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Reduced Differential Transform Method for the Generalized Ito System

Ann J. Al-Sawoor[1], Mohammed O. Al-Amr[2]

[1,2]Department of Mathematics, College of Computer Sciences and Mathematics,
University of Mosul, Mosul 41002, Iraq
[1]annsawoor@yahoo.com, [2]malamroo@yahoo.com, alamr@uomosul.edu.iq

Abstract: In this paper, Reduced differential transform method (RDTM) is employed to obtain the solution of the generalized Ito system. The efficiency of the proposed method is illustrated by three test examples. The results obtained by employing RDTM are compared with exact solutions to reveal that the RDTM is very accurate, effective, and convenient to handle a wide range of nonlinear equations.

Keywords: Reduced differential transform method (RDTM); partial differential equations; generalized Ito system.

Introduction

Nonlinear partial differential equations (NPDEs) play an important role in such various fields as physics, chemistry, biology, mathematics and engineering. There are many interesting and useful features of physical systems hidden in their nonlinear behavior. The investigation of exact solutions of NPDEs is becoming an important aspect since it can help us well to understand the mechanism of the complicated physical phenomena modeled by NPDEs.

The investigation of exact solutions of NPDEs plays an important role in the study of nonlinear physical phenomena. Many methods, exact, approximate and purely numerical are available in literature [1-9] for the solution of NPDEs.

In this paper, we apply the reduced differential transform method for solving the generalized Ito system [10]

$$\begin{aligned} u_t &= v_x, \\ v_t &= -2v_{xxx} - 6(uv)_x + aww_x + bpw_x + cwp_x + dpp_x + fw_x + gp_x, \\ w_t &= w_{xxx} + 3uw_x, \\ p_t &= p_{xxx} + 3up_x. \end{aligned} \tag{1}$$

where a, b, c, d, f and g are arbitrary constants.

In recent years, different cases of the generalized Ito system has been studied analytically and numerically by many authors [11-16].The RDTM, which first proposed by the Turkish mathematician YildirayKeskin [17-20] in 2009, has received much attention due to its applications to solve a wide variety of problems [21-28].

This paper has been organized as follows: Section 2 deals with the analysis of the method. In Section 3, we apply the RDTM to solve three special cases of the generalized Ito system, Conclusions are given in Section 4.

Analysis of the Method

Consider a function of two variables $u(x,t)$ and suppose that it can be represented as a product of two single-variable functions, i.e., $u(x,t) = f(x)g(t)$. Based on the properties of one-dimensional differential transform, the function $u(x,t)$ can be represented as

$$u(x,t) = \left(\sum_{i=0}^{\infty} F(i)x^i\right)\left(\sum_{j=0}^{\infty} G(j)t^j\right) = \sum_{k=0}^{\infty} U_k(x)t^k \tag{2}$$

where $U_k(x)$ is called t -dimensional spectrum function of $u(x,t)$.

The basic definitions of reduced differential transform method are introduced as follows [17-20]:

Definition 2.1 If function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(x,t)=\frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}u(x,t)\right]_{t=0} \quad (3)$$

where the t -dimensional spectrum function $U_k(x)$ is the transformed function. In this paper, the lowercase $u(x,t)$ represent the original function while the uppercase $U_k(x)$ stand for the transformed function.

Definition 2.2 The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t)=\sum_{k=0}^{\infty}U_k(x)t^k \quad (4)$$

Then combining equation (3) and (4) we write

$$u(x,t)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}t^k \quad (5)$$

From the above definitions, it can be found that the concept of the reduced differential transform is derived from the power series expansion.

To illustrate the basic concepts of the RDM, consider the following nonlinear partial differential equation written in an operator form

$$Lu(x,t)+Ru(x,t)+Nu(x,t)=g(x,t), \quad (6)$$

with initial condition

$$u(x,0)=f(x), \quad (7)$$

where $L=\frac{\partial}{\partial t}$, R is a linear operator which are partial derivatives, $Nu(x,t)$ is a nonlinear operator and $g(x,t)$ is an inhomogeneous term.

According to the RDTM, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x,t)=\mathrm{G}_k(x)-RU_k(x)-NU_K(x), \quad (8)$$

where $U_k(x), RU_k(x), NU_k(x)$ and $G_k(x)$ are the transformations of the functions $Lu(x,t), Ru(x,t), Nu(x,t)$ and $g(x,t)$ respectively.

From initial condition (7), we write

$$U_0(x)=f(x), \quad (9)$$

Substituting (9) into (8) and by straightforward iterative calculation, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$ gives the n-terms approximate solution as

$$\tilde{u}_n(x,t)=\sum_{k=0}^{n}U_k(x)t^k, \quad (10)$$

Therefore the exact solution of the problem is given by

$$u(x,t)=\lim_{n\to\infty}\tilde{u}_n(x,t). \quad (11)$$

The fundamental mathematical operations performed by RDTM can be readily obtained and are listed in Table 1.

Table 1: The fundamental operations of RDTM

Functional Form	**Transformed Form**
$u(x,t)$	$U_k(x,t)=\frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}u(x,t)\right]_{t=0}$
$w(x,t)=u(x,t)\pm v(x,t)$	$W_k(x)=U_k(x)\pm V_k(x)$
$w(x,t)=\alpha u(x,t)$	$W_k(x)=\alpha U_k(x)$ (α is constant)
$w(x,t)=x^m t^n$	$W_k(x)=x^m\delta(k-n)$
$w(x,t)=x^m t^n u(x,t)$	$W_k(x)=x^m U_{k-n}(x)$

$w(x,t)=u(x,t)v(x,t)$	$W_k(x)=\sum_{r=0}^{k}V_r(x)U_{k-r}(x)=\sum_{r=0}^{k}U_r(x)V_{k-r}(x)$
$w(x,t)=\frac{\partial^k}{\partial t^k}u(x,t)$	$W_k(x)=(k+1)\cdots(k+r)U_{k+r}(x)=\frac{(k+r)!}{k!}U_{k+r}(x)$
$w(x,t)=\frac{\partial}{\partial x}u(x,t)$	$W_k(x)=\frac{\partial}{\partial x}U_k(x)$

Applications

In this section, we employ the RDTM to solve some special cases of system (1), the results of test examples are compared with exact solutions to prove the efficiency of the proposed method. These results are shown in Figs. 1-3 and detailed in Tables 2-4. We use MAPLE software to obtain the solutions from the RDTM.

