Using Adomian Decomposition Method for Solving Modified Equal Width Equation

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Abstract: In this paper, Adomian decomposition method (ADM) is applied to medium equal width (MEW) equation. The numerical results of the equation with different solitary wave, when compared with the exact solutions, show that the present method is a successfully numerical technique for solving the MEW equation.

Keywords: Adomian Decomposition Method (ADM), The Modified Equal Width (MEW) equation, Nonlinear Partial Differential Equations (NPDEs), Error Norm.

INTRODUCTION

In recent years, many phenomena in difference science such as biology, chemistry, engineer and other fields of science, are described by nonlinear partial differential equations (NPDEs).Modified Equal Width (MEW) equation, which was in introduced by Morrison et al [7].

$$u_t + 3u^2 u_x - \mu u_{xxt} = 0 \tag{1}$$

subject to the boundary conditions

$$u(a,t) = u(b,t) = 0$$
$$u_x(a,t) = u_x(b,t) = 0 \qquad , t > 0$$

and the initial condition

$$u(x,0) = f(x) \qquad , \quad a \le x \le b$$

Where t is time , x is the space coordinate , μ is a positive parameter, and $f(x)$ is a localized disturbance inside interval [a,b] [9].

Recently, various powerful methods have been applied to solve MEW equation such as homotopy perturbation method[5,10],Septic B-spline finite elements method [11], Cubic B-spline Lumped Galerkin method [2]. Adomian decomposition method was first proposed by Adomian in 1983 [2]. Javidi et al. [4] applied the Adomian decomposition method for solving the linear and nonlinear parabolic equations. [1] used Adomian decomposition method to find an analytical approximate solution for nonlinear diffusion system of Lotka-Volterea type. Mohammad et al. in [3] applied the Adomian decomposition method to solve the nonlinear wave–like equation with variable coefficient. Garg and Sharma in [6] obtained closed from solution of space –time fractional telegraph equations using Adomian decomposition method. Royawas used Adomian decomposition method to solve a system of partial differential equations [8].

The paper layout is as follows: Section 2 deals with application of ADM to MEW equation. In section 3 we present the numerical results of the MEW equation. In last section we conclude the paper.

The Adomian decomposition method to solve MEW equation

Writing (1) in an operator form

$$L_t\, u = \mu L_{xxt} u - 3u^2 u_x \tag{2}$$

Where, the differential operator L is given by

$$L = \frac{d}{dx}$$

and therefore the inverse operator L^{-1} is defined by :

$$L^{-1}(.) = \int_0^t (.)\, dt$$

applying L^{-1} to both sides of (2) we get

$$L_t^{-1}(L_t\, u) = L_t^{-1}(\mu\, L_{xxt} u - 3u^2 u_x) \tag{3}$$

and by using initial condition from the L.H.S we have:

$$L_t^{-1}(L_t\, u) = \int_0^t u_t(x,t)\,dt = u(x,t)\Big]_0^t$$
$$= u(x,t) - u(0,t)$$
$$\Rightarrow L_t^{-1}(L_t\, u) = u(x,t) - \alpha \operatorname{sec} h[k(x - x_0)] \tag{4}$$

now substituting (4) in (3) to get :

$$u(x,t) - \alpha \operatorname{sec} h[k(x - x_0)] = L_t^{-1}[\mu\, L_{xxt} u - 3u^2 u_x]$$
$$u(x,t) = \alpha \operatorname{sec} h[k(x - x_0)] + L_t^{-1}[\mu\, L_{xxt} u - 3u^2 u_x] \tag{5}$$

The Adomian decomposition method suggest that the unknown linear function u may be represent by the decomposition series,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{6}$$

substituting (6) in (5)

$$\sum_{n=0}^{\infty} u_n(x,t) = \alpha \operatorname{sec} h[k(x - x_0)] + L_t^{-1}[\mu\, L_{xxt} \sum_{n=0}^{\infty} u_n - 3\sum_{n=0}^{\infty} A_n] \tag{7}$$

from (7)

$$u_0(x,t) = \alpha \operatorname{sec} h[k(x - x_0)]$$
$$u_{k+1}(x,t) = L_t^{-1}[\mu\, L_{xxt} u_n - 3A_k]\ ,\ k \geq 0$$

Where A_n are the Adomian polynomials for the nonlinear term $F(u) = u^2 u_x$ that can be evaluated by using the following expression:

$$A_n = \frac{1}{n!}\frac{d^n}{dx^n}\left[F(\sum_{i=0}^{n} \lambda^i u_i)\right]_{\lambda=0}\ ,\ n = 0,1,2,... \tag{8}$$

