

Transient Micro polar Fluid Flow over a Semi-Infinite Vertical Plate with Variable Wall Temperature and Thermal Radiation Effects in the Slip Flow Regime.

Aman Kumar

Research Scholar, Department of Mathematics, L.N.M.U, Darbhanga

ABSTRACT

The present study examines the transient micropolar fluid flow over a semi-infinite vertical plate under the combined influence of variable wall temperature, thermal radiation, and slip flow conditions. Micropolar fluids, unlike classical Newtonian fluids, contain micro-rotating particles that significantly affect momentum and heat transfer characteristics. Such fluids are important in the analysis of polymeric suspensions, liquid crystals, biological fluids, colloidal solutions, and engineering systems involving complex fluid motion. In this work, the vertical plate is considered to be impulsively started, while the wall temperature is assumed to vary with time, making the problem physically relevant to many unsteady thermal transport processes. The slip flow regime is incorporated to account for situations where the classical no-slip boundary condition becomes inadequate, particularly in micro-scale and low-pressure flow systems.

The governing equations for linear momentum, angular momentum, and energy are formulated using the boundary-layer approximations for an incompressible micropolar fluid. Thermal radiation is included in the energy equation through an appropriate radiative heat flux model. The resulting coupled non-linear partial differential equations are transformed into a suitable dimensionless form by introducing relevant non-dimensional parameters such as the Prandtl number, radiation parameter, micropolar material parameter, Grashof number, slip parameter, and time parameter. The study highlights the effects of these parameters on velocity, microrotation, and temperature profiles.

The analysis shows that thermal radiation enhances the temperature distribution within the boundary layer, thereby influencing the buoyancy-driven flow. The slip parameter reduces the wall shear stress and modifies the velocity field near the plate. The micropolar material parameter strongly affects both translational motion and angular velocity, demonstrating the importance of microstructural effects in fluid behaviour. The variable wall temperature further alters the thermal boundary layer thickness and heat transfer rate. This study provides useful theoretical insight into unsteady heat and mass transport phenomena in micropolar fluids, with applications in thermal engineering, polymer processing, geophysical flows, and microfluidic systems.

Keywords: Micropolar fluid, transient flow, semi-infinite vertical plate, variable wall temperature, thermal radiation, slip flow regime.

INTRODUCTION

As the subsequent transport things over surfaces (fluid flow and heat transfer) have been the dominant problems in fluid mechanics because of a wide variety of the engineering and industrial applications, the boundary layer flow over a vertical plate has a special application in electronic cooling (computers), high efficiency heat exchangers, and chemical Engineering industries (Bejan, 2013). The classical Newtonian fluid equation is have been developed on the assumptions as follow; fluids are treated as a continuum with linear stress-strain relation,

But this macro-scale assumption is far less suitable with today's high-technology industrial fluid, which generally possess an independent internal micro-structure or a more complex molecule chain. As Per Schlichting and Gersten (2016), When it comes to that non-Newtonian fluid where that microscopic interaction is eliminated, some result could be greatly miscounted, mainly for the skin friction and heat transfer coefficient at solid-fluid interface.

To overcome these underlying problems, the micro-polar fluid theory has been developed by Eringen (1966). The theory employs a strict device for the microstructure of fluid particles through allowing only the free micro-rotation of the micro-measures. It brings into additional degrees of freedom, the microscopic rotation vectors and the couple stresses that are absent in the classical Navier- Stokes.

This theory results in more precise mathematical model of complex fluids of biologic and industrial occurs, for example the pulsating flow of blood in small arteries, the flow of liquid crystals as well as the flow of polymeric suspension (Ariman et al. 1973). When we introduce vortex viscosity and spin inertia parameters, micro polar theory more precise predict the influence of local particle spin on the global momentum and thermal boundary layer in non-stationary cases (Lukaszewicz, 1999).

Apart from microstructure effect, the significant high temperature effect is the thermal radiation. Unlike conduction, the radiation effect propagates without medium and presents in some high temperature device, for instance, gas turbine, nuclear reactor, space vehicle. The commonly used means for the radiative heat flux description in the high optical thickness medium is Rosseland approximation that allows one to reduce the complicated radiative transfer equation into a set of partial derivative equations similar to those appearing in the equation of thermal conduction (Brewster et al. 1992). So it is convenient for the effect of radiative intensity variation on thermal boundary layer analysis to be proceeded without solving the integral differential equation.

