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Optimization of Angular Orientation of Composite Laminates

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Abstract: A Composite material can be defined as a combination of two or more materials that results in better properties than those of the individual components used alone. In contrast to metallic alloys, each material retains its separate chemical, physical, and mechanical properties. The two constituents of composites are reinforcement and matrix. Stiffness and strength is the basic concept for underlying the mechanics of fiber-reinforced advanced composite materials. This aspect of composite materials technology is sometimes terms micromechanics, because it deals with the relations between macroscopic engineering properties and the microscopic distribution of the material's constituents, namely the volume fraction of fiber. I analyze in this report laminate composite materials by numerical methods. I write a MATLAB program that assists the user to find out the stiffness matrix of a laminate composite. The main objective of the report is to show the advantages and use of MATLAB software in composite laminates analysis. The Input parameters are four elastic constants, E_1, E_2, ν_{12}, & G_{12}. By using input parameters I find out the global strain, global stress, local strain & local stress. Finally with use of MATLAB program I find the different value of strength ratio for composite laminates. For different orientations the FOS must lies from 1.0 to 1.2. In last I obtained the optimum value of angular orientation of composite laminates by using MATLAB program.

Keywords: MATLAB, Laminates, Fiber Reinforced Composites, Matrix, Lamina Orientation, Optimization, strength ratio.

1. Introduction

Laminated Composite materials have characteristics of high modulus/weight and strength/weight ratios, excellent fatigue properties, and non-corroding behaviour. These advantages encourage the extensive application of composite materials, for example, in sports and aerospace. The understanding of the mechanical behaviour of composite materials is essential for their design and application. Although composite materials are often heterogeneous, they are presumed homogeneous from the viewpoint of macro mechanics and only the averaged apparent mechanical properties are considered[15].For a transversely isotropic composite material, five elastic constants are necessary to describe the linear stress-strain relationship. If the geometry of the material could be considered as two-dimensional, four independent constants are necessary due to the assumption about the out-of-plane shear modulus or Poisson's ratio. The most common method to determine these constants is static testing. For composite materials, three types of specimens with different stacking sequences, i.e., [0], [90], and [±45] are generally fabricated.

1.1. Laminates

When there is a single ply or a lay-up in which all of the layers or plies are stacked in the same orientation, the lay-up is called a lamina. When the plies are stacked at various angles, the lay-up is called a laminate. Continuous-fiber composites are normally laminated materials in which the individual layers, plies, or laminae are oriented in directions that will enhance the strength in the primary load direction. Unidirectional (0°) laminae are extremely strong and stiff in the 0° direction. However, they are very weak in the 90° direction because the load must be carried by the much weaker polymeric matrix. While a high-strength fiber can have a tensile strength of 500 ksi (3500 MPa) or more, a typical polymeric matrix normally has a tensile strength of only 5 to 10 ksi (35 to 70 MPa). The longitudinal tension and compression loads are carried by the fibers, while the matrix distributes the loads between the fibers in tension and stabilizes the fibers and prevents them from buckling in compression. The matrix is also the primary load carrier for interlaminar shear (i.e., shear between the layers) and transverse (90°) tension.

There can be no doubt that fibers allow us to obtain the maximum tensile strength and stiffness of a material, but there are obvious disadvantages of using a material in fiber form. Fibers alone cannot support longitudinal compressive loads and their transverse mechanical properties are generally not as good as the corresponding longitudinal properties. So they are combined with matrix materials to improve these disadvantages [15]. The matrix also serves to protect the

fibers from external damage and environmental attack. Transverse reinforcement is generally provided by orienting fibers at various angles according to the stress field in the component of interest. Filler particles are also commonly used in composites for a variety of reasons, such as weight reduction, cost reduction, flame and smoke suppression, and prevention of ultraviolet degradation due to exposure to sunlight.