The general formula (8) can be simplified assume to the nonlinear function is F(u). Therefore, by using (8) the Adomian polynomials will be:

$$A_0 = u_0^2 u_{0x}$$
$$A_1 = u_0^2\, u_{1x} + 2u_0\, u_1\, u_{0x}$$

Then we get

$$u_0(x,t) = \alpha \operatorname{sec} h[k(x - x_0)] \tag{9}$$
$$A_0 = -\alpha^3 \operatorname{sec} h[k(x - x_0)]^3 \tanh[k(x - x_0)]k$$
$$u_1(x,t) = 3\alpha^3 \operatorname{sec} h[k(x - x_0)]^3 \tanh[k(x - x_0)]k\,t \tag{10}$$
$$A_1 = 3\alpha^5 \operatorname{sec} h[k(x - x_0)]^5 \big(-2\tanh[k(x - x_0)]^2\, k^2 t$$
$$-3\, \tanh[k(x - x_0)]^2\, k^2\, t + ((1 - \tanh[k(x - x_0)]^2)k^2 t)\big)$$

and

$$u_2(x,t) = \left(27\alpha^5 \tanh[k(x - x_0)]^2\, k^2\, t^2 - \frac{9}{2}\alpha^5\, k^2\, t^2\right) \operatorname{sec} h[k(x - x_0)]^5$$
$$+\left(60\,\mu\,\alpha^3 \tanh[k(x - x_0)]^3\, k^3\, t - 33\mu\,\alpha^3 \tanh[k(x - x_0)]k^3\, t\right)\operatorname{sec} h[k(x - x_0)]^3 \tag{11}$$

Substituting relations (9-10-11) into (6) yields

$$u_{Adomian} = \alpha \operatorname{sec} h[k(x - x_0)] + 3\alpha^3 \operatorname{sec} h[k(x - x_0)]^3 \tanh[k(x - x_0)]k\,t$$
$$+\left(27\alpha^5 \tanh[k(x - x_0)]^2\, k^2\, t^2 - \frac{9}{2}\alpha^5\, k^2\, t^2\right) \operatorname{sec} h[k(x - x_0)]^5$$
$$+\left(60\,\mu\,\alpha^3 \tanh[k(x - x_0)]^3\, k^3\, t - 33\mu\,\alpha^3 \tanh[k(x - x_0)]k^3\, t\right)\operatorname{sec} h[k(x - x_0)]^3 \tag{12}$$

Which represent the approximate solution to modified equal width wave equation .

NUMERICAL RESULTS

In this paper, the boundary conditions are taken as [5]:

$u(a,t) = u(b,t) = 0$

$u_x(a,t) = u_x(b,t) = 0 \qquad , 0 \le t \le 2$

and the initial condition

$u(x,0) = \alpha \operatorname{sech}[k(x - x_0)], \qquad -30 \le x \le 30$

with the exact solution

$u(x,t) = \alpha \operatorname{sech}[k(x - x_0 - vt)]$

Which represent the motion of a single solitary wave with amplitude α , where the wave velocity $v = \alpha^2 / 2$ and $k = \sqrt{1/\mu}$.

Table 1: The exact solution and approximate solution (ADM) where $\alpha = 0.05, \mu = 1, x_0 = 30$

	Time=0.8		Time=1.6	
x	**ADM**	**Exact**	**ADM**	**Exact**
-30	0.000000000000009	0.000000000000009	0.000000000000009	0.000000000000009
-20	0.000000000206115	0.000000000205909	0.000000000206115	0.000000000205704
-10	0.000004539992965	0.000004535455243	0.000004539992962	0.000004530922055
0	0.049999100000000	0.049999975000010	0.049996400000000	0.049999900000167
10	0.000004539992969	0.000004544535231	0.000004539992971	0.000004549082039
20	0.000000000206115	0.000000000206322	0.000000000206115	0.000000000206528
30	0.000000000000009	0.000000000000009	0.000000000000009	0.000000000000009

Table 2: The absolute error for the exact solution and approximate solution (ADM) where $\mu = 1, x_0 = 30$

