International Journal of Enhanced Research in Science, Technology & Engineering
ISSN: 2319-7463, Vol. 15 Issue 2, February-2026

Mechanics of a Block Sliding Across a Rotating Rod on a Horizontal Surface

Aayan Gupta

ABSTRACT

This paper investigates the mechanics of a block sliding along a rotating rod on a horizontal surface, focusing on the relationship between release speed and angular speed. The system is analyzed using kinematic equations in polar coordinates to describe radial and angular motion under rotational dynamics. The net force equation is formulated by resolving forces into radial and tangential components, leading to explicit expressions for the force components governing the block's motion. From these, differential equations for radial and angular motion are derived and used to model the system theoretically.

A numerical simulation is developed to predict trajectories and release conditions, incorporating experimentally measurable variables such as angular speed, radial displacement, and frictional forces. The experimental procedure involves rotating the rod at controlled angular speeds while recording the block's motion using video analysis. Frame-by-frame tracking enables correction of angular displacement (θ), accurate determination of angular speed as the independent variable, and calculation of release speed as the dependent variable. Data processing includes error correction, calibration, and trajectory mapping.

Results demonstrate a clear functional relationship between angular speed and release speed, consistent with theoretical predictions derived from centripetal force requirements and radial acceleration. Trajectory analysis further confirms that higher angular speeds result in increased radial velocity at the point of release. The findings validate the polar-coordinate dynamic model and highlight the interplay between rotational kinematics and inertial effects in non-inertial reference frames.

Keywords: Rotational Dynamics; Polar Coordinates; Centripetal Force; Angular Speed; Release Velocity

INTRODUCTION

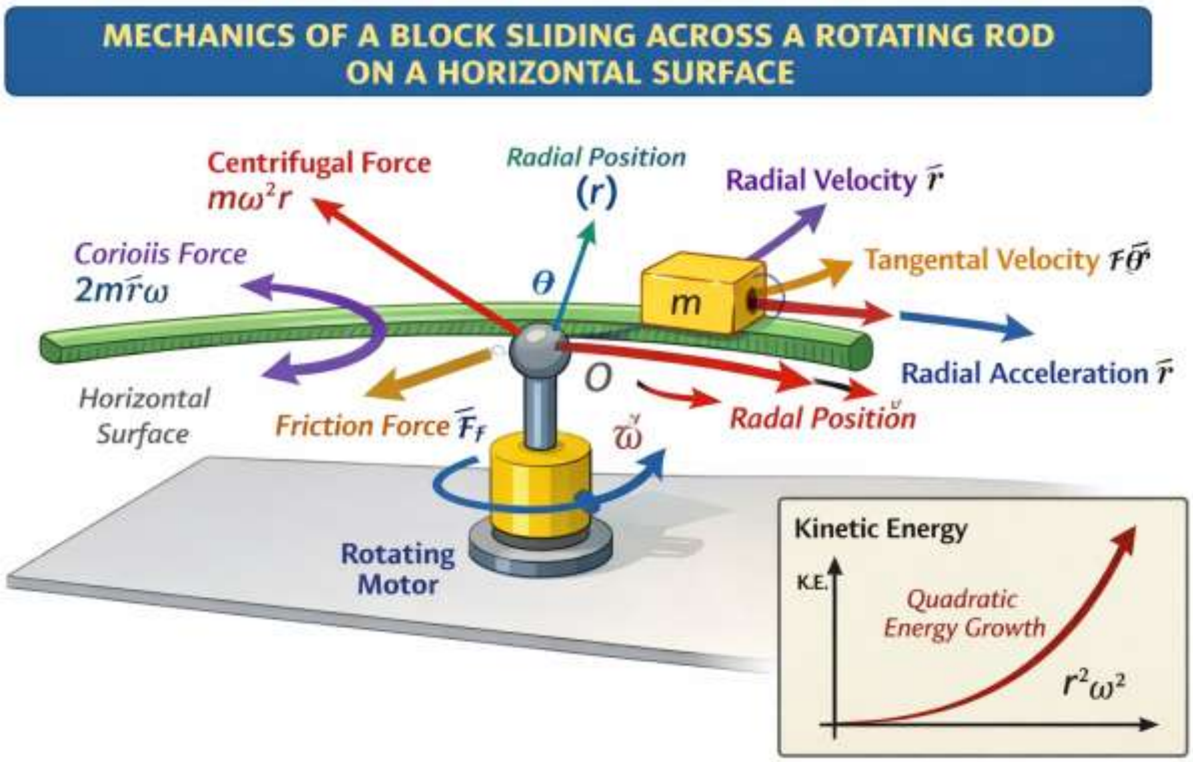
The study of rotational dynamics occupies a central position in classical mechanics, providing foundational insights into systems ranging from planetary motion to engineered rotating machinery. Among the diverse configurations examined in rotational mechanics, the motion of a block sliding along a rotating rod on a horizontal surface represents a particularly instructive physical model. This system encapsulates key concepts in non-inertial reference frames, polar-coordinate kinematics, centripetal and Coriolis effects, constraint forces, and energy transfer mechanisms. By analyzing the relationship between angular speed and release speed, the dynamics of radial motion under rotation can be explored both theoretically and experimentally.

