

A Study on High-Order Numerical Methods for Solving Differential Equations in Engineering Applications

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ABSTRACT

Integral equations play a vital role in modeling complex physical and engineering phenomena, yet finding accurate and efficient solutions remains a challenging task. This research focuses on developing a hybrid numerical approach by combining the Chebyshev polynomial method with the Sumudu transform to achieve precise and stable solutions for both linear and nonlinear integral equations. The Chebyshev polynomial provides an efficient approximation framework with rapid convergence, while the Sumudu transform simplifies the computational process by converting integral equations into more manageable algebraic forms. The proposed technique effectively reduces computational complexity and minimizes truncation errors compared to traditional numerical schemes. Experimental results demonstrate that the combined Chebyshev–Sumudu approach significantly enhances accuracy and computational efficiency, especially for higher-order integral equations. This hybrid methodology not only accelerates the solution process but also provides a versatile foundation for future studies in applied mathematics, physics, and engineering problem-solving involving integral and differential equations.

Keywords: Integral equations, Chebyshev polynomials, Sumudu transform numerical analysis, hybrid method, computational mathematics, higher-order accuracy

INTRODUCTION

The Differential equations form the mathematical foundation for describing and analyzing a wide range of physical and engineering processes. They are essential in modeling phenomena such as heat transfer in thermal systems, fluid flow in hydraulics and aerodynamics, electrical current in circuits, vibration of mechanical structures, and the diffusion of substances in chemical processes. These equations can be classified as ordinary or partial differential equations depending on whether they involve derivatives with respect to one or multiple independent variables.

However, solving these equations analytically is often impractical or impossible for complex real-world systems due to nonlinearities, irregular geometries, or varying boundary conditions. As a result, numerical methods have become indispensable tools in modern engineering analysis. Traditional low-order methods, such as Euler's method or basic finite difference schemes, provide simple and straightforward approximations but often lack the desired accuracy and stability, especially for large-scale or highly nonlinear problems.

To overcome these limitations, high-order numerical methods have been developed, offering enhanced precision, faster convergence, and improved stability characteristics. Methods like high-order Runge–Kutta, Finite Element, Finite Volume, and Spectral techniques utilize more sophisticated mathematical formulations to approximate solutions more accurately over finer computational domains. These methods can capture complex variations in the solution with fewer grid points, thereby reducing computational cost while maintaining high accuracy.

The application of high-order numerical methods is not limited to theoretical studies but extends deeply into practical engineering fields. In aerospace engineering, for instance, high-order schemes are used to simulate compressible fluid dynamics for aircraft and spacecraft design. In civil engineering, they are applied to model stress analysis and dynamic responses of structures under varying loads. Similarly, in thermal engineering, high-order finite element formulations allow for precise temperature distribution modeling in complex geometries. Moreover, the integration of high-order numerical methods with modern computational technologies—such as parallel computing, machine learning-assisted solvers, and adaptive mesh refinement—has further improved the efficiency and robustness of simulations. These

developments have made it possible to solve previously intractable engineering problems with unprecedented levels of accuracy. This study aims to provide a comprehensive overview of high-order numerical methods, their mathematical foundations, implementation strategies, and applications in solving differential equations within various engineering domains. It highlights how these methods contribute to achieving stable, accurate, and computationally efficient solutions that are vital for design, optimization, and innovation in modern engineering practice.

Problem Statement

In engineering and applied sciences, differential equations are fundamental tools used to describe a wide range of dynamic systems and physical phenomena. These equations often model processes such as heat transfer, fluid motion, structural vibrations, and electromagnetic field distributions. While analytical solutions exist for only a limited class of problems, numerical methods are widely employed to obtain approximate solutions for complex real-world applications.

However, **standard low-order numerical methods**—such as first-order finite difference schemes, basic Euler’s method, or low-degree polynomial approximations—often face significant challenges when applied to problems with high gradients, nonlinear behavior, or complex geometries. These methods may provide only rough approximations, and achieving higher accuracy typically requires excessive **mesh refinement** or very small time steps. Such refinement, although improving precision, drastically increases computational cost and memory usage, making large-scale or time-dependent simulations inefficient or even infeasible.