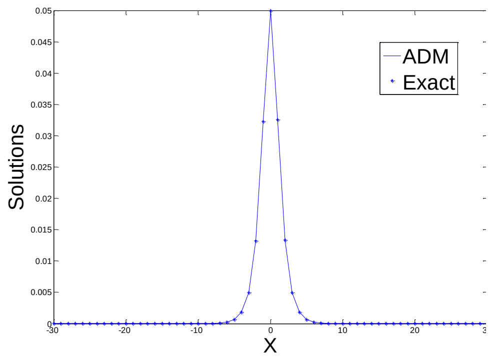
x / t	**0.0**	**0.4**	**0.8**	**1.2**	**1.6**	**2.0**
$\alpha = 0.05$						
-30	0.0	0.0004678 e-14	0.0009353 e-14	0.0014031 e-14	0.001870 e-14	0.0023360 e-14
-20	0.0	0.1030319 e-12	0.20601234 e-12	0.3089411 e-12	0.4118188 e-12	0.5146448 e-12
-10	0.0	0.2269429 e-08	0.45377225 e-08	0.6804881 e-08	0.9070908 e-08	0.1133580 e-7
0	0.0	0.2187500 e-06	0.87500001 e-06	1.9687501 e-06	3.5000002 e-06	5.4687504 e-06
10	0.0	0.2270563 e-08	0.45422615 e-08	0.6815096 e-08	0.9089068 e-08	0.1136418 e-07
20	0.0	0.1030835 e-12	0.20621845 e-12	0.3094050 e-12	0.4126432 e-12	0.5159331 e-12
30	0.0	0.0004680 e-14	0.00093620 e-14	0.001405 e-14	0.001873 e-14	0.002342 e-14
$\alpha = 0.1$						
-30	0.0	0.0037390 e-14	0.0018600 e-14	0.0014900 e-14	0.0011200 e-12	0.0007470 e-12
-20	0.0	0.8236375 e-12	0.4101764 e-12	0.3284690 e-12	0.2465979 e-10	0.1645629 e-10
-10	0.0	0.1814182 e-07	0.9034732 e-08	0.7235007 e-08	0.5431678 e-06	0.3624738 e-06
0	0.0	0.7000001 e-05	0.1750002 e-04	0.1120001 e-04	0.6300003 e-03	0.2800001 e-03
10	0.0	0.1817815 e-07	0.9125533 e-08	0.7293119 e-08	0.5464366 e-06	0.3639266 e-06
20	0.0	0.8252865 e-12	0.4142988 e-12	0.3311072 e-12	0.2480819 e-10	0.1652225 e-10
30	0.0	0.0037470 e-14	0.0018800 e-14	0.0015000 e-14	0.0011260 e-12	0.0007500 e-12
$\alpha = 0.25$						
-30	0.0	0.0000581 e-11	0.0001000 e-11	0.0002000 e-11	0.0002000 e-11	0.0003000 e-11
-20	0.0	0.1280203 e-10	0.2544500 e-10	0.3793100 e-10	0.5026200 e-10	0.6243900 e-10
-10	0.0	0.2819834 e-06	0.5604639 e-06	0.8354850 e-06	0.1107090 e-06	0.1375321 e-05
0	0.0	0.6835950 e-03	0.2734395 e-02	0.6152447 e-02	0.1093783 e-01	0.1709064 e-01
10	0.0	0.2855303 e-06	0.5746521 e-06	0.8674106 e-06	0.1163852 e-06	0.1464021 e-05
20	0.0	0.1296306 e-10	0.2608900 e-10	0.3938000 e-10	0.5283900 e-10	0.6646600 e-10
30	0.0	0.0000589 e-11	0.0001000 e-11	0.0002000 e-11	0.0002000 e-11	0.0003000 e-11

When we compute the error norm E :

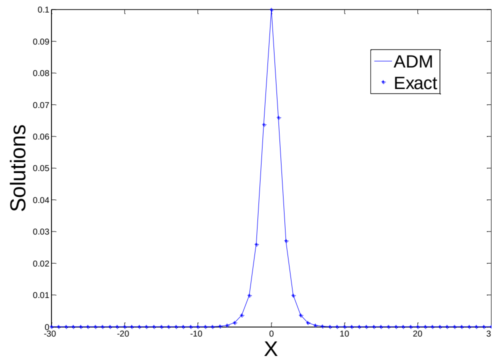
$$E = \frac{1}{N} \left\| u^{exact} - u^{ADM} \right\|_2 = \frac{1}{N} \sqrt{\sum_{j=0}^{N} \left| u_j^{exact} - u_j^{ADM} \right|^2}$$

Table 3: The error norm for the exact solution and approximate solution (ADM) where $\mu = 1, x_0 = 30$, $x = [-30,30]$

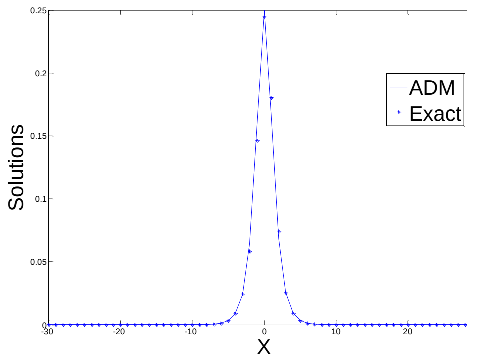
α /t	0.0	0.4	0.8	1.2	1.6	2.0
0.05	**0.0**	3.1253365 e-08	1.2500337 e-07	2.8125337 e-07	5.0000339 e-07	7.8125342 e-07
0.1	**0.0**	1.0000068 e-06	4.0000075 e-06	9.0000106 e-06	1.6000019 e-05	2.5000037 e-05
0.25	**0.0**	9.7656449 e-05	3.9062792 e-04	8.7892097 e-04	0.1562547 e-02	0.2441520 e-02



