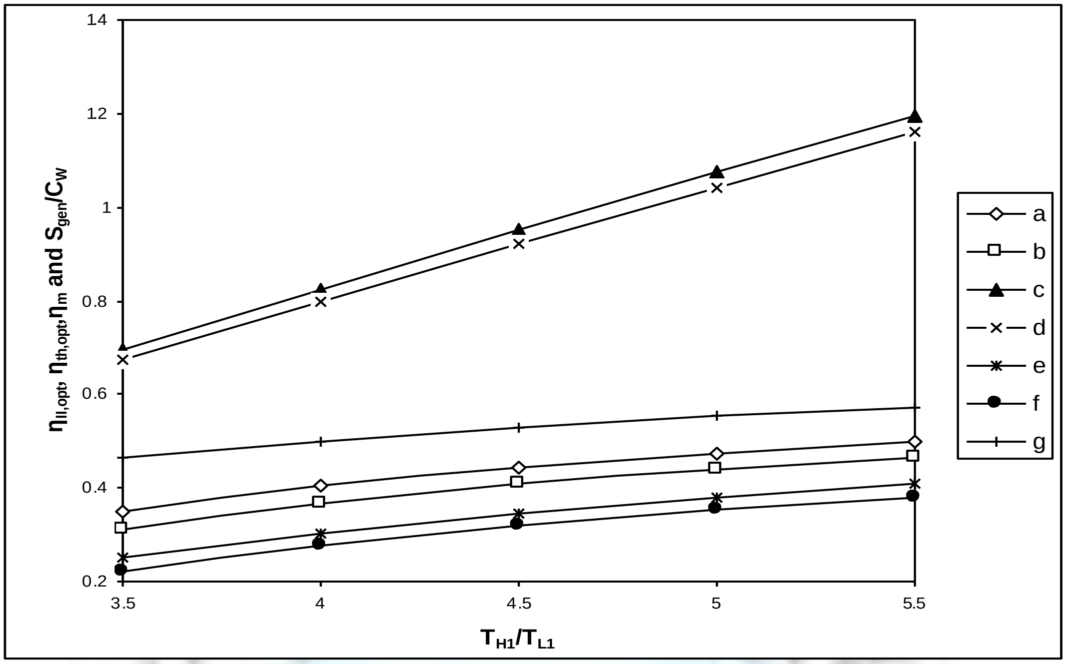

Fig. 3 Variation of second law efficiency, dimensionless entropy generation rate, optimum thermal efficiency, and efficiency at maximum power at maximum ecological function with respect to the hot-cold side temperature ratio T_{H1}/T_{L1} for $N_R = 4$, x = y = 0.94 (a) $\eta_{II,opt}$ at $N_H = N_L=4$, (b) $\eta_{II,opt}$ at $N_H = N_L=3$, (c) Dimensionless entropy generation rate at $N_H = N_L=4$ (d) Dimensionless entropy generation rate at $N_H = N_L=3$ (e) $\eta_{th,opt}$ at $N_H = N_L=4$, (f) $\eta_{th,opt}$ at $N_H = N_L=3$, (g) η_m

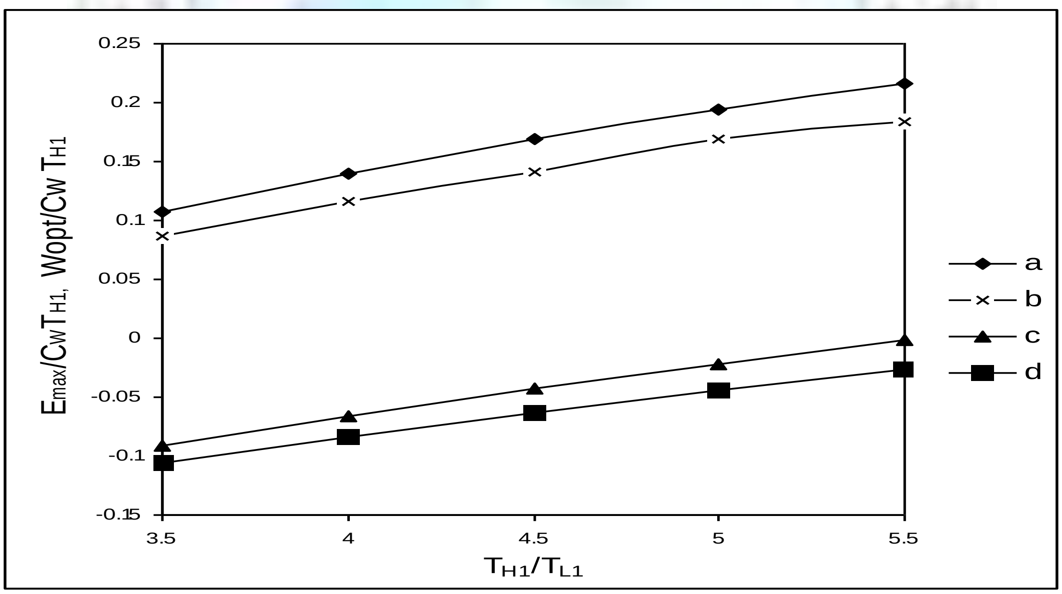

Fig. 4: Variation of maximum ecological function and dimensionless optimum power output with heat reservoir temperature ratios T_{H1}/T_{L1} for x = y = 0.94, $N_R = 4$, $\dot{C}_I/\dot{C}_W = 0.02$, $T_O=T_{L1}$ (a)Dimensionless power output at $N_H = N_L= 4$ (b) Dimensionless power output at $N_H = N_L = 3$ (c) Maximum ecological function at $N_H = N_L= 4$ (d) Maximum ecological function at $N_H = N_L= 3$