Example 1

Consider the system (1) for $a=-37, b=2, c=1/2, d=-1, f=2,$ and $g=2$, which yields [13]

$$\begin{aligned}
&u_t=v_x,\\
&v_t=-2v_{xxx}-6(uv)_x-37ww_x+2pw_x+\frac{1}{2}wp_x-pp_x+2w_x+2p_x,\\
&w_t=w_{xxx}+3uw_x,\\
&p_t=p_{xxx}+3up_x.
\end{aligned}\tag{12}$$

Subject to initial conditions

$$\begin{aligned}
&u(x,0)=\frac{7}{12}-2\tanh^2(x),\ v(x,0)=-\frac{7}{48}+\frac{1}{2}\tanh^2(x),\\
&w(x,0)=-\frac{36}{37}+\frac{1}{6}\tanh(x),\ p(x,0)=4+\frac{37}{12}\tanh(x).
\end{aligned}\tag{13}$$

According to the RDTM and Table 1, the differential transform of Eqs. (13)read

$$\begin{aligned}
&(k+1)U_{k+1}(x)=\frac{\partial}{\partial x}V_k(x),\\
&(k+1)V_{k+1}(x)=-2\frac{\partial^3}{\partial x^3}V_k(x)-6\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}V_{k-r}(x)-6\sum_{r=0}^{k}V_r(x)\frac{\partial}{\partial x}U_{k-r}(x)-37\sum_{r=0}^{k}W_r(x)\frac{\partial}{\partial x}W_{k-r}(x)\\
&\qquad+2\sum_{r=0}^{k}P_r(x)\frac{\partial}{\partial x}W_{k-r}(x)+\frac{1}{2}\sum_{r=0}^{k}W_r(x)\frac{\partial}{\partial x}P_{k-r}(x)-\sum_{r=0}^{k}P_r(x)\frac{\partial}{\partial x}P_{k-r}(x)+2W_k(x)+2P_k(x),\\
&(k+1)W_k(\mathrm{x})=\frac{\partial^3}{\partial x^3}W_k(x)+3\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}W_{k-r}(x),\\
&(k+1)P_k(\mathrm{x})=\frac{\partial^3}{\partial x^3}P_k(x)+3\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}P_{k-r}(x).
\end{aligned}\tag{14}$$

where the t -dimensional spectrum functions $U_k(x), V_k(x), \mathrm{W}_k(x)$ and $P_k(x)$ are the transformed functions.

From initial conditions (13), we have

$$\begin{aligned}
&U_0(x)=\frac{7}{12}-2\tanh^2(x), V_0(x)=-\frac{7}{48}+\frac{1}{2}\tanh^2(x),\\
&W_0(x)=-\frac{36}{37}+\frac{1}{6}\tanh(x),\ P_0(x)=4+\frac{37}{12}\tanh(x).
\end{aligned}\tag{15}$$

Substituting Eqs.(15)intoEqs. (14)and by straightforward iterative steps, we can obtain

$$U_1(x)=\frac{\sinh(x)}{\cosh(x)^3}, V_1(x)=-\frac{1}{4}\frac{\sinh(x)}{\cosh(x)^3}, W_1(x)=-\frac{1}{24\cosh(x)^2},\ P_1(x)=-\frac{37}{48\cosh(x)^2},$$

$$U_2(x)=\frac{1}{8}\frac{2\cosh(x)^2-3}{\cosh(x)^4}, V_2(x)=-\frac{1}{32}\frac{2\cosh(x)^2-3}{\cosh(x)^4}, W_2(x)=-\frac{1}{96}\frac{\sinh(x)}{\cosh(x)^3},\ P_2(x)=-\frac{37}{192}\frac{\sinh(x)}{\cosh(x)^3},$$

$$U_3(x)=\frac{1}{24}\frac{\sinh(x)\left(\cosh(x)^2-3\right)}{\cosh(x)^5},\ V_3(x)=-\frac{1}{96}\frac{\sinh(x)\left(\cosh(x)^2-3\right)}{\cosh(x)^5},\ W_3(x)=-\frac{1}{1152}\frac{2\cosh(x)^2-3}{\cosh(x)^4},$$

$$P_3(x)=-\frac{37}{2304}\frac{2\cosh(x)^2-3}{\cosh(x)^4}.$$

$\vdots$

and so on, in the same manner, the rest of components can be easily obtained.

Taking the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$, $\{V_k(x)\}_{k=0}^{n}$, $\{W_k(x)\}_{k=0}^{n}$ and $\{P_k(x)\}_{k=0}^{n}$ gives n-terms approximate solutions as

$$\tilde{u}_n(x,t)=\sum_{k=0}^{n}U_k(x)t^k=\frac{7}{12}-2\tanh^2(x)+\frac{\sinh(x)}{\cosh(x)^3}t+\frac{1}{8}\frac{2\cosh(x)^2-3}{\cosh(x)^4}t^2+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(\frac{7}{12}-2\tanh^2(x-\frac{t}{4})\right)\right]_{t=0}t^n,$$

$$\tilde{v}_n(x,t)=\sum_{k=0}^{n}V_k(x)t^k=-\frac{7}{48}+\frac{1}{2}\tanh^2(x)-\frac{1}{4}\frac{\sinh(x)}{\cosh(x)^3}t-\frac{1}{32}\frac{2\cosh(x)^2-3}{\cosh(x)^4}t^2$$
$$+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(-\frac{7}{48}+\frac{1}{2}\tanh^2(x-\frac{t}{4})\right)\right]_{t=0}t^n,$$