1.2. Constituent materials for composites:

Composites incorporating glass or other relatively low modulus fibers (less than about 12 X 10^6 psi) are used in many high-volume applications such as automotive vehicles because of their low cost, and are sometimes referred to as "basic" composites[16]. The so-called "advanced" composites made from graphite, silicon carbide, aramid polymer, boron, or other higher modulus fibers are used mainly in more exotic applications such as aerospace structures where their higher cost can be justified based on improved performance.

2. Theory Formulation

The basic building block of a composite structure is the lamina. For the purposes of mechanics analysis, however, the "unidirection-ally reinforced," or "unidirectional" lamina with an arrangement of parallel, continuous fibers is generally used. A composite material is heterogeneous at the constituent material level, with properties changing from point to point. For example, the stress-strain relationships at a point are different for a point in the fiber material from how they are for a point in the matrix material. The properties of a composite associated with an axis passing through a point in the material generally depend on the orientation of the axis.So they are anistropic in behaviour. Each type of composite has characteristic material property symmetries that make it possible to simplify the general anisotropic stress-strain relationships. In particular, the symmetry possessed by the unidirectional lamina makes it a so-called orthotropic material. In the analysis of fiber-reinforced composite materials, the assumption of plane stress is usually used for each layer (lamina). This is mainly because fiber reinforced materials are utilized in beams, plates, cylinders, and other structural shapes which have at least one characteristic geometric dimension in an order of magnitude less than the other two dimensions.

Assumptions[18]:

- The layers are perfectly bonded
- The material of each layer is linearly elastic and has two planes of material symmetry
- The strains and displacements are small
- Deflection is wholly due to bending strains only
- Plane sections originally perpendicular to the longitudinal plane of the plate remain plane, but not necessarily perpendicular to longitudinal plane
- The transverse shearing strains (stresses) are assumed to be constant along the plate thickness

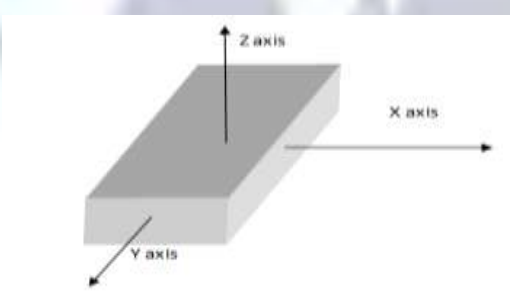
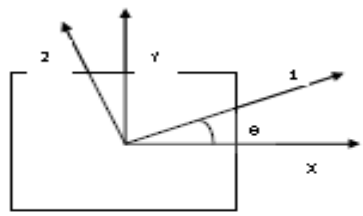


Fig 1: Schematic representation of a composite lamina.

2.1. Hooke's Law for composite laminates

Figure 1 shows a schematic representation of a composite lamina. The direction along the fiber axis is designated 1 (x axis). The direction transverse to the fiber axis but in the plane of the lamina is designated 2 (y axis). The direction transverse to both the fiber axis and the plane of the lamina is designated 3 (z axis). This direction is not shown in the figure as it only becomes necessary in three-dimensional cases.
The 1-2 co-ordinate system can be considered to be local co-ordinates based on the fiber direction. However this system is inadequate as fibers can be placed at various angles with respect to each other and the structure. Therefore a new co-ordinate system needs to be defined that takes into account the angle the fiber makes with its surroundings as shown in fig 2. This new system is referred to as global co-ordinates (x-y system) and is related to the local co-ordinates (1-2 system) by the angle θ.



1-2 : Local Coordinates
x-y : Global Coordinates

Fig 2: Global co-ordinate system in relation to local co-ordinate system.