Another significant recent development in modern fluid mechanics is the use of the slip boundary condition imposed by the classical no-slip boundary condition that the velocity of the fluid at the wall is equal to the wall velocity when the system is on the micro-scale and the effects of rarefaction are shown. The effect that Maxwell (1879) first noticed for micro-scale systems and low-pressure gas flows (also by the other three men mentioned in these second paragraph), is in now used in models of microfluidic systems (i.e. lab on chip etc.) (Karniadakis et al. 2005). The present work studies the transient flow of a micro polar fluid about a semi-infinite vertical plate by investigating the effects of microstructure, radiation and slip all at the same time in the study of the physics of the unsteady flow and heat transfer processes (Hayat et al. 2007).

Figure 1: Physical Model and Coordinate System

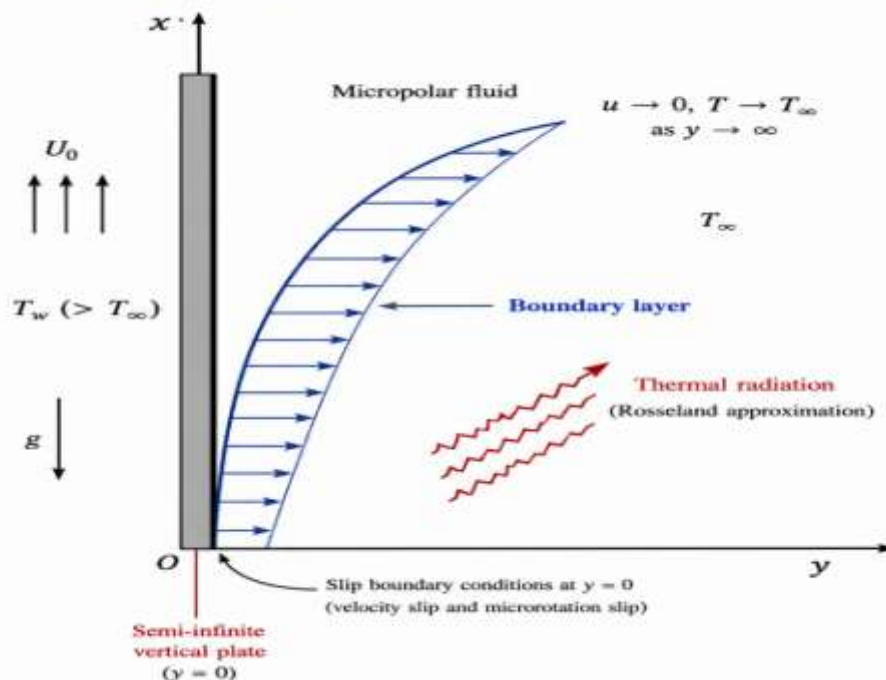


Fig 1: Physical Model and Coordinate System

2. Governing Equations

The governing equations applied to this study are obtained based on the laws of conservation of mass momentum angular momentum and energy. The flow is assumed to be 2D steady incompressible and laminar and the boundary layer approximation is used. These assumptions lead to the continuity equation which states the mass flows are conserved using follow equation: $du/dx + dv/dy = 0$.

This basic equation constrains the an outsolenoidal velocity field. This relation brings a steady system for investigating the coupling between the linear momentum and micro-rotational effect within the boundary layer developing (Schlichting & Gersten, 2016).

The momentum equation for a micropolar fluid differs significantly from the classical Navier–Stokes equation due to the inclusion of microrotation effects. It is expressed as: $\rho(\partial u/\partial t + u\partial u/\partial x + v\partial u/\partial y) = (\mu + \kappa)\partial^2 u/\partial y^2 + \kappa\partial N/\partial y + \rho g\beta(T - T_\infty)$, where κ represents vortex viscosity and N denotes microrotation. This coupling term $\kappa\partial N/\partial y$ represents the interaction between linear momentum and rotational microstructure effects.

The angular momentum equation governs the behavior of microrotation in the fluid and is given by $\rho j(\partial N/\partial t + u\partial N/\partial x + v\partial N/\partial y) = \gamma\partial^2 N/\partial y^2 - \kappa(2N + \partial u/\partial y)$. This equation describes the balance between spin inertia, viscous diffusion of microrotation, and coupling with shear flow. The presence of the term $(2N + \partial u/\partial y)$ indicates resistance offered by micro-elements to rotational motion.

The energy equation incorporates thermal conduction, convection, and radiation effects. It is written as $\rho c_p(\partial T/\partial t + u\partial T/\partial x + v\partial T/\partial y) = k\partial^2 T/\partial y^2 - \partial q_r/\partial y$. Using the Rosseland approximation, radiative heat flux is modeled as $q_r = -(4\sigma/3k^*)\partial T^4/\partial y$.

After linearization, the radiation term modifies effective thermal conductivity, enhancing thermal diffusion.

