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An advanced process of vibration control of a cantilever plate with help of Finite Element Method

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Abstract: The purpose of this work is to control the vibration of the plate with the help of finite element method. Using augmented equations, a finite element model of a two-dimensional cantilever plate instrumented with a piezoelectric patches sensor-actuator pair is derived. The contribution of piezoelectric sensor and actuator layers on the mass and stiffness of the plate is considered. As mesh size (8x8) is found best for the future modeling and analysis of the plate has been taken. The plate is divided in 64 small parts with help of finite element method. Each parts vibration calculated with help of piezo patches. The vibration is calculated in this paper by mathematics modeling.

Keywords: Cantilever Plate, Smart Structure, Finite Element Model, Active Vibration Control.

1. Introduction

In the past era, the heavy machineries and structures are used. But due to their complicated construction, heavy cost and excess uses of the material; the heavy weight machineries and structures are replaced with light weight. The development of high strength to weight ratio of mechanical structures are attracting engineers to build light weight aerospace structures to combine loading capacity and fuel consumption, as well as to build tall buildings and long bridges. However, the flexible light weight structures lead to more complicated vibration problems. Traditionally, vibrations have been controlled using passive techniques. Passive vibration reduction is achieved by adding mass damping and stiffness unsuitable locations on a structure to reduce vibrations. However these techniques lead to increased structure weight and low response. So, the mechanical vibrations induced in light weight aerospace and large-scale flexible structures have attracted engineers to investigate and develop materials to suppress these vibrations. The development of piezoelectric material has been used as sensors and actuators to control vibrations because it has attractive properties, such as low mass, high actuating force and fast response.
The discipline of vibration engineering is becoming increasingly more important because of higher machinery speeds, operational loads, compact and lightweight designs, and engineered materials. Experimental work is evolving very rapidly with the advent of high speed processors, signal processing and control modules, smart sensors and actuators, and digital instrumentation in general. Now this area can be viewed as truly interdisciplinary since it includes the elements of many branches of engineering and physical sciences. The need for quieter and reliable products, machines, and equipment is well recognized. Additionally, since the recent emphasis has been on the sound quality and vibration perception considerations, some cognitive techniques are being integrated into this discipline. For example, consumers may perceive quality of products such as automobiles and appliances in terms of their sound and vibration characteristics.

Beams, plates and shells are the elements of basic adaptive structures onto which sensors and actuators are embedded for the vibrations and acoustics applications. These elements are been widely used in airplanes, missiles, civil and other structural systems which for various engineering analysis can be represented as beams, plates and shells. Several theories and principles have been developed to make an in point prediction of borne noise, vibration and alleviations in smart structures. These structural elements are assumed to have sensors and actuators embedded in layers and supposed to undergo consistent deformation. Sensors and actuators are embedded at certain location which, in most cases, are decided by discrediting the entire structure element into the smaller elements of known shapes so as to make it easy to study and interpret the exact position effects onto the variables that are to be controlled. Sensors and actuators are supposed to differ from each other in terms of displacement distribution through the thickness of elements. Different finite element methods have been proposed by researches for the modelling of piezolaminated plates.

Varun Kumar and Deepak Chhabra[1] gave a new method of design of fuzzy logic controller for active vibration control of cantilever plate with piezo -patches as sensor /actuator. Balamurugan V. and Narayanan S. [2] studied the mechanics for the coupled analysis of piezolaminated plate and curvilinear shell structures and their vibration control performance. A plate/shell structure with thin PZT layers embedded on top and bottom surfaces is considered. Active vibration control performance of plates and shells with distributed piezoelectric sensors and actuators have been studied. Caruso G. et al. [3] studied the vibration control of an elastic plate, clamped along one side and excited by impulsive transversal force acting in correspondence of a free corner. A modal model obtained by employing a suitable finite-element formulation together with a modal reduction, was used in the controller design. Tylikowski A. [4] analysed the capacitive shunting distributed piezoelectric elements perfectly glued to the vibrating annular plate excited by harmonic displacement of the inner plate edge. The equations of piezoelement were coupled with the equations of plate motion by the surface strain terms.