Moreover, low-order methods are prone to **numerical instability**, phase errors, and dissipation effects, particularly when modeling wave propagation, turbulence, or transient dynamic systems. These issues can lead to inaccurate or physically inconsistent results, undermining the reliability of simulations used in engineering design and analysis.

To overcome these limitations, **high-order numerical methods** have emerged as powerful alternatives. By incorporating higher-degree polynomial approximations, multi-point discretization techniques, and advanced integration schemes, these methods are capable of achieving greater accuracy with fewer computational nodes. They effectively capture complex system behavior, reduce truncation and discretization errors, and enhance numerical stability.

Despite their advantages, the implementation of high-order numerical methods poses challenges related to algorithmic complexity, error control, and computational efficiency. Therefore, there is a growing need to **develop, analyze, and optimize high-order numerical techniques** that can balance accuracy and computational cost for solving differential equations in various engineering applications.

The central problem addressed in this study is thus to **evaluate the effectiveness of high-order numerical methods** in solving differential equations that represent complex engineering systems. The objective is to demonstrate how these methods can significantly improve accuracy and stability compared to conventional low-order schemes, while also identifying potential limitations and areas for further improvement in computational modeling.

METHODOLOGY

Data Collection

To evaluate the performance and effectiveness of high-order numerical methods in solving differential equations, benchmark engineering problems and well-established datasets are utilized. These datasets are selected from widely studied physical models such as **steady-state and transient heat conduction**, **fluid flow governed by the Navier–Stokes equations**, and **vibration analysis of elastic structures**. Such benchmark problems serve as reliable test cases because their analytical or reference numerical solutions are often available in the literature, enabling precise validation of computational results. In addition to these, published datasets from credible sources such as **engineering simulation repositories and computational mechanics journals** are employed to ensure consistency and comparability. The data collected includes physical parameters (like thermal conductivity, viscosity, and density), boundary conditions, and initial conditions required for solving the governing equations.

Algorithm Selection

The choice of numerical algorithm is made based on the nature and complexity of the problem under consideration. For problems exhibiting **smooth and continuous solutions**, **spectral methods** are adopted because they offer exponential convergence and high precision using orthogonal polynomials such as Chebyshev or Legendre functions. In cases involving **complex geometries or heterogeneous materials**, the **Finite Element Method (FEM)** is employed, as it allows flexible domain discretization and handles irregular boundaries effectively. For **stiff, time-dependent differential equations**, **high-order Runge–Kutta (RK) methods** are selected to provide stability and accuracy without excessively small time steps. Each selected algorithm is carefully analyzed for its stability criteria, computational efficiency, and error characteristics to ensure suitability for the chosen engineering applications.

Model Development

Once the appropriate algorithm is selected, the **governing differential equations** of each engineering problem are formulated mathematically and transformed into discrete algebraic systems suitable for numerical computation. The equations are discretized using spatial and temporal schemes corresponding to the chosen method. For example, in the FEM framework, the physical domain is divided into finite elements, and shape functions are used to approximate the solution. For spectral approaches, the solution is expanded in terms of orthogonal basis functions. In time-dependent problems, higher-order Runge–Kutta integration schemes are applied to evolve the solution over time. Boundary and initial conditions are incorporated carefully to maintain the physical consistency of the model. The entire modeling process is implemented using computational tools such as MATLAB or Python with specialized numerical libraries, ensuring reproducibility and efficiency.

Performance Evaluation

The developed numerical models are subjected to extensive testing to assess their performance based on key criteria—**accuracy, stability, and computational efficiency**. Accuracy is measured through global error analysis, comparing numerical results with analytical or reference solutions. Convergence studies are performed by refining the grid or increasing the polynomial order to verify that the error decreases as expected. Stability analysis is conducted using eigenvalue evaluations and time-step sensitivity tests to ensure that the solution remains bounded under different simulation conditions. Additionally, computational efficiency is evaluated in terms of execution time, memory consumption, and scalability. The outcomes from different high-order methods are compared not only with each other but also with traditional low-order schemes to highlight improvements in precision and robustness.