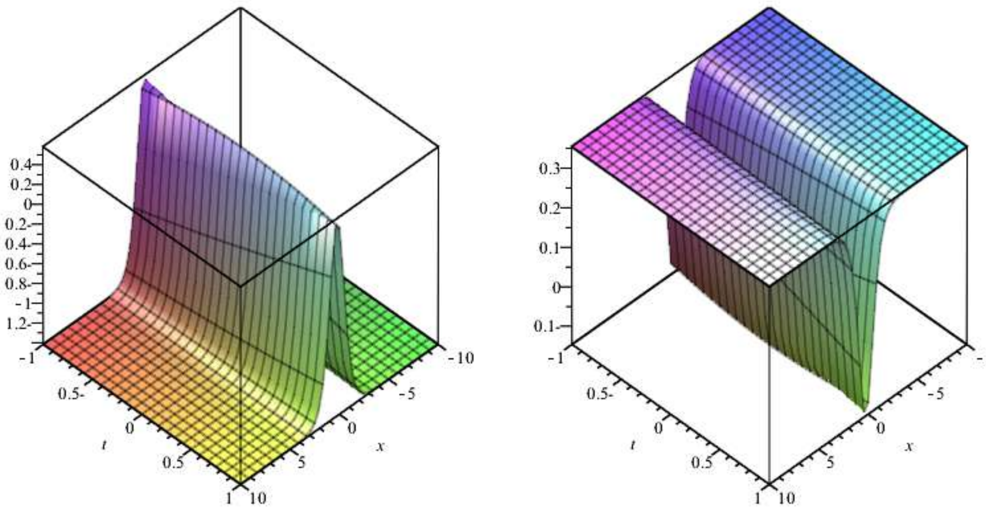
$$\tilde{w}_n(x,t)=\sum_{k=0}^{n}W_k(x)t^k=-\frac{36}{37}+\frac{1}{6}\tanh(x)-\frac{1}{24\cosh(x)^2}t-\frac{1}{96}\frac{\sinh(x)}{\cosh(x)^3}t^2+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(-\frac{36}{37}+\frac{1}{6}\tanh(x-\frac{t}{4})\right)\right]_{t=0}t^n,$$

$$\tilde{p}_n(x,t)=\sum_{k=0}^{n}P_k(x)t^k=4+\frac{37}{12}\tanh(x)-\frac{37}{48\cosh(x)^2}t-\frac{37}{192}\frac{\sinh(x)}{\cosh(x)^3}t^2+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(4+\frac{37}{12}\tanh(x-\frac{t}{4})\right)\right]_{t=0}t^n.$$

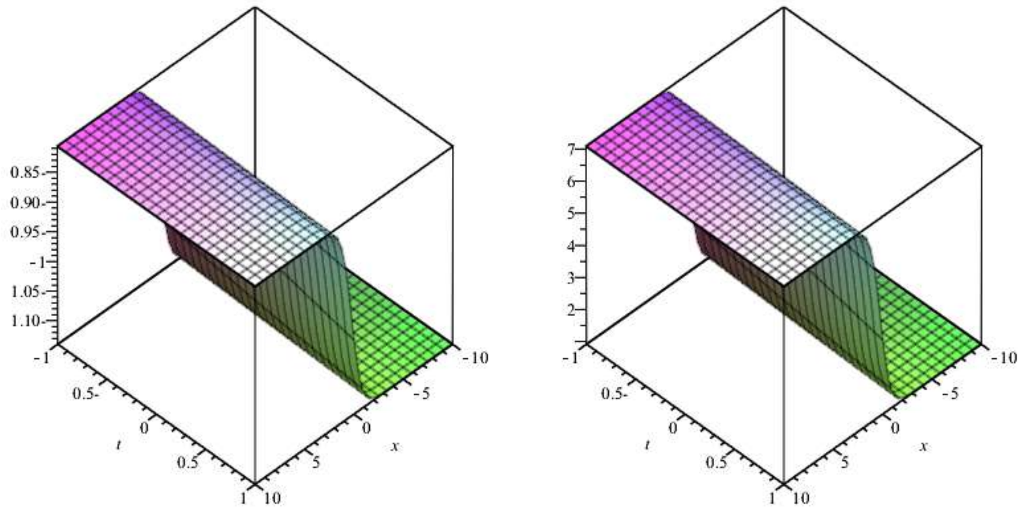
Therefore, the exact solution of problem is readily obtained as

$$\begin{aligned}u(x,t)&=\lim_{n\to\infty}\tilde{u}_n(x,t)=\frac{7}{12}-2\tanh^2(x-\frac{t}{4}),\ v(x,t)=\lim_{n\to\infty}\tilde{v}_n(x,t)=-\frac{7}{48}+\frac{1}{2}\tanh^2(x-\frac{t}{4}),\\ w(x,t)&=\lim_{n\to\infty}\tilde{w}_n(x,t)=-\frac{36}{37}+\frac{1}{6}\tanh(x-\frac{t}{4}),\ p(x,t)=\lim_{n\to\infty}\tilde{p}_n(x,t)=4+\frac{37}{12}\tanh(x-\frac{t}{4}).\end{aligned}\qquad(16)$$

Fig.1 shows the 15-terms approximate solutions of problem (12) obtained by RDTM. The absolute errors of these solutions for different values of t at $x=20$ are detailed in Table 2.