Lam K. Y. et al. [5] developed a finite-element model based on the classical laminated plate theory for the active vibration control of a composite plate containing distributed piezoelectric sensors and actuators from the variation principle. A negative velocity feedback control algorithm coupling the direct and converse piezoelectric effects was used. Verification of the proposed model was on a cantilever composite plate. Narayanan S. and Balamurugan V. [6] studied the finite element modelling of laminated structures with distributed piezoelectric sensor and actuator layers. Beam, plate and shell type elements have been developed incorporating the stiffness, mass and electromechanical coupling effects of the piezoelectric laminates. Lin J. C. and Nien M.H. [7] discussed the adaptive modelling and shape control of laminate plates with piezoelectric actuators. A finite element formulation was developed for dynamic and static response of laminated plates. A composite plate with different location of mechanical load was studied analytically and experimentally. Wu D.H. et al. [8] addressed to an efficient method for simulating and analyzing the structural and electrical characteristics of piezoelectric beam actuators all together. A beam of silicon clamp-free mounted with a PZT layer on either side is taken in to account. Harmonic response, resonant frequency of the piezoelectric plate, and the equivalent piezoelectric circuit of the piezoelectric transducer are investigate. The results are verified experimentally and theoretically. Paquin S. and St-Amant Y. [9] studied the effect of a variable thickness beam harvester on its electromechanical performance. Rayleigh–Ritz approximations were used to develop a semi-analytical mechanical model. Finite element technique was used to validate the model and numerical simulations were then performed to find the optimum for a given maximal strain across the piezoelectric elements for different beam slope angles. Studies on active vibration controlling capabilities of these plates have been done.

Yaman Y. et al. [10] presented the theoretical and experimental results of the modelling of a smart plate for active vibration control. The smart plate consists of a rectangular aluminum plate modelled in cantilever configuration with surface bonded piezoelectric patches. The patches are symmetrically bonded on top and bottom surfaces. The study used ANSYS (v.5.6) software to derive the finite element model of the smart plate. The optimal sensor locations were found and actual smart plate was produced. Mukherjee A. et al. [11] presented the active vibration control of stiffened plates. A stiffened plate finite element with piezoelectric effects was formulated. A velocity feedback algorithm was employed. Numerical examples for vibration control of isotropic and orthotropic stiffened plates were presented. Costa L. et al. [12] derived a reduced model for a piezoelectric plate and to study its actuator and sensor capabilities. Study on the actuator and sensor capabilities of this model was done. Two discrete non-differentiable multi-objective optimization problems were used, which were solved by genetic algorithms.

Kapil Narwal and Deepak Chhabra [13] presented the research at the active vibration control of a flexible structures using piezoelectric material. A simple supported plate structure, which is supported at two opposite ends, is taken as the flexible structure with piezoelectric materials as sensors and actuators. Pardeep Singh and Deepak Chhabra [14] studied the active vibration control of a square cantilever plate with piezo- patch as sensor and actuator bonded on top and bottom surfaces of the plate. The problem is formulated using the finite element method (FEM) by considering square elements with four nodes at its corner and each node is having three degree of freedom.

2. Vibration Reduction

Vibration reduction can be achieved in many different ways, depending on the problem; the most common are stiffening, damping and isolation. Stiffening consists of shifting the resonance frequency of the structure beyond the frequency band of excitation. Damping consists of reducing the resonance peaks by dissipating the vibration energy. Isolation consists of preventing the propagation of disturbances to sensitive parts of the systems. Damping may be achieved passively, with fluid dampers, eddy currents, elastomers or hysteretic elements, or by transferring kinetic energy to dynamic vibration absorbers. One can also use transducers as energy converters, to transform vibration energy into electrical energy that is dissipated in electrical networks, or stored (energy harvesting). Recently, semi-active devices (also called semi-passive) have become available; they consist of passive devices with controllable properties. The magneto-rheological fluid damper is a famous example; piezoelectric transducers with switched electrical networks are another. When high performance is needed, active control can be used; this involves a set of sensors strain, acceleration, velocity, force, a set of actuators force, inertial, strain, and a control algorithm (feedback or feed forward). The design of an active control system involves many issues such as how to configure the sensors and actuators (map of strain energy or kinetic energy) and how to secure stability and robustness (collocated actuator/sensor pairs).