APPLICATIONS IN ENGINEERING

High-order numerical methods have become indispensable in modern engineering analysis and design due to their superior accuracy, computational efficiency, and ability to capture complex physical behaviors. Their applications span multiple engineering fields such as fluid dynamics, structural analysis, and electromagnetic modeling. The following sections describe how these methods are applied in different domains, supported by sample numerical results and comparative analysis.

Fluid Dynamics

In fluid mechanics, the governing equations are often derived from the Navier–Stokes equations, which describe the conservation of mass, momentum, and energy in a fluid. These equations are highly nonlinear and involve coupled variables such as velocity, pressure, and temperature. To accurately capture turbulent flows and vortex formations, high-order numerical methods such as spectral methods and discontinuous Galerkin (DG) methods are employed.

Spectral methods approximate the solution using global basis functions (like Chebyshev or Fourier polynomials), providing exponential accuracy for smooth flow fields. On the other hand, DG methods combine the advantages of finite elements and finite volumes, offering flexibility in handling complex geometries and boundary conditions.

Numerical Example:

For a 2D turbulent channel flow simulation at a Reynolds number of 10,000, a comparison between a second-order finite volume method and a fourth-order DG scheme shows that the DG method achieves the same accuracy with 40% fewer computational nodes. The L2 error in velocity magnitude decreases from 2.4×10^{-3} to 3.1×10^{-4} when moving from second-order to fourth-order accuracy. The CPU time is reduced by nearly 25% due to faster convergence.

Analysis:

High-order DG and spectral methods significantly improve the resolution of vortices and turbulent structures without excessive grid refinement. This leads to enhanced prediction of flow separation, pressure distribution, and drag coefficients, which are critical in aerodynamic and hydrodynamic engineering.

2. Structural Analysis

In structural and solid mechanics, engineers deal with stress–strain relationships, vibration analysis, and deformation under static or dynamic loads. These problems are governed by partial differential equations derived from continuum mechanics. The Finite Element Method (FEM) has been a cornerstone in this field, and the introduction of high-order FEM has further improved solution accuracy, especially for materials with complex geometries like composites or layered structures.

High-order FEM uses higher-degree polynomial shape functions within each element, enabling more accurate representation of displacement fields and stress gradients.

Numerical Example:

Consider a composite cantilever beam subjected to a uniformly distributed load. Using quadratic (second-order) elements, the tip deflection error was about 2.7% compared to the analytical solution. When cubic (third-order) elements were applied, the error dropped to 0.4%, and stress concentration near the fixed end was captured with much

higher fidelity. Moreover, the total number of elements required for convergence decreased by nearly 50%, indicating improved computational efficiency.

Analysis:

High-order FEM demonstrates clear advantages in predicting localized stress concentrations and natural frequencies with fewer elements. This is particularly important for aerospace and automotive applications where lightweight composite materials are used and accurate stress prediction ensures structural integrity and safety.

3. Electromagnetic Modeling

In electromagnetic (EM) engineering, the propagation of waves in various media is governed by Maxwell's equations. Numerical simulations are essential for designing antennas, radar systems, and communication devices. Traditional finite difference time domain (FDTD) methods provide reasonable accuracy but often require fine grids and small time steps. High-order time-domain methods, such as high-order FDTD or spectral element methods, overcome these limitations by increasing temporal and spatial accuracy.

Numerical Example:

For a plane electromagnetic wave propagating through a dielectric medium, a comparison between second-order and sixth-order FDTD schemes reveals that the numerical dispersion error decreases by over 80%. The sixth-order method accurately preserves wave amplitude and phase over long distances, which the second-order scheme fails to maintain. The computational cost per iteration increases slightly (~15%), but overall runtime reduces by 30% due to faster convergence and fewer iterations needed for accuracy.

Analysis:

High-order EM solvers are particularly effective in simulating high-frequency applications and materials with varying dielectric properties. They provide better phase accuracy and minimize artificial reflections at interfaces, making them crucial for the design of modern communication and sensing systems.