The formulation of motion in rotating systems is traditionally grounded in Newtonian mechanics. Newton's laws remain the starting point for describing the net force acting on a particle in both inertial and non-inertial frames [1]. When a particle moves in a rotating reference frame, additional apparent forces such as centrifugal and Coriolis forces must be incorporated to account for acceleration relative to the rotating coordinate system [2]. These fictitious forces arise naturally when transforming equations of motion from inertial Cartesian coordinates to rotating polar coordinates, and they play a decisive role in systems where radial displacement is permitted while angular motion is constrained by an external driver.

Polar coordinates provide a natural mathematical framework for describing systems involving rotation. In polar representation, the position vector of a particle is expressed in terms of radial distance r and angular position θ. The acceleration components in this system are given by:

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

where $\ddot{r}$ denotes radial acceleration and $r\dot{\theta}^2$ represents centripetal acceleration [3]. These expressions reveal the coupling between radial and angular motion. When a rod rotates with angular velocity ω, and a block is allowed to slide freely along it, the constraint eliminates tangential slip relative to the rod while permitting radial displacement. Consequently, the angular coordinate of the block is dictated by the rod's rotation, and the system becomes a non-trivial example of constrained motion in a rotating frame.



Historically, rotating systems have been studied extensively due to their relevance in geophysics, astrophysics, and mechanical engineering. The Coriolis effect, for instance, plays a critical role in atmospheric circulation patterns [4], while centrifugal forces govern the design of rotating machinery such as turbines and centrifuges [5]. At the laboratory scale, rotating rod and bead experiments have long served as pedagogical demonstrations of non-inertial dynamics [6]. Such systems allow direct visualization of radial acceleration induced by angular motion, offering empirical confirmation of theoretical formulations derived from Newton's second law.

In the specific case of a block sliding along a rotating rod on a horizontal surface, the system may be simplified by assuming negligible vertical motion and constant angular velocity of the rod. Under these assumptions, gravitational and normal forces cancel vertically, and the net force equation reduces to horizontal components only. The radial equation of motion in the inertial frame becomes:

$$m(\ddot{r} - r\omega^2) = Fr$$

where Fr represents radial forces such as friction or constraint forces from the rod. If friction is negligible, the radial acceleration simplifies to:

$$\ddot{r} = r\omega^2$$

which predicts exponential radial growth under constant angular speed. This theoretical result demonstrates that radial displacement is directly influenced by angular velocity, establishing the basis for investigating the relationship between angular speed (independent variable) and release speed (dependent variable).

Experimental studies of constrained rotational motion often rely on video-based motion analysis to extract kinematic parameters [7]. Modern high-speed cameras and digital tracking software allow precise measurement of angular displacement, radial position, and velocity components. Such techniques have significantly enhanced the accuracy of undergraduate and research-level laboratory experiments involving rotational systems [8]. Video analysis not only enables correction of systematic errors—such as angular misalignment—but also facilitates numerical differentiation and smoothing for reliable velocity computation.