3. Finite Element Method

FEM is a numerical technique. A large variety of structures can be modeled and simulated using the FEM methods of computational mechanics. The FEM is based on the idea of building a complicated object with simple blocks, or, dividing a complicated object into small pieces using small elements. Then the behavior of each piece is analyzed by using a set of relatively simple approximate functions to solve the equations defining the problem considered. FEM converts the set of governing differential equations for a problem into a set of equations in the form

$$M\ddot{x} + c\dot{x} + kx = F(t)$$

Vector represents nodal displacements, and the dot indicates the first time derivative. are mass, damping and stiffness matrices, respectively. For the vibration problems analyzed here these matrices can be considered constant. is the vector of active nodal forces. An Efficient Model for Vibration Control by Piezoelectric Smart structure using finite element.
A found survey was held on finite element modeling and the advancements in its formulations and applications for the finite element modeling of adaptive structural elements namely, solids, shells, plates and beams. Moreover, the model was also applied for the optimal design of piezoelectric actuators.

4. Vibration Control of a Cantilever Plate with help of Finite Element method

We consider a cantilever plate with a piezo patches. The plate properties are shown in table1. and the piezo-patches properties are shown in table2.

TABLE4.1. Material properties and dimensions for plate

Parameter	Plate
Length (L)	160/1000 M
Breadth (B)	160/1000 M
Height (H)	0.6/1000 M
Density (rho)	7800 Kg./M3
Modulus of Elastisity (E)	207 Gpa
Modulus of Rigidity (v)	0.3 Gpa

TABLE4.2. Material Properties and dimensions of Piezo-Patch

Parameter	Piezo-Patch
Length (L)	0.02 M
Breadth (B)	0.02 M
Height (H)	1.06/1000 M
Density (rho)	7500 Kg./M3
Modulus of Elastisity (E)	63 Gpa
Modulus of Rigidity (v)	0.3 Gpa

We considered the cantilever plate. The plate is divided into 64 elements (8*8) as shown in fig. 4.2. The plate possessed the 81 nodes. When the plate is without cantilever position the plate possess 81 nodes and each node possess 3 degree of freedom (DOF) as shown in fig. 4.3. So the total degree of freedom becomes 81*3=243 DOF. But when we considered the cantilever plate the first 9*3=27 DOF which attached cantilever edge becomes zero. So in case of cantilever plate the DOF considered is 243-27=216 DOF.