(a) (b)



(c) (d)

Fig.1 The approximate solution (a) $\tilde{u}_{15}(x,t)$ (b) $\tilde{v}_{15}(x,t)$ (c) $\tilde{w}_{15}(x,t)$ (d) $\tilde{p}_{15}(x,t)$ obtained by RDTM

Table 2: The absolute error of $\tilde{u}_{15}(x,t),\tilde{v}_{15}(x,t),\tilde{w}_{15}(x,t)$ and $\tilde{p}_{15}(x,t)$ for different values of t at $x=20$

t	$\lvert u(x,t)-\tilde{u}_{15}(x,t)\rvert$	$\lvert v(x,t)-\tilde{v}_{15}(x,t)\rvert$	$\lvert w(x,t)-\tilde{w}_{15}(x,t)\rvert$	$\lvert p(x,t)-\tilde{p}_{15}(x,t)\rvert$
0.2	1.63400×10^{-46}	4.08500×10^{-47}	6.80834×10^{-48}	1.25954×10^{-46}
0.4	1.07723×10^{-41}	2.69307×10^{-42}	4.48845×10^{-43}	8.30363×10^{-42}
0.6	7.11797×10^{-39}	1.77949×10^{-39}	2.96582×10^{-40}	5.48676×10^{-39}
0.8	7.14460×10^{-37}	1.78615×10^{-37}	2.97692×10^{-38}	5.50729×10^{-37}
1.0	2.55361×10^{-35}	6.38402×10^{-36}	1.06400×10^{-36}	1.96841×10^{-35}
1.2	4.74990×10^{-34}	1.18747×10^{-34}	1.97912×10^{-35}	3.66138×10^{-34}
1.4	5.62959×10^{-33}	1.40740×10^{-33}	2.34566×10^{-34}	4.33948×10^{-33}
1.6	4.79741×10^{-32}	1.19935×10^{-32}	1.99892×10^{-33}	3.69800×10^{-32}
1.8	3.17775×10^{-31}	7.94438×10^{-32}	1.32406×10^{-32}	2.44952×10^{-31}
2.0	1.72555×10^{-30}	4.31387×10^{-31}	7.18978×10^{-32}	1.33011×10^{-30}

Example 2

Consider the system (1) when $a=6, b=7, c=1, d=1, f=1,$ and $g=1,$ which yields [15]

$$\begin{aligned} &u_t = v_x, \\ &v_t = -2v_{xxx} - 6(uv)_x + 6ww_x + 7pw_x + wp_x + pp_x + w_x + p_x, \\ &w_t = w_{xxx} + 3uw_x, \\ &p_t = p_{xxx} + 3up_x. \end{aligned} \tag{17}$$

with initial conditions

$$\begin{aligned} &u(x,0) = \frac{31}{12} - 4\tanh^2(x), v(x,0) = -\frac{31}{48} + \tanh^2(x), \\ &w(x,0) = 4 + 2\tanh^2(x),\ p(x,0) = \frac{99}{8} - 12\tanh^2(x). \end{aligned} \tag{18}$$

Taking the differential transform of Eqs.(17), we have

$$(k+1)U_{k+1}(x) = \frac{\partial}{\partial x}V_k(x),$$

$$(k+1)V_{k+1}(x) = -2\frac{\partial^3}{\partial x^3}V_k(x) - 6\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}V_{k-r}(x) - 6\sum_{r=0}^{k}V_r(x)\frac{\partial}{\partial x}U_{k-r}(x) + 6\sum_{r=0}^{k}W_r(x)\frac{\partial}{\partial x}W_{k-r}(x)$$
$$+7\sum_{r=0}^{k}P_r(x)\frac{\partial}{\partial x}W_{k-r}(x) + \sum_{r=0}^{k}W_r(x)\frac{\partial}{\partial x}P_{k-r}(x) + \sum_{r=0}^{k}P_r(x)\frac{\partial}{\partial x}P_{k-r}(x) + W_k(x) + P_k(x),$$
$$(k+1)W_k(\mathrm{x}) = \frac{\partial^3}{\partial x^3}W_k(x) + 3\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}W_{k-r}(x), \tag{19}$$
$$(k+1)P_k(\mathrm{x}) = \frac{\partial^3}{\partial x^3}P_k(x) + 3\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}P_{k-r}(x).$$

where the t -dimensional spectrum functions $U_k(x), V_k(x), \mathrm{W}_k(x)$ and $P_k(x)$ are the transformed functions.

From initial conditions(18), we write

$$U_0(x) = \frac{31}{12} - 4\tanh^2(x), V_0(x) = -\frac{31}{48} + \tanh^2(x),$$
$$W_0(x) = 4 + 2\tanh^2(x),\ P_0(x) = \frac{99}{8} - 12\tanh^2(x). \tag{20}$$

Substituting Eqs.(20)intoEqs. (19)and by straightforward iterative steps, we can obtain

$$U_1(x) = \frac{2\sinh(x)}{\cosh(x)^3}, V_1(x) = -\frac{1}{2}\frac{\sinh(x)}{\cosh(x)^3}, W_1(x) = -\frac{\sinh(x)}{\cosh(x)^3},\ P_1(x) = \frac{6\sinh(x)}{\cosh(x)^3},$$
$$U_2(x) = \frac{1}{4}\frac{2\cosh(x)^2-3}{\cosh(x)^4}, V_2(x) = -\frac{1}{16}\frac{2\cosh(x)^2-3}{\cosh(x)^4}, W_2(x) = -\frac{1}{8}\frac{2\cosh(x)^2-3}{\cosh(x)^4},\ P_2(x) = \frac{3}{4}\frac{2\cosh(x)^2-3}{\cosh(x)^4},$$
$$U_3(x) = \frac{1}{12}\frac{\sinh(x)\left(\cosh(x)^2-3\right)}{\cosh(x)^5}, V_3(x) = -\frac{1}{48}\frac{\sinh(x)\left(\cosh(x)^2-3\right)}{\cosh(x)^5}, W_3(x) = -\frac{1}{24}\frac{\sinh(x)\left(\cosh(x)^2-3\right)}{\cosh(x)^5},$$
$$P_3(x) = \frac{1}{4}\frac{\sinh(x)\left(\cosh(x)^2-3\right)}{\cosh(x)^5}.$$
$$\vdots$$

and so on, in the same manner, the rest of components can be easily computed.

