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Numerical Investigation of Mixed Convection in a Water–Silver Nanofluid Filled Lid-Driven Cavity in the Presence of Magnetic Field

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ABSTRACT

This paper examines the laminar mixed convection in two dimensional lid-driven square cavity filled with water-silver nanofluid and influenced by uniform magnetic field. The horizontal walls are insulated while the vertical walls are maintained at constant but different temperatures. The thermal conductivity and viscosity of nanofluid have been calculated by Maxwell–Garnetts and Brinkman models, respectively. The dimensionless governing equations are solved using finite volume method and SIMPLE algorithm. Comparisons with previously published work are performed and found to be in good agreement. Calculations were performed for Rayleigh number 10^4 to 10^6, Hartmann number 0 to 60, solid volume fraction 0 to 0.05 and constant Reynolds and Prandtl numbers. It is found that heat transfer increases with an increase of Rayleigh number, whereas it decreases with the increase of Hartmann number. Furthermore, it is found that in the presence of the magnetic field, the effect of the solid volume fraction on the heat transfer depends on the value of Rayleigh number.

Keywords: Heat transfer, Lid-driven cavity, Magnetic field, Mixed convection, Water-Silver nanofluid.

1. INTRODUCTION

The main limitation of conventional fluids such as water, oil, and ethylene glycol is their low thermal conductivity. Nanofluid, which is a dilute liquid suspensions of nanoparticles suspended in base fluid, has a superior thermal conductivity compared to the base fluid. This higher thermal conductivity enhances rate of heat transfer in this fluid. Due to the ability of high thermal conductivity, Nanofluids are preferred in many engineering area such as cooling of electronics, cooling of diesel electric generators, cooling and heating in buildings, etc. In recent decades, many researchers have investigated convective heat transfer in nanofluids. Khanafer et al. [1] were the first to investigate the problem of buoyancy-driven heat transfer enhancement of nanofluids in a two-dimensional enclosure. The impact of nanoparticle volume fraction on the buoyancy-driven heat transfer process was discussed. Consequently, at any given Grashof number, the heat transfer rate increases as solid volume fraction is increased. Putra et al. [2] experimentally studied the heat transfer characteristics of nanofluids under natural convection inside a horizontal cylinder heated and cooled from the two ends respectively. Their results showed that a paradoxical behaviour of heat transfer deterioration. The deterioration is dependent on parameters such as particle density, concentration as well as the aspect ratio of the cylinder. Jou and Tzeng [3] numerically investigated the heat transfer performance of nanofluids inside two dimensional rectangular enclosures. They indicated that increasing the nanoparticle concentration causes an increase in the average heat transfer coefficient. Oztop and Abu-Nada [4] investigated heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using various types of nanoparticles. The results indicated heat transfer enhancement by using nanofluids and was more pronounced at low aspect ratio than at high aspect ratio and the mean Nusselt number increased as increasing the volume fraction of nanoparticles for the whole range of Rayleigh number.

Mixed convection is a result of interaction between the forced convection, due to the moving wall of the cavity, and free convection, due to the temperature gradients within the cavity. Analysis of mixed convection in a lid-driven cavity has important applications in many branches of engineering. These applications include cooling of electronic devices, multi-shield structures used for nuclear reactors, chemical processing equipment and drying technologies, high-performance building insulation, glass production, solar power collectors, etc. Fluid flow and heat transfer due to mixed convection driven by buoyancy and shear in a cavity filled with either pure fluid or nanofluid have been the subjects of some studies. Tiwari and Das [5] studied the behaviour of nanofluids inside a two-sided lid-driven differentially heated square cavity.

They have observed that the presence of nanoparticles in a base fluid is capable of increasing the rate of heat transfer of the base fluid. Numerical study of mixed convection of nanofluid in a square lid-driven cavity partially heated from below has been performed by Mansour et al. [6]. Their results show that the average Nusselt number increases on increase in the volume fraction of nanoparticles. Nemati et al. [7] has been used a new numerical strategy to show the ability of Lattice Boltzmann Method for simulation of nanofluid in a lid-driven square cavity. Chamkha and Abu-Nada [8] numerically studied steady mixed convection flow in single and double-lid square cavities filled with a water-Al_2O_3 nanofluid. They found that significant heat transfer enhancement can be obtained due to the presence of nanoparticles and that this was accentuated by increasing the nanoparticle volume fractions at moderate and large Richardson numbers. Pourmahmoud et al. [9] investigated numerically the laminar mixed convection in lid- driven cavity filled with water-Al_2O_3, water-Cu or water-TiO_2 nanofluids. The results illustrated that the type of nanofluid is a key factor to heat transfer enhancement.