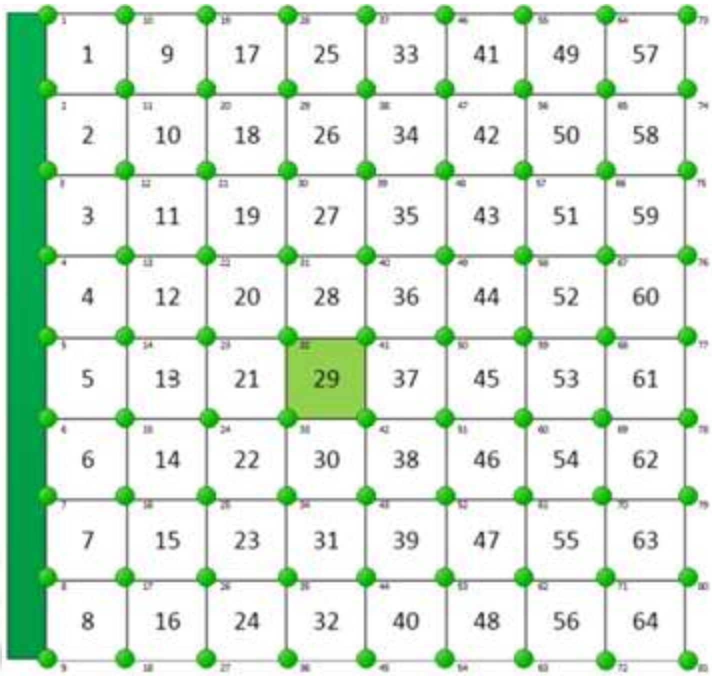

Fig.1: Cantilever Plate with 64 elements and 81 Nodes

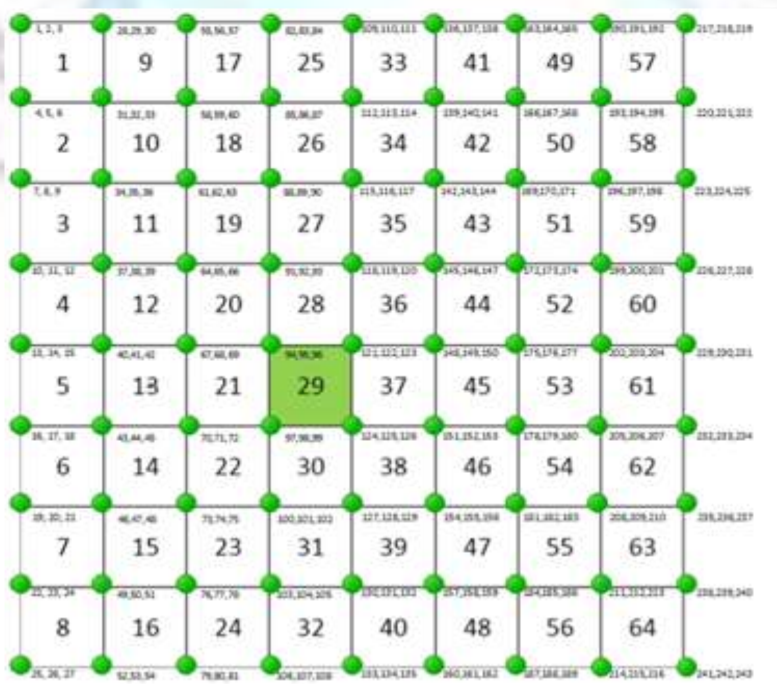

Fig. 2: Plate with 64 elements and 243 deegre of freedoms.

5. Mathematical Calculation

The plate is modeled using the finite element method. It is divided into discrete finite elements where 'ζ' and 'η' are the natural coordinates of the finite element and they are related to global coordinates (x, y) as:

$$\zeta = \frac{x}{a} \quad \text{And} \quad \eta = \frac{y}{b} \qquad (1)$$

Each finite element has four nodes and each node has three degrees of freedom: one translational ω and two rotational θ_x and θ_y . If $\{u_e\}$ is the displacement vector of an element then displacement in the z-direction can be interpolated as:

$$\omega = [N]_{1\times12}\{u_e\}_{12\times1} \qquad (2)$$

Where $[N]_{1\times12}$ is Hermite's interpolation function .
Ignoring shear deformations in the plate and using Kirchhoff's classical plate theory , strains $\{\boldsymbol{\varepsilon}\}$ developing in the plate can be written as:

$$\{\varepsilon\} = \{\frac{\partial u}{\partial x} \ \frac{\partial v'}{\partial y} \ \frac{\partial v'}{\partial x} + \frac{\partial u}{\partial x}\}^T \qquad (3)$$