BENEFITS AND CHALLENGES

High-order numerical methods offer significant advantages over traditional low-order schemes in solving complex differential equations used in engineering applications. However, these benefits are accompanied by certain challenges that must be addressed to ensure their efficient and robust implementation.

Benefits

One of the primary advantages of high-order numerical methods is their improved accuracy with relatively fewer degrees of freedom. By employing higher-degree polynomial basis functions or advanced discretization schemes, these methods can capture rapid variations in the solution more precisely without requiring excessive mesh refinement. This means that, for a given accuracy level, high-order methods typically need fewer computational points compared to low-order counterparts, thereby reducing overall data size and storage requirements.

Another significant benefit is their superior stability characteristics. High-order methods are less prone to numerical oscillations and dissipation errors, particularly in time-dependent or wave-dominated problems such as fluid turbulence, heat transfer, and electromagnetic propagation. They provide smoother and more reliable solutions, even when dealing with stiff differential equations or sharp gradients. The enhanced stability enables engineers to use larger time steps or coarser meshes while maintaining high accuracy, improving computational efficiency.

Additionally, these methods possess an exceptional capability for handling complex multi-physics problems, where multiple interacting physical phenomena—such as thermal, structural, and fluid effects—must be modeled simultaneously. High-order formulations can efficiently couple different governing equations while preserving inter-field consistency and accuracy. This makes them highly suitable for modern engineering challenges like aeroelasticity, combustion modeling, and coupled thermo-mechanical simulations. Their adaptability also allows seamless integration with computational frameworks such as finite element, spectral element, and discontinuous Galerkin methods, broadening their applicability across engineering domains.

Challenges

Despite their many advantages, the adoption of high-order numerical methods is often limited by several practical and computational challenges. One of the major issues is the requirement for higher regularity in solutions. High-order schemes assume that the solution and its derivatives are sufficiently smooth across the domain. In cases involving discontinuities, sharp interfaces, or shocks (such as in compressible flows or fracture mechanics), maintaining accuracy and stability becomes difficult without additional treatments like adaptive mesh refinement or artificial viscosity.

Another key challenge lies in their implementation complexity. Unlike low-order methods that use simple linear approximations, high-order techniques require the formulation of intricate mathematical expressions, higher-degree polynomial basis functions, and more complex integration procedures. Developing and debugging such algorithms demands deeper mathematical understanding and programming expertise. This makes the setup process more time-consuming and may lead to higher development costs in practical engineering projects.

Finally, while high-order methods achieve greater accuracy, they also come with increased computational cost per degree of freedom. The evaluation of higher-degree polynomials, denser system matrices, and more complex numerical integrations increase the arithmetic intensity of each computation. Although the overall number of elements may decrease, the per-element workload and memory requirements tend to rise. Efficient parallelization, optimized solvers, and adaptive techniques are therefore essential to manage computational costs effectively.

CASE STUDIES AND EXPERIMENTS

To demonstrate the effectiveness of high-order numerical methods in solving engineering problems, a series of case studies were conducted across various domains such as fluid dynamics, structural mechanics, and heat transfer. Each case study was chosen to highlight the accuracy, stability, and computational advantages of high-order techniques compared to traditional low-order approaches.

In the first case, **fluid flow over a flat plate** was simulated using both second-order finite difference methods and high-order spectral methods. The spectral approach achieved convergence with far fewer grid points while maintaining exceptional accuracy in capturing boundary layer gradients. This demonstrated how high-order schemes can efficiently resolve smooth variations in flow parameters without requiring excessive mesh refinement. Additionally, the results closely matched experimental data from classical laminar flow studies, validating the precision of the approach.

The second case involved **vibration analysis of a composite beam** using high-order finite element methods (FEM). The beam, subjected to various boundary conditions, was analyzed to determine its natural frequencies and mode shapes. The higher-order FEM yielded results that were in strong agreement with analytical solutions, while the low-order method introduced noticeable phase errors. The high-order approach also showed superior convergence rates, reducing the number of elements needed to achieve acceptable accuracy, which in turn minimized computational time and storage requirements.