Taking the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$, $\{V_k(x)\}_{k=0}^{n}$, $\{W_k(x)\}_{k=0}^{n}$ and $\{P_k(x)\}_{k=0}^{n}$ gives n-terms approximate solutions as

$$\tilde{u}_n(x,t) = \sum_{k=0}^{n}U_k(x)t^k = \frac{31}{12} - 4\tanh^2(x) + \frac{2\sinh(x)}{\cosh(x)^3}t + \frac{1}{4}\frac{2\cosh(x)^2-3}{\cosh(x)^4}t^2 + \cdots + \frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(\frac{31}{12} - 4\tanh^2(x - \frac{t}{4})\right)\right]_{t=0} t^n,$$
$$\tilde{v}_n(x,t) = \sum_{k=0}^{n}V_k(x)t^k = -\frac{31}{48} + \tanh^2(x) - \frac{1}{2}\frac{\sinh(x)}{\cosh(x)^3}t - \frac{1}{16}\frac{2\cosh(x)^2-3}{\cosh(x)^4}t^2$$
$$+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(-\frac{31}{48} + \tanh^2(x - \frac{t}{4})\right)\right]_{t=0} t^n,$$
$$\tilde{w}_n(x,t) = \sum_{k=0}^{n}W_k(x)t^k = 4 + 2\tanh^2(x) - \frac{\sinh(x)}{\cosh(x)^3}t - \frac{1}{8}\frac{2\cosh(x)^2-3}{\cosh(x)^4}t^2 + \cdots + \frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(4 + 2\tanh^2(x - \frac{t}{4})\right)\right]_{t=0} t^n,$$
$$\tilde{p}_n(x,t) = \sum_{k=0}^{n}P_k(x)t^k = \frac{99}{8} - 12\tanh^2(x) + \frac{6\sinh(x)}{\cosh(x)^3}t + \frac{3}{4}\frac{2\cosh(x)^2-3}{\cosh(x)^4}t^2 + \cdots + \frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(\frac{99}{8} - 12\tanh^2(x - \frac{t}{4})\right)\right]_{t=0} t^n.$$

Therefore, the exact solution of problem is readily obtained as

$$u(x,t) = \lim_{n\to\infty}\tilde{u}_n(x,t) = \frac{31}{12} - 4\tanh^2(x - \frac{t}{4}), v(x,t) = \lim_{n\to\infty}\tilde{v}_n(x,t) = -\frac{31}{48} + \tanh^2(x - \frac{t}{4}),$$
$$w(x,t) = \lim_{n\to\infty}\tilde{w}_n(x,t) = 4 + 2\tanh^2(x - \frac{t}{4}),\ p(x,t) = \lim_{n\to\infty}\tilde{p}_n(x,t) = \frac{99}{8} - 12\tanh^2(x - \frac{t}{4}). \tag{21}$$

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In Fig. 2, the 15-terms approximate solutions of problem (17) obtained by RDTM are shown graphically. The absolute errors of these solutions for different values of t at $x = 20$ are detailed in Table 3.