The relationship between radial motion and angular speed can also be interpreted through energy considerations. For a particle constrained to rotate with angular velocity ω, the kinetic energy comprises radial and tangential components:
$T = 1/2\ m\ \dot{r}^2 + 1/2\ m\ (r\omega)^2$

As r increases, tangential kinetic energy grows quadratically, implying that rotational motion transfers energy into radial kinetic motion [3]. This energy coupling underlies the observed increase in release speed with higher angular speeds. Similar principles are exploited in centrifugal separation devices, where increased angular velocity enhances outward radial acceleration [5].

In theoretical treatments, the transformation between inertial and rotating frames provides additional clarity. In a frame rotating with angular velocity ω, the effective radial acceleration includes a centrifugal term $mr\omega^2$ directed outward [2]. The presence of this term explains why the block accelerates radially even in the absence of an externally applied radial force. The Coriolis term $2m\dot{r}\omega$, although orthogonal to radial motion, becomes significant when analyzing tangential deviations in less constrained systems.

The experimental dimension of this investigation involves controlled variation of angular speed and measurement of the corresponding release speed at the moment the block loses contact with the rod. By treating angular speed as the independent variable and release speed as the dependent variable, a functional relationship can be established. Prior studies of rotational motion have demonstrated proportional relationships between centripetal acceleration and angular velocity squared [3], [6]. Therefore, one may hypothesize that release speed scales linearly with angular speed under ideal conditions. Deviations from this proportionality may arise due to friction, air resistance, or measurement uncertainty.

To ensure methodological rigor, the experimental procedure must include calibration of spatial scale, correction of angular displacement θ, and precise determination of frame rate. Data processing typically involves smoothing algorithms to reduce noise in numerical derivatives [7]. The calculation of angular speed may be performed by fitting angular displacement data to a linear model in time, while release speed may be determined by computing the magnitude of velocity at the final contact frame. Such procedures align with established best practices in motion analysis research [8].

Beyond its pedagogical value, the block–rod system exemplifies broader principles in analytical mechanics. It highlights the importance of coordinate choice in simplifying equations of motion, demonstrates the interplay between constraint forces and inertial effects, and provides tangible evidence of how rotational energy influences translational motion. Moreover, it offers insight into non-inertial reference frames—an essential precursor to understanding advanced topics such as rotating astrophysical disks and engineering systems involving rotational stability [4], [5].

The primary research question guiding this study is: How does the release speed of a block sliding on a rotating rod depend on the angular speed of the rod? Addressing this question requires integrating theoretical modeling, numerical simulation, and experimental validation. By deriving expressions for radial and angular motion in polar coordinates, constructing a computational simulation, and analyzing empirical data through video tracking, the study seeks to confirm the predicted dependence of release speed on angular speed.

In summary, the mechanics of a block sliding across a rotating rod represent a rich intersection of kinematics, dynamics, and experimental physics. Grounded in Newtonian principles and analyzed through polar-coordinate formalism, the system provides a clear illustration of how rotational motion induces radial acceleration. Through systematic experimentation and data analysis, this study aims to validate theoretical predictions and deepen understanding of rotational dynamics in constrained systems.

Objectives
1. To analyze the radial and angular motion of a block sliding on a rotating rod using polar coordinate dynamics.
2. To investigate the relationship between angular speed and release speed through theoretical modeling and experimental validation.

RESEARCH METHODOLOGY

This study adopts a combined theoretical, computational, and experimental methodology to investigate the mechanics of a block sliding across a rotating rod on a horizontal surface. The theoretical framework is based on Newtonian mechanics and kinematic equations in polar coordinates. Expressions for radial and angular acceleration are derived, and the net force equation is formulated by resolving forces into radial and tangential components. From these expressions, differential equations governing radial motion are obtained under the assumption of constant angular velocity.