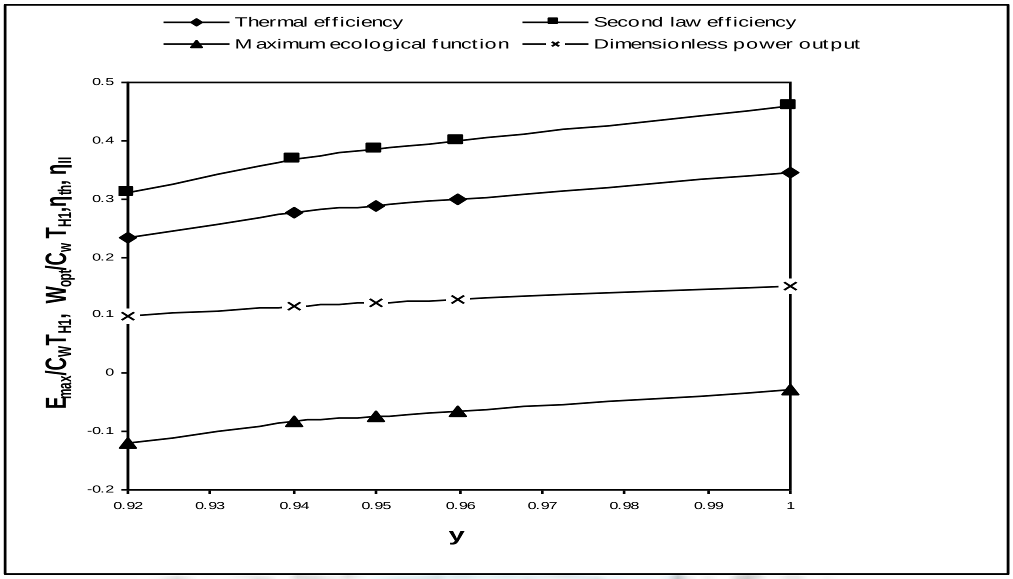

Fig. 5: Effect of variation of turbine irreversibility parameter y on maximum ecological function, Dimensionless power output at maximum ecological function, η_{th} & $\eta_{II,opt}$ at maximum ecological function for $T_{H1}/T_{L1} = 4$, x = 0.94, $N_H = N_L = 3$, $N_R = 4$, $\dot{C}_I/\dot{C}_W = 0.02$ & $T_O = T_{L1}$

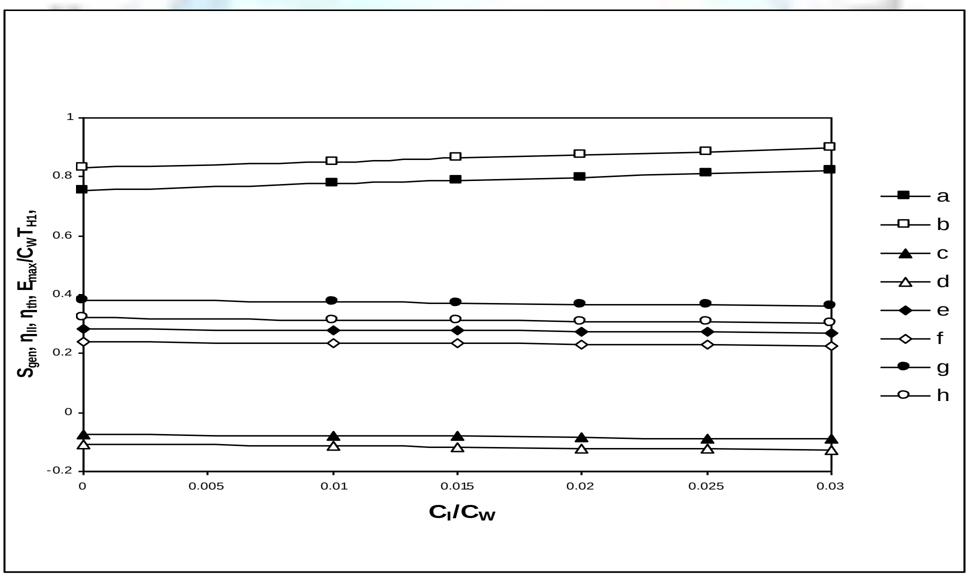

Fig.6: Effect of variation of heat transfer rate between the two reservoirs per unit temperature difference $\dot{C}_I/\dot{C}_W$ on (a) Entropy generation y = 0.94 (b) Entropy generation y = 0.92 (c) Dimensionless power output at maximum ecological function y = 0.94 (d) Dimensionless power output at maximum ecological function y = 0.92 (e) Thermal efficiency y = 0.94 (f) Thermal efficiency y = 0.92 (g) Exegetic efficiency y = 0.94 (h) Exegetic efficiency y = 0.92; at maximum ecological function at compressor irreversibility parameter x = 0.94, $T_O = T_{L1}$, $T_{H1}/T_{L1} = 4$, $N_H = N_L = 3$, $N_R = 4$