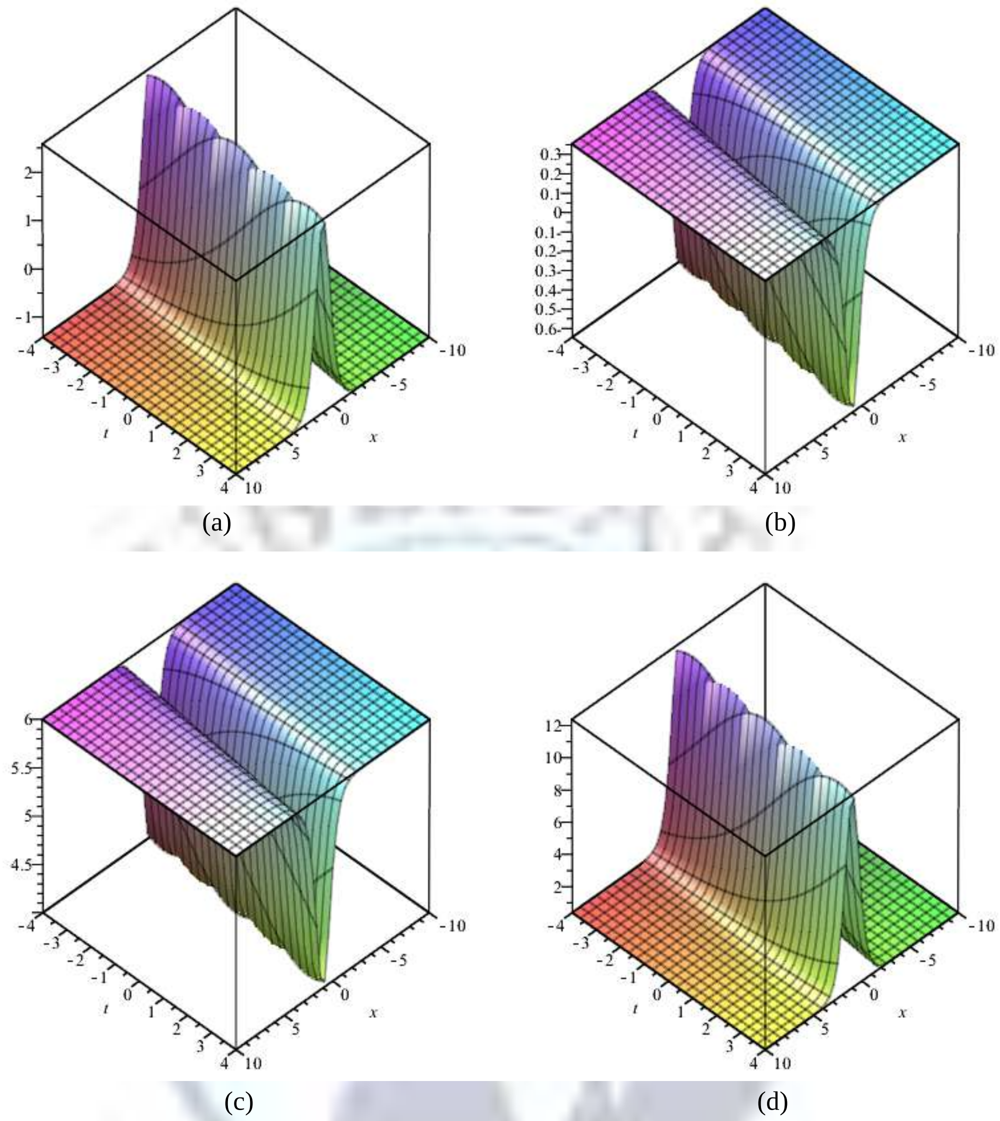


Fig.2 The approximate solution (a) $\tilde{u}_{15}(x,t)$ (b) $\tilde{v}_{15}(x,t)$ (c) $\tilde{w}_{15}(x,t)$ (d) $\tilde{p}_{15}(x,t)$ obtained by RDTM

Table 3: The absolute error of $\tilde{u}_{15}(x,t), \tilde{v}_{15}(x,t), \tilde{w}_{15}(x,t)$ and $\tilde{p}_{15}(x,t)$ for different values of t at $x = 20$

t	$\lvert u(x,t)-\tilde{u}_{15}(x,t)\rvert$	$\lvert v(x,t)-\tilde{v}_{15}(x,t)\rvert$	$\lvert w(x,t)-\tilde{w}_{15}(x,t)\rvert$	$\lvert p(x,t)-\tilde{p}_{15}(x,t)\rvert$
0.1	4.97187×10^{-51}	1.24297×10^{-51}	2.48594×10^{-51}	1.49156×10^{-50}
0.2	3.26800×10^{-46}	8.17001×10^{-47}	1.63400×10^{-46}	9.80401×10^{-46}
0.3	2.15292×10^{-43}	5.38231×10^{-44}	1.07646×10^{-43}	6.45877×10^{-43}
0.4	2.15445×10^{-41}	5.38614×10^{-42}	1.07723×10^{-41}	6.46336×10^{-41}
0.5	7.67697×10^{-40}	1.91924×10^{-40}	3.83849×10^{-40}	2.30309×10^{-39}
0.6	1.42359×10^{-38}	3.55898×10^{-39}	7.11797×10^{-39}	4.27078×10^{-38}
0.7	1.68203×10^{-37}	4.20508×10^{-38}	8.41015×10^{-38}	5.04609×10^{-37}
0.8	1.42892×10^{-36}	3.57230×10^{-37}	7.14460×10^{-37}	4.28676×10^{-36}
0.9	9.43527×10^{-36}	2.35882×10^{-36}	4.71764×10^{-36}	2.83058×10^{-35}
1.0	5.10721×10^{-35}	1.27680×10^{-35}	2.55361×10^{-35}	1.53216×10^{-34}

Example 3

Consider the system (1) for $a=0, b=-6, c=-6, d=0, f=0,$ and $g=0$, which yields [11]

$$\begin{aligned} u_t &= v_x, \\ v_t &= -2v_{xxx} - 6(uv)_x - 6pw_x - 6wp_x, \\ w_t &= w_{xxx} + 3uw_x, \\ p_t &= p_{xxx} + 3up_x. \end{aligned} \tag{22}$$

with initial conditions

$$\begin{aligned} u(x,0) &= \frac{-b_2+4\mu^4}{6\mu^2} - 2\mu^2\tanh^2(\mu x), v(x,0) = \frac{-b_2{}^2+4f_1t_1\mu^2-8b_2\mu^4}{8\mu^4} + b_2\tanh^2(\mu x), \\ w(x,0) &= -\frac{f_1t_0}{t_1} + f_1\tanh(\mu x),\ p(x,0) = t_0 + t_1\tanh(\mu x). \end{aligned} \tag{23}$$

where μ, b_2, t_0, t_1 and f_1 are arbitrary constants.

Applying the RDTM to Eqs.(22), we obtain

$$\begin{aligned} (k+1)U_{k+1}(x) &= \frac{\partial}{\partial x}V_k(x), \\ (k+1)V_{k+1}(x) &= -2\frac{\partial^3}{\partial x^3}V_k(x) - 6\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}V_{k-r}(x) - 6\sum_{r=0}^{k}V_r(x)\frac{\partial}{\partial x}U_{k-r}(x) - 6\sum_{r=0}^{k}P_r(x)\frac{\partial}{\partial x}W_{k-r}(x) \\ &\quad - 6\sum_{r=0}^{k}W_r(x)\frac{\partial}{\partial x}P_{k-r}(x), \\ (k+1)W_k(x) &= \frac{\partial^3}{\partial x^3}W_k(x) + 3\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}W_{k-r}(x), \\ (k+1)P_k(x) &= \frac{\partial^3}{\partial x^3}P_k(x) + 3\sum_{r=0}^{k}U_r(x)\frac{\partial}{\partial x}P_{k-r}(x). \end{aligned} \tag{24}$$

where the t -dimensional spectrum functions $U_k(x), V_k(x), W_k(x)$ and $P_k(x)$ are the transformed functions.