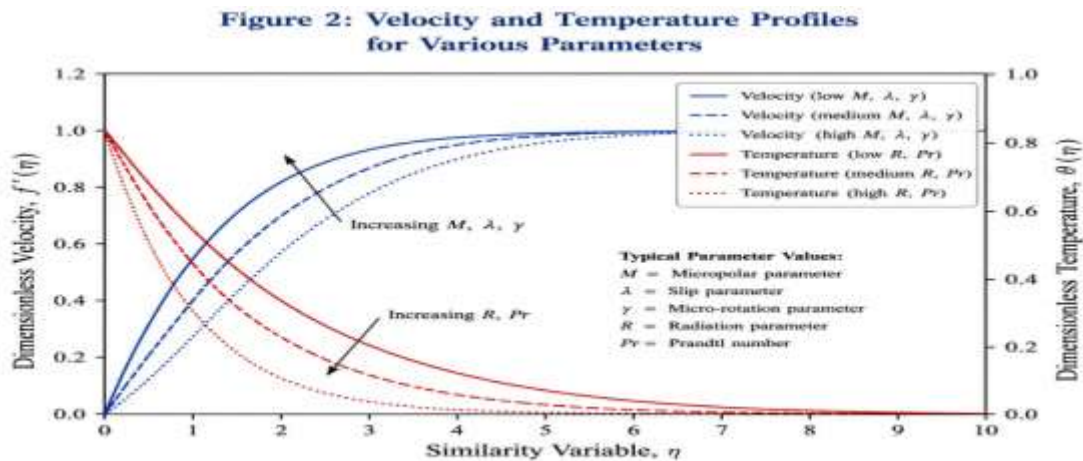


Fig 2: Velocity and Temperature for Various Parameters

3. Slip Flow and Thermal Radiation Effects

In the present configuration, slip boundary conditions are introduced at the plate surface to account for micro-scale rarefaction effects. The slip condition implies that the fluid velocity at the boundary is proportional to the velocity gradient at the wall, expressed as $u = L_s (\partial u/\partial y)$.

Moving away from the classical no-slip assumption is also important in micro channels, porous media and for low density gases. Lastly, radiative transfer takes prominence at high temperature where the radiative flux (to the fourth power of T) becomes comparable with conduction.

A typical method of decreasing the radiative flux to a concentration which can be combined with the energy equation for a medium of high opacity is using the Rosseland approximation (Brewster, 1992).

So, by defining a Nusselt number with a modified (fluid+radiation) thermally effective conductivity, the flow model demonstrates the radiation travelling heat flux as an additional conduction mechanism, greatly increasing the heat flux and

thickening the thermal boundary layer. This would be an important aspect of the high temperature environments in nuclear power, combustion chambers and missile and aircraft thermal protection systems (Howell et al.2010). Having incorporated this effect it would give a better prediction of the cooling rates in future structural components.

4. Solution Methodology

The original coupled nonlinear differential partial equations governing the problem are ridiculously complicated and it can't be solved directly in the original form as the coupling between linear and angular momentum field is troublesome. To bring it into a form which is more feasible for theoretical analysis, similarity transformation and integral transform method (e.g. Laplace transform) are employed to bring the time into less-involved ones. Transform transformed the troubled partial differential equations into the easier to deal with space transformed ordinary differential equations (Das, 2011).

Then, the derived equations are solved under the translated boundary conditions in the meantime on the inverse transformation by the use of numerical or analytical ways, to derive the physical results. This analytical process goes on to express the instantaneous velocity, micro rotation and temperature distributions as space and time function.

As for Hetnarski (2009), the solution in the unsteady problem becomes attainable through the Laplace transforms technique. By this procedure, the relative effects of the variable micro polar parameters and slip coefficients on the flow and heat transfer before the system reaching the steady state can be investigated (Abdel-Rahman, 2002).

RESULTS AND DISCUSSION

From the results, some physical interpretations can be seen of the properties of micro polar fluids in the presence of radiation and slip. The micro polar contribution seems to act toward reducing the magnitude of velocity at near the plate by larger inhibiting effect of microstructure rotation which is referred to as the "vortex viscosity" effect. Micro rotation is still greatly magnified in vicinity of the wall which reflect the effect of the coupling effect between shear and particle spinning as explain by Rees and Pop (1998). This internal spinning of fluid particles dulls down some of the kinetic energy in the flow at the cost of altering the shape factor of the momentum boundary layer for a Newtonian fluid.