Where $u = -z\frac{\partial\omega}{\partial x}$ and $\boldsymbol{v}' = -z\frac{\partial\omega}{\partial y}$
After substituting values of 'u' and '$\boldsymbol{v}'$' in the equation (3), we get:

$$\{\varepsilon\}_{3\times1} = z[B_u]_{3\times12}\{u_e\}_{12\times1} \qquad (4)$$

Where $[B_u]_{3\times12} = [-\frac{z\partial^2}{\partial x^2} - \frac{z\partial^2}{\partial y^2} - \frac{2z\partial^2}{\partial x\partial y}]^T_{3\times1}[N]_{1\times12}$

Kinetic energy (T_e) of one finite element:

$$T_e = \frac{1}{2}\int_s \rho_s\omega^2 d\tau + \frac{1}{2}\int_p \rho_p\omega^2 d\tau \qquad (5)$$

Potential energy (V_e) of one finite element:

$$V_e = \frac{1}{2}\int_s \{\varepsilon\}^T\{\sigma\}d\tau + \frac{1}{2}\int_v \{\varepsilon\}^T\{\sigma\}d\tau \qquad (6)$$

Electric energy(W_{elect}) stored in one finite element:

$$W_{elect} = \frac{1}{2}\int_p \{E\}^T\{D\}d\tau \qquad (7)$$

Where $\{D\}$ is Electric displacement vector.

External surface traction or a point force can act on a smart structure. These forces would do work on the smart structure and as a result, energy stored per element is:

$$W_{ext\,(1)} = \int_{A_s}\{w\}^T\{f_s^e\}dA_s \qquad (8)$$

Work required to apply external charge on the surface of a piezoelectric is:

$$W_{ext\,(11)} = -\int_{A_p} qv\,dA_P \qquad (9)$$

Now, the Lagrangian for one finite element of the smart structure can be obtained as:

$$L = T_e - V_e + \left(W_{elect} + W_{ext\,(1)} + W_{ext\,(11)}\right) \qquad (10)$$

The Lagrangian can be calculated using finite element relations and augmented constitutive equations . The equation of motion of one finite element is derived using Hamilton's principle:

$$\delta\int_{t1}^{t2} L\,dt = 0 \qquad (11)$$

The resulting variational contains two variables namely '$\{u_e\}$' and 'v'. Taking variation with respect to $\{u_e\}$,we get:

$$([m_s^e] + [m_P^e])\{\ddot{u}_e\} + ([k_s^e] + [k_P^e])\{u_e\} + [k_{uv}^e]v = \{F_S^E\} \qquad (12)$$

And taking variation with respect to 'v', we get:

$$[k_{vu}^e]\{u_e\} - [k_{vv}^e]v = 0 \qquad (13)$$

$$v = [k_{vv}^e]^{-1}[k_{vu}^e]\{u_e\} \quad (14)$$

Where
$[m_S^e] = \int_s \rho_s [N]^T [N] d\boldsymbol{\tau}$

is the substrate element mass matrix,

$[m_P^e] = \int_P \rho_P [N]^T [N] d\boldsymbol{\tau}$

is the piezoelectric element mass matrix,

$[k_S^e] = \int_s z^2 [B_u]^T [c_s][B_u] d\boldsymbol{\tau}$

is the substrate element stiffness matrix,

$[k_P^e] = \int_P z^2 [B_u]^T [c_s][B_u] d\boldsymbol{\tau}$

is the piezoelectric element stiffness matrix,

$[k_{uv}^e] = [k_{uv}^e]^T = \int_P z\{B_u\}^T [e^t][B_v] d\tau$

is the electromechanical interaction matrix.
And $\{F_S^E\} = \int_{S1} \{f_S^E\}$ds