From initial conditions (23), we write

$$\begin{aligned} U_0(x) &= \frac{-b_2+4\mu^4}{6\mu^2} - 2\mu^2\tanh^2(\mu x), V_0(x) = \frac{-b_2{}^2+4f_1t_1\mu^2-8b_2\mu^4}{8\mu^4} + b_2\tanh^2(\mu x), \\ W_0(x) &= -\frac{f_1t_0}{t_1} + f_1\tanh(\mu x),\ P_0(x) = t_0 + t_1\tanh(\mu x). \end{aligned} \tag{25}$$

Substituting Eqs.(25)intoEqs. (24)and by straightforward iterative steps, we can obtain

$$U_1(x) = \frac{2b_2\mu\sinh(\mu x)}{\cosh(\mu x)^3}, V_1(x) = -\frac{\sinh(\mu x)b_2{}^2}{\cosh(\mu x)^3\mu}, W_1(x) = -\frac{1}{2}\frac{f_1b_2}{\cosh(\mu x)^2\mu},\ P_1(x) = -\frac{1}{2}\frac{t_1b_2}{\cosh(\mu x)^2\mu},$$

$$U_2(x) = \frac{1}{2}\frac{b_2{}^2\left(2\cosh(\mu x)^2-3\right)}{\cosh(\mu x)^4}, V_2(x) = -\frac{1}{4}\frac{b_2{}^3\left(2\cosh(\mu x)^2-3\right)}{\mu^2\cosh(\mu x)^4}, W_2(x) = -\frac{1}{4}\frac{f_1b_2{}^2\sinh(\mu x)}{\cosh(\mu x)^3\mu^2},$$

$$P_2(x) = -\frac{1}{4}\frac{t_1b_2{}^2\sinh(\mu x)}{\cosh(\mu x)^3\mu^2}, U_3(x) = \frac{1}{3}\frac{b_2{}^3\sinh(\mu x)\left(\cosh(\mu x)^2-3\right)}{\mu\cosh(\mu x)^5}, V_3(x) = -\frac{1}{6}\frac{\sinh(\mu x)b_2{}^4\left(\cosh(\mu x)^2-3\right)}{\mu^3\cosh(\mu x)^5},$$

$$W_3(x) = -\frac{1}{24}\frac{f_1b_2{}^3\left(2\cosh(\mu x)^2-3\right)}{\cosh(\mu x)^4\mu^3},\ P_3(x) = -\frac{1}{24}\frac{t_1b_2{}^3\left(2\cosh(\mu x)^2-3\right)}{\cosh(\mu x)^4\mu^3}.$$

$\vdots$

and so on, in the same manner, the rest of components can be easily computed.

Taking the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^n$, $\{V_k(x)\}_{k=0}^n$, $\{W_k(x)\}_{k=0}^n$ and $\{P_k(x)\}_{k=0}^n$ gives n-terms approximate solutions as

$$\tilde{u}_n(x,t)=\sum_{k=0}^{n}U_k(x)t^k=\frac{-b_2+4\mu^4}{6\mu^2}-2\mu^2\tanh^2(\mu x)+\frac{2b_2\mu\sinh(\mu x)}{\cosh(\mu x)^3}t+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(\frac{-b_2+4\mu^4}{6\mu^2}-2\mu^2\tanh^2\left(\mu(x-\frac{b_2t}{2\mu^2})\right)\right)\right]_{t=0}t^n,$$

$$\tilde{v}_n(x,t)=\sum_{k=0}^{n}V_k(x)t^k=\frac{-b_2^{\,2}+4f_1t_1\mu^2-8b_2\mu^4}{8\mu^4}+b_2\tanh^2(\mu x)-\frac{\sinh(\mu x)b_2^{\,2}}{\cosh(\mu x)^3\mu}t+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(\frac{-b_2^{\,2}+4f_1t_1\mu^2-8b_2\mu^4}{8\mu^4}+b_2\tanh^2\left(\mu(x-\frac{b_2t}{2\mu^2})\right)\right)\right]_{t=0}t^n,$$

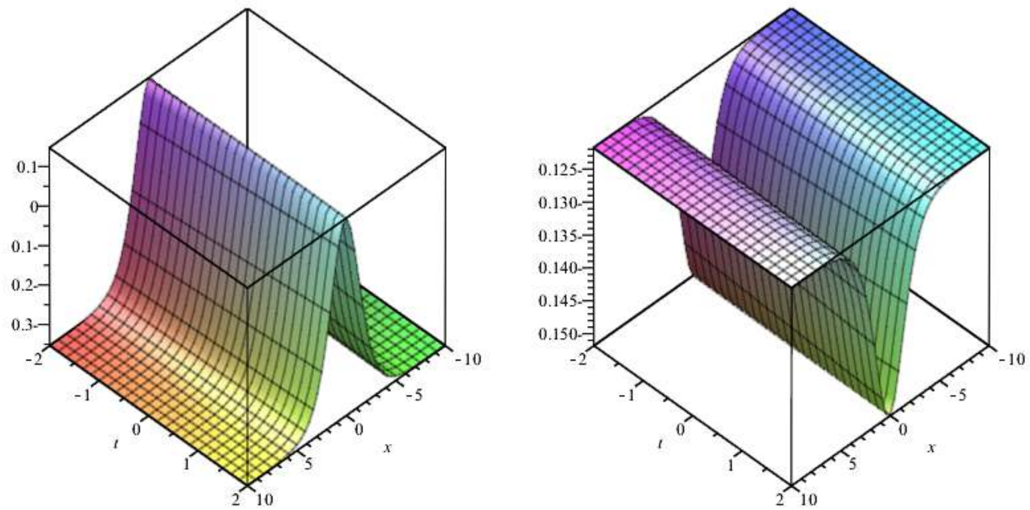
$$\tilde{w}_n(x,t)=\sum_{k=0}^{n}W_k(x)t^k=-\frac{f_1t_0}{t_1}+f_1\tanh(\mu x)-\frac{1}{2}\frac{f_1b_2}{\cosh(\mu x)^2\mu}t+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(-\frac{f_1t_0}{t_1}+f_1\tanh\left(\mu(x-\frac{b_2t}{2\mu^2})\right)\right)\right]_{t=0}t^n,$$

$$\tilde{p}_n(x,t)=\sum_{k=0}^{n}P_k(x)t^k=t_0+t_1\tanh(\mu x)-\frac{1}{2}\frac{t_1b_2}{\cosh(\mu x)^2\mu}t+\cdots+\frac{1}{n!}\left[\frac{\partial^n}{\partial t^n}\left(t_0+t_1\tanh\left(\mu(x-\frac{b_2t}{2\mu^2})\right)\right)\right]_{t=0}t^n.$$