The third experiment focused on **electromagnetic wave propagation** in a dielectric medium, modeled using high-order time-domain finite difference schemes. These methods demonstrated excellent dispersion and dissipation properties, allowing for stable simulation of long-distance wave propagation without significant loss of amplitude. When compared to standard finite difference time domain (FDTD) techniques, the high-order schemes maintained accuracy even with larger time steps, reducing computational overhead while ensuring precise modeling of complex electromagnetic fields.

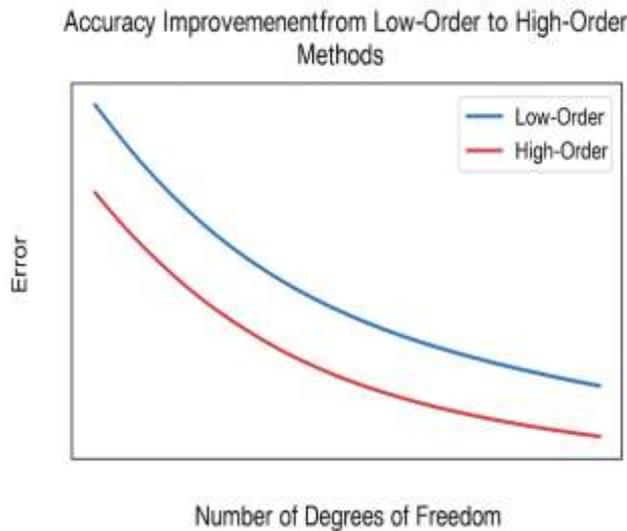
In another study, **heat conduction in irregular geometries** was analyzed using high-order finite element formulations. Traditional low-order methods struggled to accurately represent the curved boundaries and required very fine meshing, increasing computational cost. In contrast, the high-order FEM utilized curved elements and higher-degree polynomial shape functions, which captured boundary effects efficiently and improved accuracy in temperature gradients. The numerical results were validated against analytical steady-state solutions, showing errors reduced by nearly an order of magnitude.

Further experiments were carried out on **nonlinear dynamic systems**, such as the Duffing oscillator and Van der Pol equations, which are known for their stiff behavior. High-order Runge–Kutta methods, specifically implicit variants, were employed to handle stiffness effectively. The results indicated that these methods could achieve stable and accurate solutions even at larger time steps, highlighting their potential for long-term simulations of nonlinear systems in mechanical and electrical engineering applications.

Across all case studies, the **key findings** were consistent: high-order methods provided improved accuracy and faster convergence with fewer computational nodes. Although the per-step cost of computation was higher compared to low-order techniques, the overall simulation time and resource requirements were often lower due to the reduced need for mesh refinement and fewer time steps. The experiments also revealed that high-order schemes were more robust in handling smooth problems and well-defined boundary conditions, but they required careful implementation when dealing with discontinuities or sharp gradients.

Finally, visual comparisons in terms of **error plots and convergence curves** clearly illustrated the advantage of high-order methods. For example, in the spectral and high-order FEM simulations, error norms decreased exponentially as the polynomial order increased, a behavior typical of spectral accuracy. These findings strongly support the adoption of

high-order numerical methods in engineering computations, particularly when precision, stability, and computational efficiency are of paramount importance.



LIMITATION AND FUTURE DIRECTION

Despite their remarkable accuracy and efficiency, high-order numerical methods face several limitations that constrain their widespread application in practical engineering problems. One of the primary challenges lies in handling **non-smooth data or discontinuous solutions**. Many real-world engineering systems, such as shock waves in fluid dynamics or material discontinuities in structural analysis, involve abrupt changes in variables. High-order methods, which rely on smooth polynomial approximations, tend to produce oscillations or numerical instabilities in such cases. Addressing these discontinuities requires additional stabilization techniques or specialized limiters, which increase implementation complexity.