A composite material is not isotropic and therefore its stresses and strains cannot be related by the simple Hooke's Law ($\sigma = \varepsilon E$). This law has to be extended to two-dimensions and redefined for the local and global co-ordinate systems [Fig: 3.4]. The result is Equations (1) and (2).

$$\begin{bmatrix} \sigma 1 \\ \sigma 2 \\ \tau 12 \end{bmatrix} = \begin{bmatrix} Q11 & Q12 & 0 \\ Q21 & Q22 & 0 \\ 0 & 0 & Q33 \end{bmatrix} \begin{bmatrix} \in 1 \\ \in 2 \\ \gamma 12 \end{bmatrix} \quad \text{....(1)}$$

Where,
$\sigma 1,2$ are the normal stresses in directions 1 and 2 .
$\tau 12$ is the shear stress in the 1-2 plane;
$\varepsilon 1,2$ are the normal strains in directions 1 and 2;
$\gamma 12$ is the shear strain in the 1-2 plane;
[Q] is the reduced stiffness matrix;

$$\begin{bmatrix} \sigma x \\ \sigma y \\ \tau xy \end{bmatrix} = \begin{bmatrix} Q'11 & Q'12 & Q'16 \\ Q'21 & Q'22 & Q'26 \\ Q'16 & Q'26 & Q'66 \end{bmatrix} \begin{bmatrix} \in x \\ \in y \\ \gamma xy \end{bmatrix} \quad \text{....(2)}$$

Where,
$\sigma x,y$ are the normal stresses in directions x and y;
τxy is the shear stress in the x-y plane;
$\varepsilon x,y$ are the normal strains in directions x and y;
γxy is the shear strain in the x-y plane;
[Q] is the transformed reduced stiffness matrix.

The elements of Q matrix in equation are dependent on material constants and may be calculated using equation (3).

$$Q11=\frac{E1}{1-v12v21}, \; Q12=\frac{v12E2}{1-v12v21}, \; Q11=\frac{E2}{1-v12v21}, \; Q66=G12 \quad \text{.....(3)}$$

where
E1,2 are Young's modulus in directions 1 and 2;
G12 is the shear modulus in the 1-2 plane;
v12, are Poisson's ratios in the 1-2 and 2-1 planes.

The [Q'] matrix in Equation (2) may be determined by Equation (4).

$$[Q']=[T]^{-1}[Q][R][T][R]^{-1} \quad \text{.....(4)}$$

Where
[T] is the transformation matrix;
[R] is the Reuter matrix.

These matrices are given by:

$$T=\begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \quad R=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{....(5)}$$

Here
$c = \cos\theta$ and $s=\sin\theta$

The local stresses and strains in Equation (1) are related to the global stresses and strains in Equation (2) by Equation (6).

$$\begin{bmatrix}\sigma x\\ \sigma y\\ \tau xy\end{bmatrix} = [T]^{-1}\begin{bmatrix}\sigma 1\\ \sigma 2\\ \tau 12\end{bmatrix} \text{and} \begin{bmatrix}\epsilon 1\\ \epsilon 2\\ \gamma 12\end{bmatrix} = [R][T][R]^{-1}\begin{bmatrix}\epsilon x\\ \epsilon y\\ \gamma xy\end{bmatrix} \qquad \ldots\ldots(6)$$

Equations (1) to (6) are used to determine the stresses and strains for a single composite layer. Since composites are multi-layered entities, equations for this case must also be set up. The result is equation (7).

$$\begin{bmatrix}N\\ M\end{bmatrix} = \begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{bmatrix}\epsilon^{\theta}\\ \kappa\end{bmatrix} \qquad \ldots..(7)$$

where
N is the vector of resultant forces;
M is the vector of resultant moments;
ϵ^{θ} is the vector of the mid-plane strains;
κ is the vector of mid-plane curvatures.

Vectors ε and κ are related to the global co-ordinates by Equation 7.

$$\begin{matrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{matrix} = \begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix} + z\begin{bmatrix}\kappa_x\\ \kappa_y\\ \kappa_{xy}\end{bmatrix} \qquad \ldots.(8)$$

here z is an arbitrary distance from the mid-plane.