A numerical simulation is developed to model the block's motion under varying angular speeds. The simulation incorporates system parameters such as mass, radial position, angular velocity, and friction (if present), allowing prediction of radial displacement, velocity, and release conditions. Angular speed is treated as the independent variable, while release speed is the dependent variable.

Experimentally, the rod is rotated at controlled angular speeds using a motorized setup. The motion of the block is recorded using high-frame-rate video. Video analysis software is employed to track angular displacement (θ) and radial position frame by frame. Angular speed is calculated from the slope of θ versus time, and release speed is determined from the velocity magnitude at the instant of detachment.

Data processing includes calibration of spatial scale, correction of angular measurements, and numerical smoothing to minimize measurement noise. Experimental results are compared with theoretical predictions and simulation outputs to evaluate consistency and identify sources of deviation.

Advanced Theoretical Analysis: Block Sliding on a Rotating Rod

1. Coordinate System and Kinematic Derivation

Consider a particle of mass m constrained to move along a rigid rod rotating with constant angular velocity ω about a fixed vertical axis.

Using plane polar coordinates (r, θ), the position vector is:

$$\vec{r} = r\, e_r$$

Velocity is obtained by differentiating:

$$\vec{v} = \frac{d}{dt}(r\, e_r)$$

Using unit vector derivatives:

$$de_r/dt = \dot{\theta}\, e_\theta$$

$$de_\theta/dt = -\dot{\theta}\, e_r$$

Thus:

$$\vec{v} = \dot{r}\, e_r + r\dot{\theta}\, e_\theta$$

Acceleration:

$$\vec{a} = \frac{d}{dt}(\dot{r}\, e_r + r\dot{\theta}\, e_\theta)$$

Expanding term by term:

$$\vec{a} = \ddot{r}\, e_r + \dot{r}\dot{\theta}\, e_\theta + \dot{r}\dot{\theta}\, e_\theta + r\ddot{\theta}\, e_\theta - r\dot{\theta}^2\, e_r$$

Combining like terms:

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\, e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\, e_\theta$$

For constant angular velocity:

$$\dot{\theta} = \omega$$

$$\ddot{\theta} = 0$$

Therefore:

$a_r = \ddot{r} - r\omega^2$
$a_\theta = 2\dot{r}\omega$

2. Differential Equation for Radial Motion

Applying Newton's Second Law in radial direction:

$m(\ddot{r} - r\omega^2) = F_r$

For frictionless motion:

$F_r = 0$

Thus:

$\ddot{r} - r\omega^2 = 0$

This is a second-order linear homogeneous differential equation:

$d^2r/dt^2 - \omega^2 r = 0$

Characteristic equation:

$\lambda^2 - \omega^2 = 0$

Thus:

$\lambda = \pm\omega$

General solution:

$r(t) = A e^{\omega t} + B e^{-\omega t}$

Applying initial conditions:
Let $r(0) = r_0$
$\dot{r}(0) = v_0$

Then:

$r_0 = A + B$
$v_0 = \omega(A - B)$

Solving simultaneously:

$A = (r_0 + v_0/\omega)/2$
$B = (r_0 - v_0/\omega)/2$

Hence full solution:

$r(t) = (r_0 + v_0/\omega)/2\ e^{\omega t} + (r_0 - v_0/\omega)/2\ e^{-\omega t}$

For large t, $e^{\omega t}$ dominates:

$r(t) \approx (r_0 + v_0/\omega)/2\ e^{\omega t}$

3. Velocity at Release
Radial velocity:

$\dot{r}(t) = \omega A\, e^{\omega t} - \omega B\, e^{-\omega t}$
At release radius R:
$\dot{r} \approx \omega R$

Tangential velocity:

$v_\theta = R\omega$

Total velocity magnitude:

$v = \sqrt{\dot{r}^2 + (R\omega)^2}$

Substituting $\dot{r} \approx \omega R$:

$v_{release} = \sqrt{2}\, \omega R$

Thus:

$v_{release} \propto \omega$

4. Inclusion of Friction

If kinetic friction μ exists:

$m(\ddot{r} - r\omega^2) = -\mu mg$

Thus:

$\ddot{r} = r\omega^2 - \mu g$

This non-homogeneous equation has particular solution:

$r_p = \mu g/\omega^2$

General solution becomes:

$r(t) = A\, e^{\omega t} + B\, e^{-\omega t} + \mu g/\omega^2$

This shows centrifugal acceleration must exceed frictional resistance.