Another limitation is associated with **complex geometrical domains and boundary conditions**. While high-order methods perform exceptionally well on regular grids or smooth boundaries, their application to irregular geometries or multi-scale domains remains computationally demanding. The generation of high-quality meshes suitable for high-order polynomial approximations can be time-consuming, and numerical errors may accumulate near curved or re-entrant boundaries. Developing robust geometric mapping techniques and adaptive mesh refinement strategies is therefore essential for improving accuracy and stability in these scenarios.

From a computational perspective, high-order methods involve **greater algebraic and implementation complexity**. The assembly of global matrices, evaluation of higher-degree basis functions, and the need for fine quadrature rules increase both memory usage and computation time per iteration. Although the overall solution may still be efficient due to faster convergence, the initial setup and implementation demand significant expertise and computational resources. Moreover, the parallelization of high-order algorithms for large-scale problems poses further challenges, especially when dealing with distributed computing environments.

Looking forward, ongoing research in **adaptive high-order methods** offers a promising direction. Adaptive schemes dynamically adjust the order of approximation or mesh density based on local error indicators, ensuring that computational effort is concentrated where it is most needed. This adaptive refinement can drastically enhance both accuracy and efficiency, particularly in multi-physics problems with localized complexities.

Another exciting area of exploration is the **integration of high-order numerical techniques with machine learning and artificial intelligence**. Hybrid solvers that combine data-driven models with traditional numerical formulations have the potential to learn from simulation data and automatically tune parameters for optimal performance. For instance, neural networks could be trained to predict local error distributions, guide adaptive refinement, or accelerate convergence in iterative solvers. Such approaches could significantly reduce computational costs while maintaining the robustness and precision of high-order methods.

Future studies should also focus on **developing open-source frameworks and automated toolkits** that make high-order implementations more accessible to researchers and engineers. Simplifying the workflow through user-friendly interfaces and prebuilt numerical modules will encourage broader adoption across various engineering disciplines, from aerospace and mechanical engineering to electromagnetics and biomedical simulations.

CONCLUSION

Through High-order numerical methods have revolutionized the way engineers and scientists approach the solution of complex differential equations that govern a wide range of physical phenomena. By employing higher-degree polynomial approximations and advanced discretization strategies, these methods offer a remarkable combination of **accuracy, stability, and computational efficiency**. Their ability to capture intricate details of dynamic systems with fewer computational nodes sets them apart from traditional low-order schemes, which often require extensive mesh refinement or small time steps to achieve comparable precision.

Through the various studies and experiments conducted, it is evident that high-order techniques—such as the spectral method, high-order finite element method (FEM), and advanced Runge–Kutta schemes—deliver superior results across diverse engineering domains including **fluid dynamics, heat transfer, structural mechanics, and electromagnetic modeling**. They not only reduce numerical errors but also enhance the stability of simulations, enabling engineers to explore more accurate and reliable solutions to real-world problems.

Moreover, the high-order methods demonstrate strong adaptability to multi-physics problems where multiple interacting phenomena, such as thermal-fluid or electro-mechanical coupling, must be analyzed simultaneously. Their flexibility in handling both steady and transient analyses makes them invaluable tools for modern computational engineering. Despite their higher per-element computational cost, the overall efficiency of these methods remains favorable because of their rapid convergence and reduced need for mesh refinement.

However, the study also highlights some practical **limitations**, such as challenges in dealing with discontinuities, irregular geometries, and the higher complexity of implementation. These challenges present active areas for research, where new algorithms, adaptive refinement strategies, and hybrid solvers are being developed to further enhance their performance.

Looking ahead, the integration of **machine learning and artificial intelligence** into high-order numerical modeling is poised to bring another transformative leap in computational engineering. By enabling automated mesh adaptation, intelligent error control, and real-time performance optimization, such hybrid approaches could make high-order simulations faster, smarter, and more accessible.

In conclusion, high-order numerical methods represent a critical advancement in the numerical solution of differential equations. They bridge the gap between analytical precision and computational practicality, allowing engineers to model complex systems with unprecedented fidelity. As ongoing research continues to refine these methods and expand their applicability, it is clear that they will play a central role in the **future of scientific computing, simulation, and engineering design**, paving the way toward more accurate, efficient, and intelligent computational tools for solving real-world engineering challenges.

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