The [A], [B], and [D] matrices in Equation (7) are known as the extensional, coupling, and bending stiffness matrices, respectively. The elements of these matrices may be determined from Equations (9) to (11).

$$A_{ij} = \sum_{k=0}^{N} [\overline{Q_{ij}}]_K (h_k - h_{k-1}) \quad i=1,2,3 \ \& \ j=1,2,3 \qquad \ldots..(9)$$

$$B_{ij} = \frac{1}{2}\sum_{k=0}^{N} [\overline{Q_{ij}}]_K (h_k^2 - h_{k-1}^2) \quad i=1,2,3 \ \& \ j=1,2,3 \qquad \ldots.(10)$$

$$D_{ij} = \frac{1}{3}\sum_{k=0}^{N} [\overline{Q_{ij}}]_K (h_k^3 - h_{k-1}^3) \quad i=1,2,3 \ \& \ j=1,2,3 \qquad \ldots(11)$$

where n is the number of layers; is the i-th, j-th element of the [Q] matrix of the k-th layer; $h_k$is the distance of the top or bottom of the k-th layer from the mid-plane of the composite. Figure 3 illustrates how to determine the distance hk from the mid-plane.

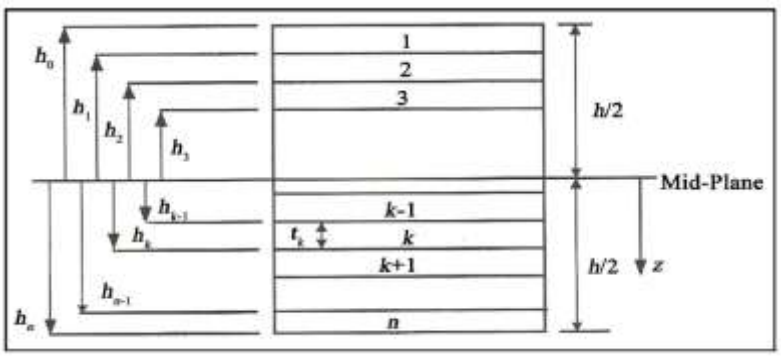


Fig: 3: Locations of layers in a composite structure[21]

2.2. Tsai-Wu Failure Theory

The Tsai-Wu failure theory is based on a total strain energy failure theory. The final aspect involved in the design of a composite structure is the failure analysis. There are various failure theories; however, the Tsai-Wu failure criterion is only one that closely correlates with experimental data. This failure theory is given by Equation (12). In this theory, failure is assumed to occur in the lamina if the following condition is satisfied:

$(F_{11}\sigma^2_1 + F_{22}\sigma^2_2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2) \leq 1$ …..(12)

Where the coefficients F_{11}, F_{22}, F_{66}, F_1, F_2, and F_{12} are given by

The parameters for the Tsai-Wu failure criterion are given by Equation (13).

$$F_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}$$
$$F_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}$$
$$F_{11} = \frac{1}{(\sigma_1^C)_{ult} * (\sigma_1^T)_{ult}} \qquad \text{…..(13)}$$
$$F_{22} = \frac{1}{(\sigma_2^C)_{ult} * (\sigma_2^T)_{ult}}$$
$$F_{66} = \frac{1}{(\tau_{12})^2_{ult}}$$
$$F_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^C)_{ult} * (\sigma_1^T)_{ult} * (\sigma_2^C)_{ult} * (\sigma_2^T)_{ult}}}$$

$\sigma_{1.2}^T$ are the ultimate tensile stresses in direction
$\sigma_{1.2}^C$ are the ultimate compressive stresses in direction 1 and 2;
τ_{12} is the ultimate shear stress in the 1-2 plane.

In order to better facilitate the use of this failure theory, each stress component of Equation (3.14) was multiplied by a variable FOS[22]. This variable is referred to as the strength ratio (SR) and combining this in Equation (12) resulted in Equation (14).

$(F_{11}\sigma^2_1 + F_{22}\sigma^2_2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2)FOS^2 + (F_1\sigma_1 + F_2\sigma_2 + F_6\tau_{12})\, FOS \leq 1.$ …….(14)

The purpose of FOS is to directly determine by what ratio the local stresses must be increased or decreased to avoid failure. This also directly relates to the applied forces. The criterion for FOS is that it can only be positive. If FOS is less than 1, then failure occurs because it means that the loading needs to decrease to avoid failure. A FOS value of 1 implies that the composite structure is perfectly suited for the applied loading conditions. A value of greater than 1 means that the structure is more than capable of carrying the applied loading and that the loading may also be increased. For example, a FOS value of 1.5 means that the loading may be increased up to 50% without failure occurring.The advantage of this theory is that there is interaction between the stress components and the theory does distinguish between the tensile and compressive strengths. A major disadvantage of this theory is that it is not simple to use.