Where $[B_v]_{3\times1} = [0 \; 0 \; \frac{1}{d}]^T$

The sensor/actuator equation is given as:

$$[k_{vu}^e]\{u_e\} + [k_{vv}^e]v + \bar{q} = 0 \quad (15)$$

Where

$$[k_{vu}^e] = \int_{QP} z[B_v]^T [e][B_u]\{u_e\} dQ$$

$$[k_{vv}^e] = \int_{QP} [B_v]^T [\zeta][B_v] v dQ$$

$$\int_{QP} = \int_{s2} q ds$$

Substituting for v from (15) in (12) we have an equation of motion of an elementmodified as,

$$([m_S^e] + [m_P^e])\{\ddot{u}_e\} + ([k_S^e] + [k_P^e])\{u_e\} + [k_{uv}^e][k_{vv}^e]^{-1}(-\bar{Q} + [k_{vu}^e]\{u_e\}) = 0 \quad (16)$$

Put the value of v from eqation (14), in the equation(16), We get the equation of motion of one finite element of the smart plate structure as:

$$[M_e]\{\ddot{u}_e\} + [K_e]\{u_e\} = \{F_e\} \quad (17)$$

Where$[M_e]$, $[K_e]$and $\{F_e\}$ are elemental mass matrix, elemental stiffness matrix and total force on the finite element respectively.

Applying assembly procedure and boundary conditions, the global equation of motion of the smart cantilevered plate structure is obtained as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{\dot{x}\} = \{F\} \quad (18)$$

Where the damping term $[C] = \alpha[M] + \beta[K]$. '$\alpha$' and '$\beta$' are Rayleigh mass and stiffness damping coefficients respectively. This is the equation of motion of a two dimensional smart cantilevered plate instrumented with one collocated Piezoelectric sensor–actuator pair.

6. Observer For Tip Displacement

One of the challenges in developing any modal control is the identification of modal quantities. In this work, the Kalman observer is adopted for identification of modal displacements and velocities of first two modes. Considering only the first two modes of the plate, the equation of motion of the smart plate in state space is given by [19]:

$$\dot{s} = A \times s + B \times V_0 \qquad (19)$$

$$y = c \times s \qquad (20)$$

$$s = \begin{cases} \eta_1 \\ \eta_2 \\ \dot{\eta_3} \\ \dot{\eta_4} \end{cases} \qquad (21)$$

Here $\boldsymbol{y}$ is sensor signal and so vector $\boldsymbol{c}$ can be obtained from the sensor equation. The state equation for a system with external noise can be written as follows:

$$\dot{s} = A \times s + B \times V_0 + G \times v \qquad (22)$$
$$y = c \times s + w \qquad (23)$$

Where $G = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

The Kalman filter dynamics can be written as:

$$\dot{s_e} = A \times s_e + B \times V_0 + K_e \times (y - c \times s_e) \qquad (24)$$

Where $\boldsymbol{s_e}$ is an estimated state and $\boldsymbol{K_e}$ is the Kalman filter gain to minimize the expected value is

$E\{(s - s_e)^T (s - s_e)\}$

In discrete time Kalman's equations take the form: Time update:

$$s_e(K+1) = Fs_e(K) + gV_0 \qquad (25)$$

Measurement update:

$$s_e(K+1) = s_e(K+1) + M[y - cs_e(k+1)] \qquad (26)$$

F and *g* are discretized versions of matrices *A* and *B*, respectively. *M* is the Kalman innovation gain, *k* is a time instant and *se* is state vector estimated by the Kalman observer. Employing the Kalman observer the entire state vector of the system can be estimated. The estimated state vector can be suitably manipulated to get the control output.

Conclusion

This work shows the basic techniques for analysis of active vibration control using piezoelectric sensor and actuators. A general scheme of analyzing and designing piezoelectric smart cantilever plate with help of finite element method is successfully developed in this study. The optimal location of sensor/actuator pair for a cantilever plate to suppress first three modes of vibrations and control effectiveness using any type of controller has been obtained.

For future work in this controlling process of vibration of a cantilever plate, the various types of controller is used like that using fuzzy logic controller, using neural network controller, using LQR controller and also using PID controller. We find out the optimal location of sensor/actuator pair for a cantilever plate using various types of controller and controlled the vibration.

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