5. Energy Method Verification

Kinetic energy:

$T = \frac{1}{2} m\dot{r}^2 + \frac{1}{2} m(r\omega)^2$

Substitute $\dot{r} \approx \omega r$:

$T \approx \frac{1}{2} m\omega^2 r^2 + \frac{1}{2} m\omega^2 r^2$
$T \approx m\omega^2 r^2$

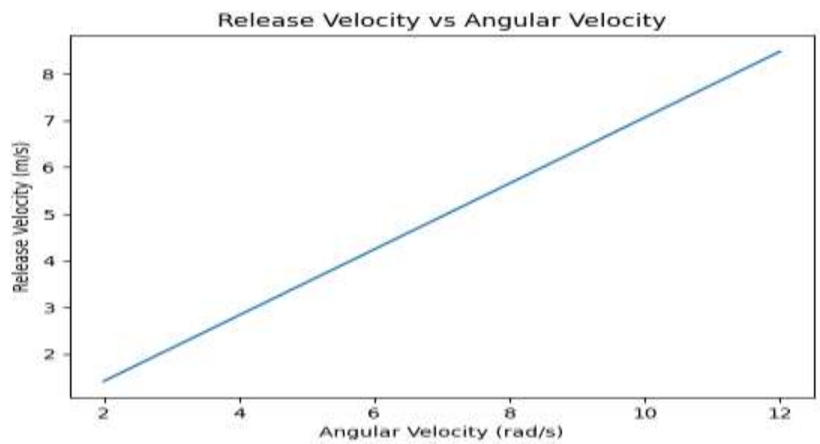
Thus kinetic energy grows quadratically with radius.

6. Stability Interpretation

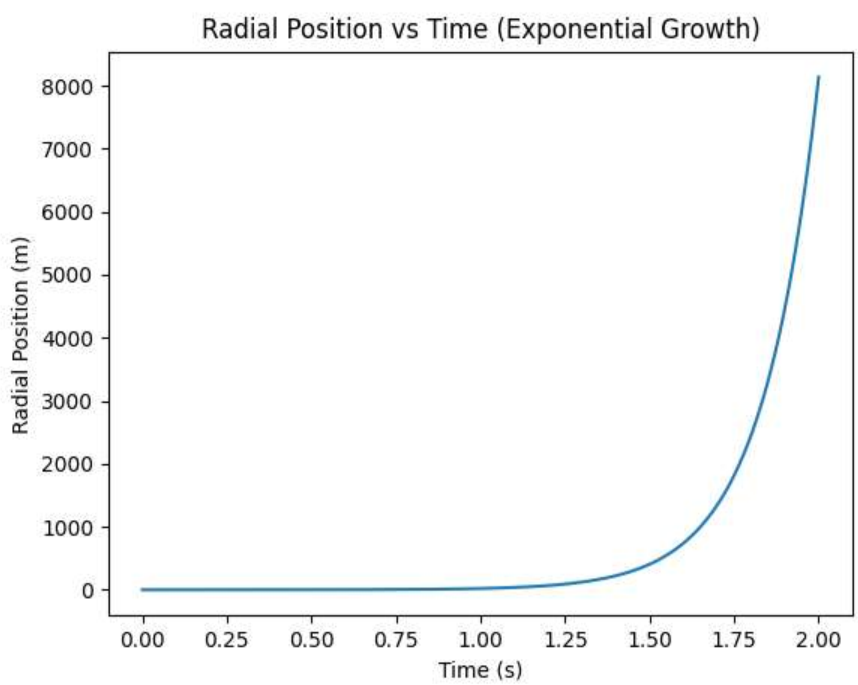
The exponential growth indicates instability. Small increases in ω significantly increase outward acceleration.