3. Modeling

A real structure will consists of multiple laminas bonded through their thickness. Since the thickness of each lamina is of order of units in mm and it will take several laminas to resist the loads. The mechanical properties of laminates are orthotropic so they can sustain unidirectional load with effectiveness but are weaker in transverse directions .So to overcome these problems it is necessary to find out optimum orientation and no of ply, for complex loading and stiffness

3.1. Structure of Program:

Initially it was decided that this program should take the applied loading conditions, material properties and material limits as inputs. The program was to assume that the number of layers equals the number of input forces, and that the fiber angle of each layer equals the direction of each force. It would then perform all the necessary matrix calculations as in the Conventional Approach. The global and local stresses and strains would be determined as before and failure analysis with the Tsai-Wu criterion may be performed.
However, the assumptions made above cannot be used in a real situation as it may result in an overdesign of a composite structure as in the case of the conventional approach. Hence a program was written that does not make any assumptions but rather builds the laminate up one layer at a time. Basically this program varies the fiber angles to achieve a composite structure that is perfectly suited to carry the applied loading conditions. The outputs of the program are the number of layers and the fiber angle of each layer.
The program asks the user to input the following:

- material properties
- material limits
- loading conditions

```
function [Z] = inputparameters()
disp('enter the values of E1,E2,G12,NU12')
global E1
E1 = input('enter the value of E1        :');
global E2
E2 = input('enter the value of E2        :');
global G12
G12 =input('enter the value of G12       :');
global NU12
NU12 = input('enter the value of NU12    :');
Z = [E1 0 0  ;E2 0 0;  0 G12 NU12];
end
```

Fig 4: Input Parameters of material Properties for Program

```
function ply = laminateinput(  )
%UNTITLED2 Summary of this function goes here
%   Detailed explanation goes here
% NOL= no of lamina
global NOL
global ang1
global ang2
global ang3
global t1
global t2
global t3

NOL=3;
t1=.005;
t2=.005;
t3=.005;
ang1=0;
ang2=30;
ang3=-45;

ply=[NOL,t1,t2,t3,ang1,ang2,ang3];

end
```

Fig 5: Lamina parameter input

The [Q] matrix and the Tsai-Wu parameters can be calculated immediately as these do not vary with fiber angle but rather with material properties and limits. The program begins with one layer at an angle of 0° and computes h_k, as well as the [T], [Q], [A], [B] and [D] matrices. Thereafter the mid-plane strains and curvatures, and, the global and local stresses and strains are calculated. The Tsai-Wu failure theory is applied and a value for SR is obtained for each layer.

```
function ply = laminateinput(  )
%UNTITLED2 Summary of this function goes here
%   Detailed explanation goes here
% NOL= no of lamina
global NOL
global ang1
global ang2
global ang3
global t1
global t2
global t3

NOL=3;
t1=.005;
t2=.005;
t3=.005;
ang1=0;
ang2=30;
ang3=-45;

ply=[NOL,t1,t2,t3,ang1,ang2,ang3];

end
```