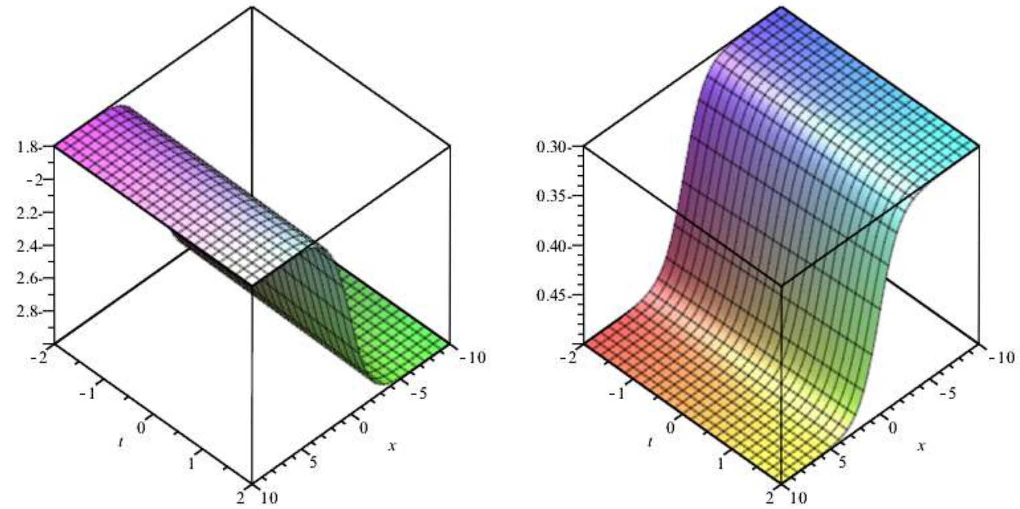
Therefore, the exact solution of problem is readily obtained as

$$\begin{aligned}
u(x,t)&=\lim_{n\to\infty}\tilde{u}_n(x,t)=\frac{-b_2+4\mu^4}{6\mu^2}-2\mu^2\tanh^2\left(\mu(x-\frac{b_2t}{2\mu^2})\right),\\
v(x,t)&=\lim_{n\to\infty}\tilde{v}_n(x,t)=\frac{-b_2^{\,2}+4f_1t_1\mu^2-8b_2\mu^4}{8\mu^4}+b_2\tanh^2\left(\mu(x-\frac{b_2t}{2\mu^2})\right),\\
w(x,t)&=\lim_{n\to\infty}\tilde{w}_n(x,t)=-\frac{f_1t_0}{t_1}+f_1\tanh\left(\mu(x-\frac{b_2t}{2\mu^2})\right),\\
p(x,t)&=\lim_{n\to\infty}\tilde{p}_n(x,t)=t_0+t_1\tanh\left(\mu(x-\frac{b_2t}{2\mu^2})\right).
\end{aligned}\qquad(26)$$

Fig.3 shows the 15-terms approximate solutions of problem (22) obtained by RDTM. In Table 4, we summarize the absolute errors of these solutions for various values of x and t with $\mu=0.5$, $b_2=0.03$, $t_0=-0.4$, $t_1=-0.1$ and $f_1=0.6$.



(a) (b)



(c) (d)

Fig.3: The approximate solution (a) $\tilde{u}_{15}(x,t)$ (b) $\tilde{v}_{15}(x,t)$ (c) $\tilde{w}_{15}(x,t)$ (d) $\tilde{p}_{15}(x,t)$ obtained by RDTM with $\mu = 0.5,\ b_2 = 0.03,\ t_0 = -0.4,\ t_1 = -0.1$ and $f_1 = 0.6$

Table 4: The absolute error of $\tilde{u}_{15}(x,t), \tilde{v}_{15}(x,t), \tilde{w}_{15}(x,t)$ and $\tilde{p}_{15}(x,t)$ for various values of x and t with $\mu = 0.5,\ b_2 = 0.03,\ t_0 = -0.4,\ t_1 = -0.1$ and $f_1 = 0.6$

t	x	$\lvert u(x,t) - \tilde{u}_{15}(x,t)\rvert$	$\lvert v(x,t) - \tilde{v}_{15}(x,t)\rvert$	$\lvert w(x,t) - \tilde{w}_{15}(x,t)\rvert$	$\lvert p(x,t) - \tilde{p}_{15}(x,t)\rvert$
0.1	0.1	1.80034×10^{-43}	1.08020×10^{-44}	1.21829×10^{-44}	2.03048×10^{-45}
	0.3	2.79404×10^{-44}	1.67643×10^{-45}	2.21409×10^{-44}	3.69015×10^{-45}
	0.5	1.64588×10^{-43}	9.87526×10^{-45}	8.59977×10^{-45}	1.43329×10^{-45}
0.3	0.1	7.77078×10^{-36}	4.66246×10^{-37}	5.21138×10^{-37}	8.68563×10^{-38}
	0.3	1.16915×10^{-36}	7.01488×10^{-38}	9.53584×10^{-37}	1.58931×10^{-37}
	0.5	7.08051×10^{-36}	4.24830×10^{-37}	3.73190×10^{-37}	6.21983×10^{-38}
0.5	0.1	2.76184×10^{-32}	1.65710×10^{-33}	1.83554×10^{-33}	3.05924×10^{-34}
	0.3	4.02489×10^{-33}	2.41493×10^{-34}	3.38181×10^{-33}	5.63636×10^{-34}
	0.5	2.50817×10^{-32}	1.50490×10^{-33}	1.33346×10^{-33}	2.22244×10^{-34}

Conclusion

In this work, we implement the reduced form of differential transform method (DTM), so-called reduced differential transform method (RDTM), to solve the generalized Ito system. The proposed technique, which does not require linearization, discretization or perturbation, gives the solution in the form of convergent power series with elegantly computed components, essentially, the accuracy of the solution increases as the number of terms increased. Three test examples are presented to demonstrate the efficiency of the present method. The results of test examples showed that the RDTM is very accurate, consistent and powerful technique to solve nonlinear problems.

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