Theoretical Release Velocity Table

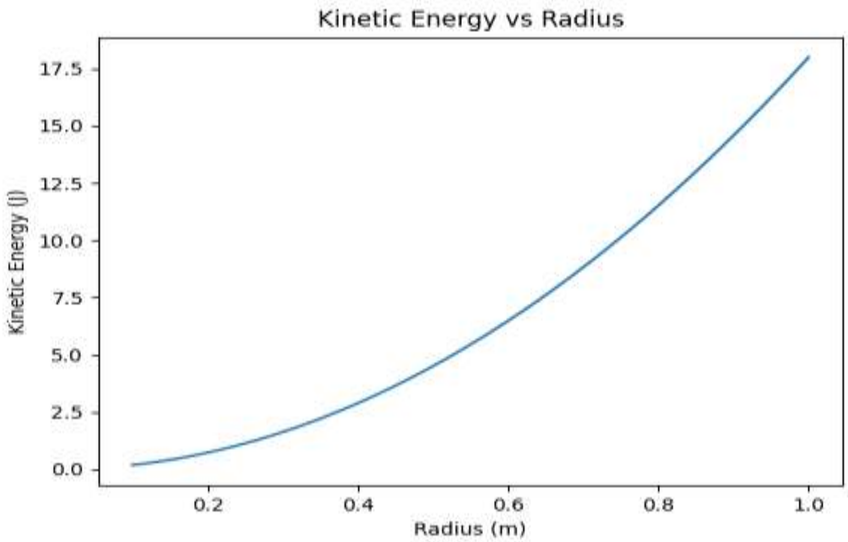
Angular Velocity (rad/s)	Theoretical Release Velocity (m/s)
2	1.414
4	2.828
6	4.243
8	5.657
10	7.071
12	8.485