Fig 6: load parameters input for lamina

The program then analyzes the FOS values and confirms whether it is in a certain range. The lower limit of this range is 1 as any value below this would mean failure. The upper limit in this range is 1.2 and this is to avoid overdesigning. If the FOS value for each layer is between 1 and 1.2, then the design of the composite structure is complete. On the other hand, if the SR value for any of the layers is out of this range, the program varies that particular fiber angle to obtain a FOS value between 1 and 1.2. However, due to the angle change all the matrix calculations have to be redone. A new FOS value is obtained for each layer and this is compared to the old values. If the new FOS value for a particular layer is higher than the old FOS value, then the angle of that layer is changed to the new one. For example, after the initial calculation a layer with fiber angle of 30° has a FOS value of 0.8. This is not acceptable and the program changes the angle to obtain a better FOS value. Say at 35° the FOS value is 0.9, the program will then change the angle of the 30° layer to 35°. If at 35° the FOS value was 0.7, then the program will retain the 30° angle. The program continues to change angles and compare old and new FOS values until the FOS value of each layer is between 1 and 1.2, in which case the design is completed. However, if this range cannot be achieved, a new layer is added on. The program starts from the beginning and recalculates the various matrices. FOS values are obtained and the comparison between new and old values resumes. New layers will be added on until the FOS value for each layer is between 1 and 1.2. The program then outputs the number of layers and the fiber angle of each layer. The results, namely the number of layers and the fiber angles, needed to be verified. Therefore these were used as inputs in the Conventional Approach program. The Tsai-Wu failure theory was used and in each case there was no failure. As an example, consider the graphite/epoxy composite laminate examined earlier.

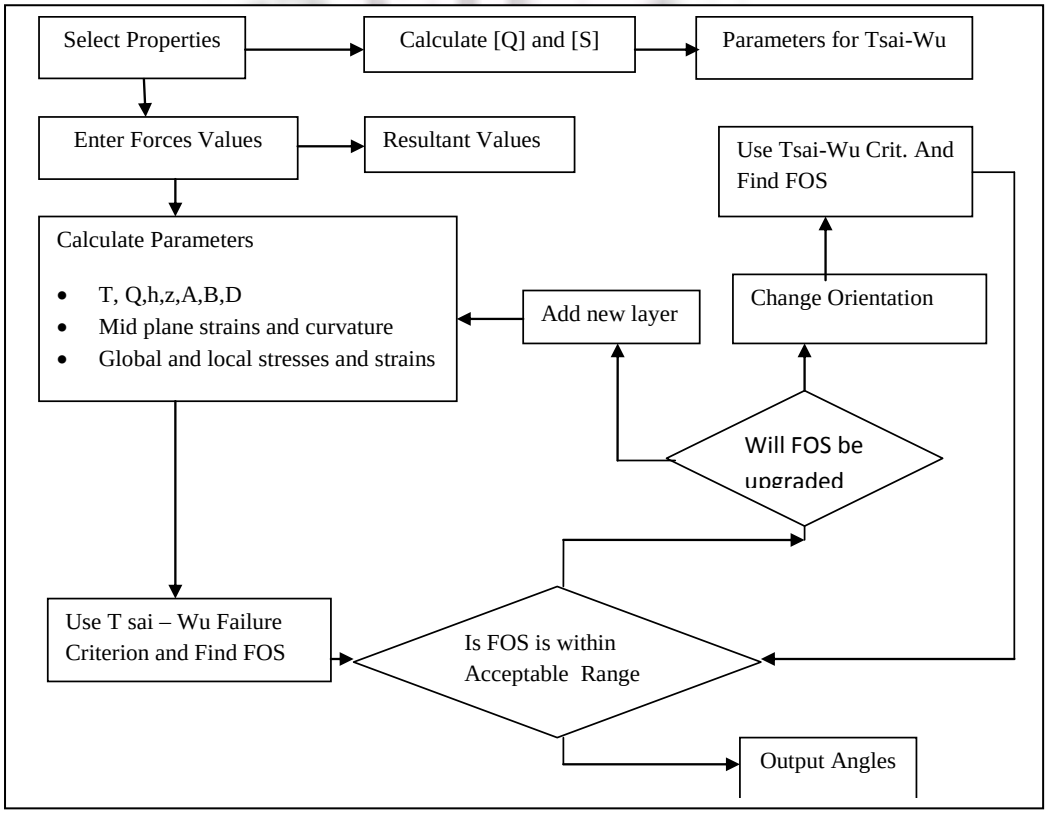


Figure 7: Flowchart describing the program

4. Results

The program was subjected to several runs. The results, namely the number of layers and the fiber angles, needed to be verified. Therefore these were used as inputs in the program. The Tsai-Wu failure theory was used and in each case there was no failure.