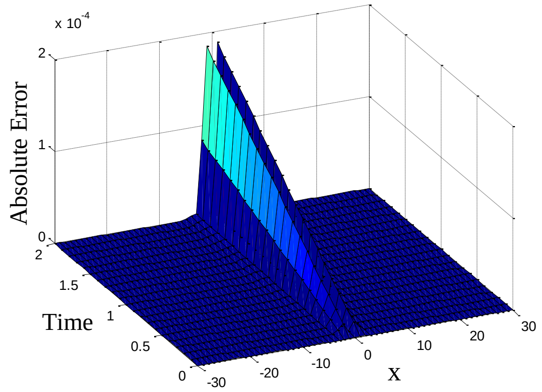
At $t = 1.1, \mu = 1, x_0 = 30, \alpha = 0.05$



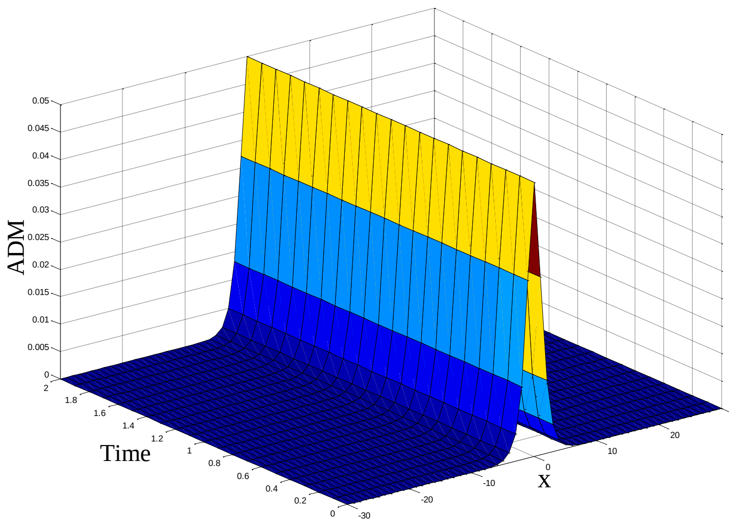
At $t = 1.1, \mu = 1, x_0 = 30, \alpha = 0.1$



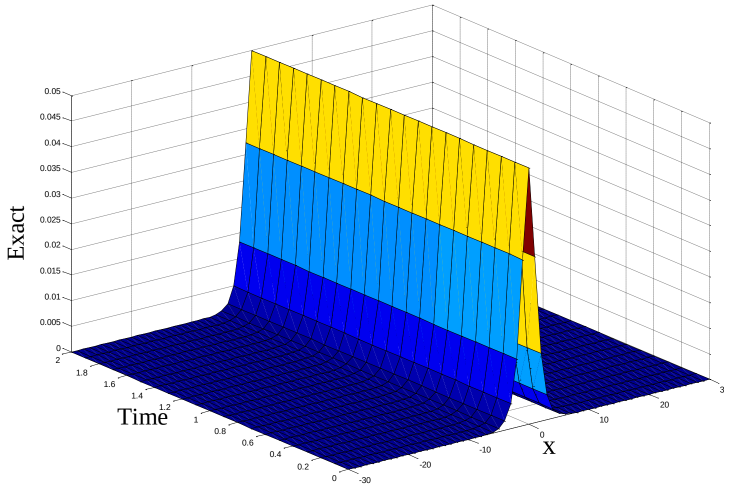
At $t = 1.1, \mu = 1, x_0 = 30, \alpha = 0.25$



Absolute Error at $\mu = 1, x_0 = 30, \alpha = 0.05$



The approximate solution (ADM)
$\mu = 1, x_0 = 30, \alpha = 0.05$



The Exact solution

$\mu = 1, x_0 = 30, \alpha = 0.05$

CONCLUSION

The ADM with Adomian Polynomials has been applied for solving Modified Equal width (MEW) equation. The results obtained by ADM applied directly without transform the nonlinear part to linear is convergence to the exact solution as shown in table 1,by using absolute error that shown in table 2 and computing the error norm in table 3 . Then the proposed method is a very efficient and simple tool to solve MEW equation . We used Matlab (R2010a) codes to obtain the solutions.

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