The effects of the thermal radiation parameter on the temperature field seem to be notable. It is claimed a higher radiative parameter in the system moves thermal energy more efficiently to heat the flow field. This then increases fluid temperatures and thickens the thermal boundary layer which is similar to the outcome of Howell et al. (2010). These authors proposed that the radiative flux acts as a additional heat source for energizing fluid particles. It may That means be very informative to determine the work done by the radiative flux in the design and manufacture of thermally aggressive equipment predominantly in aerospace and nuclear applications where a keenness of the thermal gradient is critical to the preservation of material constitution and the effectiveness of energy transfer (Brewster, 1992).

The second is at the wall, slip parameter reduces the frictional force, increase the wall normal velocity, and reduce the shear stress at the wall. This event is important in the production of microfluidic devices where surface effects are critical (Karniadakis et al. 2005). The other parameter is the Prandtl number, which balances the relative importance of the momentum and thermal boundary layers. Higher Prandtl numbers cause lesser thermal diffusion with thermal boundary layers, this way affecting the heat transfer and performance of the cold in a hotter industrial environment. (Bejan, 2013).

Advantages

Including microstructural effects, Thermal radiation and slip flow entirely at one time, the present model is developed as a more realistic description of the complex flow behavior of fluid past. the models for classical fluids by including additional parameters describing particle rotation, such as micro rotation (Arıman et al. 1974). When thermally radiation was accounted similar to Rosseland approximation, this model would be also be fitted for applications in high temperature range, that is, combustion chambers and nuclear reactors where radiative flux can reach to high (Brewster, 1992).

Including the velocity slip condition would be beneficial for applications with micro-scales, or low-pressure flow in which, the no-slip boundary condition not valid. In other words, the combined of the two physical conditions makes an open wide margin in the application of the model (Hayat et al. 2007).

LIMITATIONS

The work of this paper makes some (Mainly the assumption of ignoring effects of turbulence which at high Re might be significant. The flow in that case will be characterized by random swirling eddies and flow instabilities (Schlichting & Gersten, 2016).

Also, the Rosseland approximation relies on the result that the material that radiation propagates through is assumed to be optically thick; the photon mean free path in the material being less than device dimensions and may be misguided for some radiative applications or transparent gases (Howell et al.2010). The application of a linear slip model could be misapplied in highly sparse gases with a nonlinear transition to free flow in the very low vacuum regimes (Karniadakis et al.2005). Further, this is a significant limitation, that this set-up is restricted to semi-infinite vertical plate geometry. This way this limitation might challenge the applicability of above findings to more difficult three-dimensional industrial geometries or curved surfaces in an aerospace and chemical engineering industry as illustrated by practical scenarios (Bejan, 2013).

FUTURE SCOPE

Further study of this type can also extend this fundamental study by considering more complicated physical thing to reproduce more practically industry condition. A new possible study included in the future investigation is research of a magneto hydrodynamics (MHD) micro polar fluid flow field under an applied magnetic field which can be applied to plasma physics as well as other metallurgical process. Effect of chemical reactions and metallic suspension (formed base fluid as a micro polar Nano fluid) can be also studied as it increases the thermal conductivity and mass transfer of the system (Hayat et al. 2007)

To circumvent the limitations of the current approach, the models might have to adopt the use of the nonlinear models of radiation heat transfer for any optical thickness instead of the Rosseland approximation. The laminar solutions within the models will be taken through to provide the turbulent micro polar flow insight, more appropriate to high speed aerospace applications. Numerical simulations (FEM or FVM) will need to be performed to efficiently arrive at solutions for more complex geometries far more practically than purely analytical approaches. The theory might in the end have to be experimentally validated to make it more an engineering solution rather than it being a mathematical theory (Bejan, 2013).

CONCLUSION

The present work provided different inertial micro polar fluid dynamics between a semi-infinite vertical plate with the combined effect of slip and thermal radiation. It was observed that the micro polar effects for both velocity and micro rotation had improved compared to the Newtonian case (Eringen, 1966) due to the vortex viscosity term offering additional resistance to the flow process. And, the thermal radiation heat transfer effect introduces an additional flow mechanism by increasing the fluxing of the flow through the increased effective thermal conductivity coefficient thickening the thermal boundary layer energizing the flow field (Howell et al. 2010).

The slip condition is an important control parameter which decrease the wall shear stress and A lot alter the velocity distribution near the boundary to avoid the exaggeration of the skin friction at the micro-scale (Karniadakis et al. 2005). Because of this the combined effect of the micro polarity, radiation and slip causes the more complex flow features, which are relevant in today's applications such as micro-electromechanical systems (MEMS), nuclear cooling and the high temperature aerospace applications where thermo physical modeling is Quite a bit essential (Bejan, 2013).

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