As an example, consider the Boron Epoxy composite laminate.

Problem:

Consider a 15-mm cube made of Boron-reinforced polymer composite material that is subjected to a tensile force of 100 kN perpendicular to the fiber direction, directed along the 2-direction. The cube is free to expand or contract. The cube is made of 3 laminas of equal thickness and ply angles for each lamina is given as,(0,30,-45).
The material properties, material limits and loading conditions for this laminate were given in Table 1.

Material Properties				Material Limits					Forces	Moments
E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	$(\sigma_1^T)_{ult}$ (MPa)	$(\sigma_1^C)_{ult}$ (MPa)	$(\sigma_2^T)_{ult}$ (MPa)	$(\sigma_2^C)_{ult}$ (MPa)	$(\tau_{12})_{ult}$ (MPa)	N	M
181	10.3	7.17	0.28	1500	1500	40	246	68	1000 1000 0	0 0 0

Table 1. Material properties, limits and loading conditions for Boron Epoxy Composite

The values in Table 1 were input into the program. The output of the program is shown in Figure 8. the FOS values are all between 1 and 1.2 indicating no failure or overdesign. The fiber angles (0, 30 ,-45) along with the material properties, material limits and loading conditions in Table 1 were then input into the program.

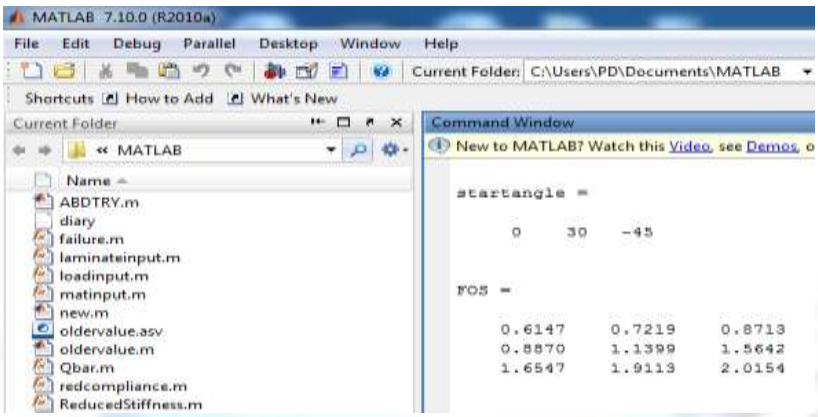


Fig 8: initial solution for given angles of FOS

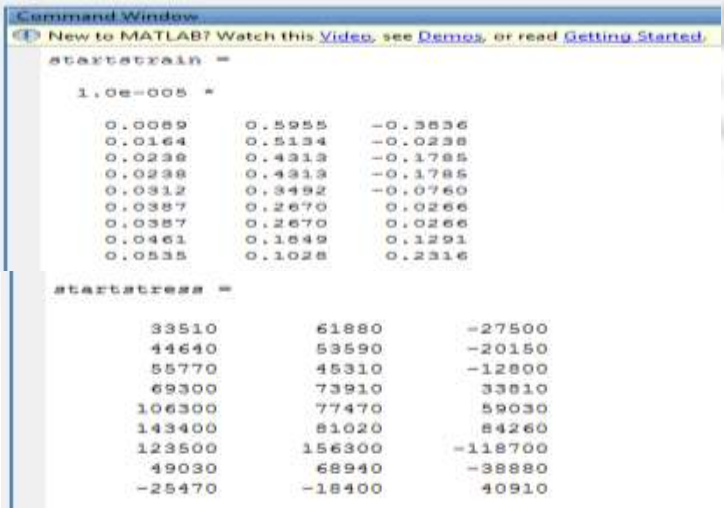


Fig 9: initial solution of stress and strains for lamina

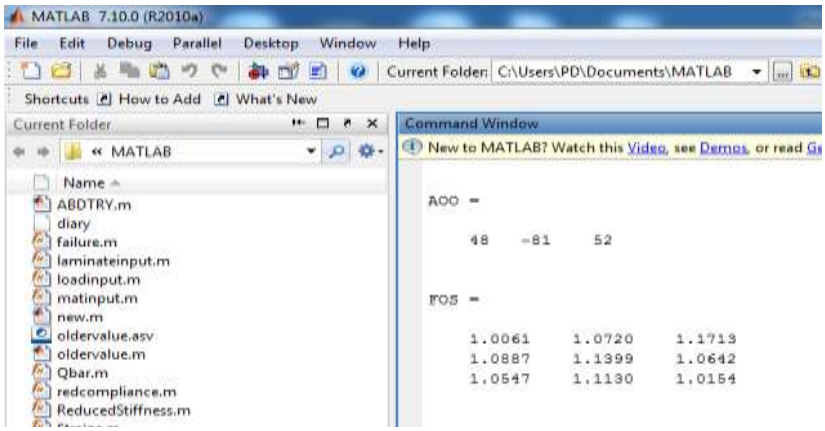