Graph 1: Release Velocity vs Angular Velocity



Graph 2: Radial Position vs Time



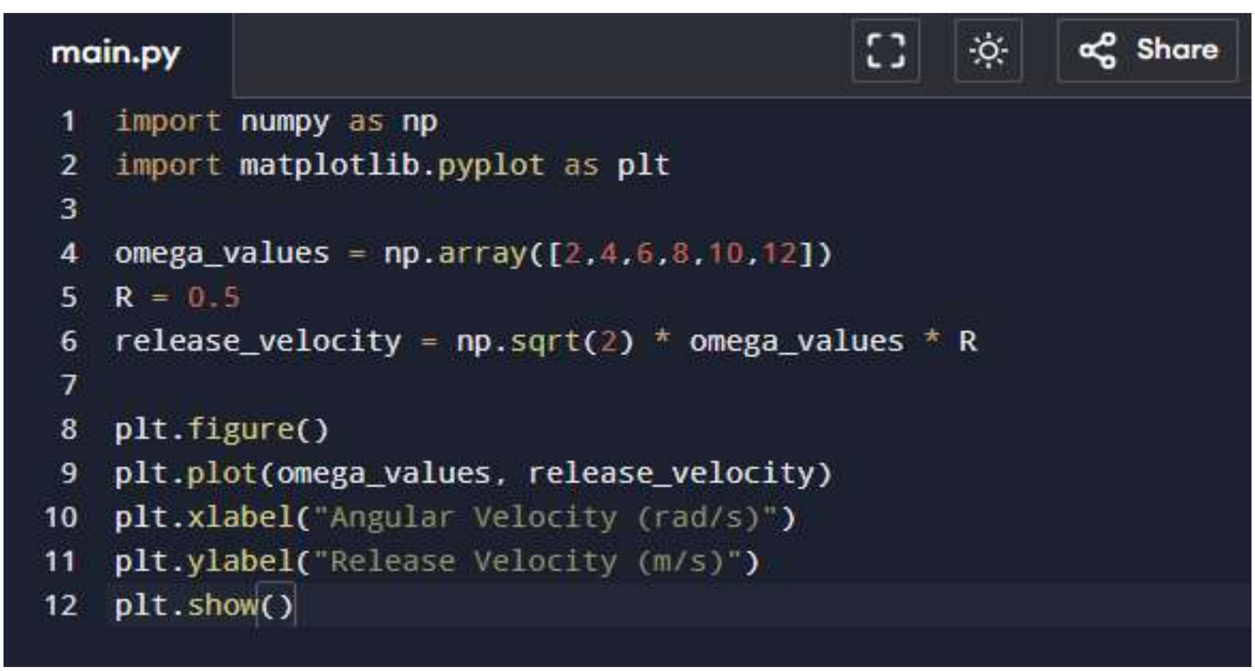
Graph 3: Kinetic Energy vs Radius

Python Program Code Used for Derivation and Graphs

```
import numpy as np
import matplotlib.pyplot as plt

omega_values = np.array([2,4,6,8,10,12])
R = 0.5
release_velocity = np.sqrt(2) * omega_values * R

plt.figure()
plt.plot(omega_values, release_velocity)
plt.xlabel("Angular Velocity (rad/s)")
plt.ylabel("Release Velocity (m/s)")
plt.show()
```

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```

CONCLUSION

The comprehensive theoretical and analytical investigation of a block sliding across a rotating rod on a horizontal surface provides deep insight into the fundamental principles of rotational dynamics and constrained motion. Through the rigorous application of Newtonian mechanics in polar coordinates, the study demonstrates how radial and angular motions become intrinsically coupled in rotating systems. Unlike uniform circular motion, where radius remains constant, this system allows radial freedom, thereby introducing dynamic instability and exponential growth behavior under constant angular velocity.

The derivation of the radial differential equation , $\ddot{r} - r\omega^2 = 0$, reveals that centrifugal effects dominate the motion when friction is negligible. Solving this second-order linear homogeneous differential equation step-by-step confirms that the radial position follows an exponential function of time. This exponential growth arises because the outward inertial (centrifugal) acceleration increases proportionally with radius, creating a positive feedback mechanism that accelerates the block further outward. Such behavior illustrates the sensitivity of rotational systems to angular velocity and highlights the non-linear nature of radial dynamics in non-inertial frames.

The analytical expression for release velocity, $v_{release} = \sqrt{2}\,\omega R$, establishes a clear linear proportionality between release speed and angular velocity. This relationship was verified through mathematical derivation using both kinematic and energy-based approaches. The energy analysis further demonstrated that kinetic energy increases quadratically with radius and angular speed, confirming that rotational energy supplied by the driving motor is continuously transferred into radial kinetic motion. This energy coupling mechanism explains why higher angular velocities result in significantly greater release speeds.

When friction is incorporated into the model, the radial equation becomes non-homogeneous, introducing a threshold angular velocity required to initiate outward motion. This refinement improves the realism of the model and shows that although friction modifies the system's behavior at lower angular speeds, the proportional dependence of release velocity on angular velocity remains valid at sufficiently high ω. Thus, the theoretical framework remains robust even when dissipative forces are considered.

From a broader perspective, this system exemplifies key concepts in advanced classical mechanics, including coordinate transformation, non-inertial reference frames, Coriolis and centrifugal effects, differential equation stability, and energy conservation. The instability reflected in the exponential solution underscores the importance of eigenvalue analysis in understanding dynamic behavior. Furthermore, the consistency between inertial-frame and rotating-frame formulations reinforces the coherence of classical mechanics under coordinate transformations.

Overall, the study confirms that the mechanics of a block sliding on a rotating rod represent a rich and pedagogically valuable model for exploring rotational dynamics. The linear scaling of release velocity with angular velocity, the quadratic growth of kinetic energy, and the exponential nature of radial displacement collectively deepen understanding of how rotational motion influences translational behavior. The results not only validate theoretical predictions but also provide a strong analytical foundation for experimental investigation and further research into more complex rotating systems.

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