Fig 10: Improved values of angles and FOS

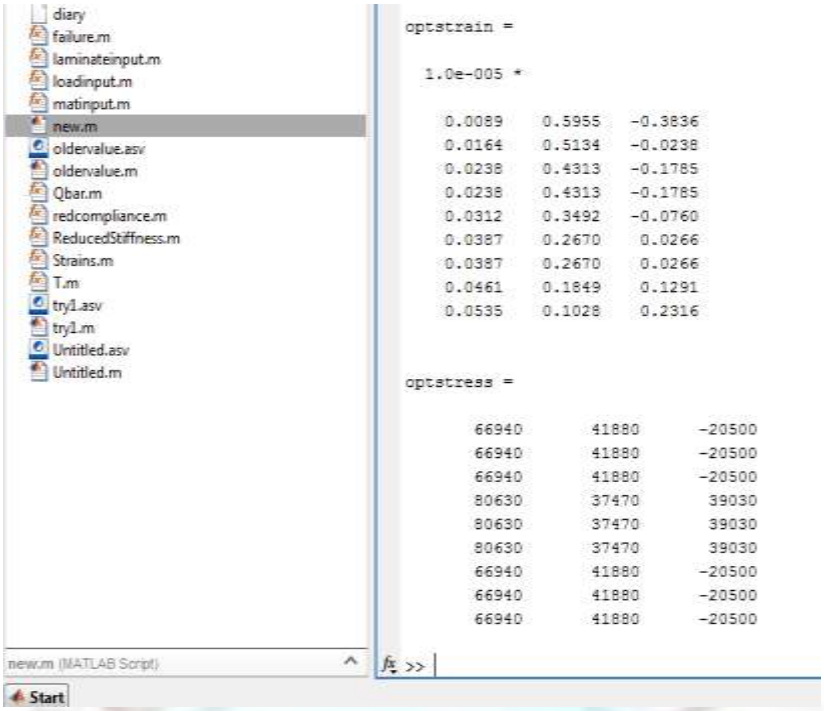


Fig 11: Final stress and strains

5. Conclusion

A MATLAB script file was generated that uses the conventional approach in the design of composite laminates. The inputs are the material properties, material limits, and thickness of each layer as well as the loading conditions. These values are then used in the governing equations, based on Hooke's law for two-dimensional unidirectional laminates, to determine the global and local stresses and strains. The local stresses are compared to allowable limits via the Tsai-Wu failure theory. The results from this program were compared to manually calculated examples in the various texts and it was found that they were comparable. the results shows that the overall stress reduced in the layers

The use of these programs will greatly reduce the design time of composite structures as the numerous computations are completed in a fraction of the time that it would take if done manually. The designer merely has to run the program, and view the number of layers required and the fiber angle for each layer. The program will also aid in reducing material costs. Presently the program is limited to flat unidirectional structures but work is underway to extend it to three dimensions. There is no limit on the type of composite material as far as the matrix and fiber reinforcement is concerned.

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