Convection due to the expose of an electrically conducting fluid to an external uniform magnetic field is called magneto-convection. The study of convection flow connected with MHD is very important in industries due to its wide variety of applications in engineering, such as electromagnetic casting, liquid-metal cooling of nuclear reactors, and plasma confinement. Many investigations have been conducted on magnetic field on the fluid flow and heat transfer using experimental or numerical methods by several researchers. The mixed convection in a square cavity in the presence of the magnetic field and an internal heat generation or absorption was examined by Chamkha [10]. The average Nusselt number decreases when the strength of a magnetic field is increased. Sivasankaran et al. [11] numerically studied mixed convection in a lid-driven square cavity when both vertical side walls are partially heated and cooled in the presence of uniform magnetic field. It was found that the average heat transfer rate enhances in the similar locations than dissimilar locations. On the other hand, the problem of mixed convection of nanofluids under the influence of a magnetic field has been the subjects of some studies. We can refer to certain works such as those of Zare Ghadi et al. [12], and Oztop et al. [13], etc.

The purpose of this study is to numerically investigate the mixed convection heat transfer in lid-driven square enclosure filled with a water–silver nanofluid and influenced by magnetic field. Both horizontal walls are insulated while the left and right walls are maintained at different temperature. The consequence of varying the Rayleigh number, Hartmann number and the solid volume fraction of nanoparticle on the hydrodynamic and thermal characteristics have been studied and discussed.

MODEL DESCRIPTION

A. Physical Model

Fig. 1 shows a two-dimensional square cavity. The width and height of the cavity are denoted as L. Side walls are the heated surfaces (hot and cold walls). Top and bottom walls of the cavity are assumed to be adiabatic. Furthermore, the top lid is assumed to have a constant speed U_0 that slides in the positive direction of x-axis. A uniform magnetic field strength of B_0 is considered in the horizontal direction. The working fluid employed in this enclosure is water based nanofluid containing Ag nanoparticles which is assumed to be Newtonian, incompressible and laminar flow. It is also assumed that, both the nanoparticles and the base fluid are in thermal equilibrium and no slip occurs between them.

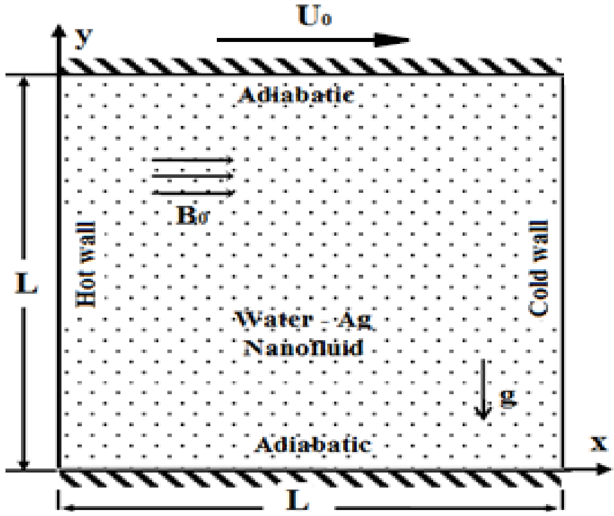


Figure 1. Schematic of the square lid-driven cavity

Table 1 presents thermophysical properties of Ag nanoparticles and water (Pr=6.2) as base fluid. It is further assumed that the Boussinesq approximation is valid for buoyancy force. The viscous dissipation and joule heating are neglect.

Table 1: Thermophysical properties of base fluid and solid nanoparticles

Physical properties	Fluid phase (water)	Ag
ρ (kgm^{-3})	997.1	10500
C_P ($Jkg^{-1}K^{-1}$)	4179	235
k ($Wm^{-1}K^{-1}$)	0.613	429
$\beta\times10^{-5}$ (K^{-1})	21	1.89
σ (Sm^{-1})	0.05	6.30×10^{7}

B. Mathematical Modeling

The governing equations of (continuity, momentum and energy equations) for a steady, two dimensional, incompressible and laminar flow in non-dimensional forms are expressed as below:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\frac{\mu_{nf}}{\rho_{nf}\nu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\frac{\mu_{nf}}{\rho_{nf}\nu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{\mathrm{Ra}}{\mathrm{Pr}\,\mathrm{Re}^2}\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\theta-\frac{\mathrm{Ha}^2}{\mathrm{Re}}V \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

Where U and V fluid velocity in X-direction and Y-direction, respectively, P is the pressure, θ is the temperature. μ_{nf}, ρ_{nf}, β_{nf}, α_{nf} and ν_f are respectively, dynamic viscosity, density, thermal expansion coefficient, thermal diffusivity and kinematic viscosity. Here, subscript *nf* and *f* indicate physical properties of nanofluid and base fluid, respectively; Re, Pr, Ra and Ha are respectively, Reynolds, Prandtl, Rayleigh and Hartmann numbers.

In equations (1) – (4), the following non-dimensional parameters are used:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{u}{U0},\quad V=\frac{v}{U0},\quad P=\frac{p}{\rho_{nf}U0^2},\quad \theta=\frac{T-T_c}{T_h-T_c},$$
$$Re=\frac{U0L}{\nu_f},\quad \mathrm{Pr}=\frac{\nu_f}{\alpha_f},\quad \mathrm{Ra}=\frac{g\,\beta_f L^3(T_h-T_c)}{\nu_f\,\alpha_f},\quad \mathrm{Ha}=B0\left(\frac{\sigma_{nf}}{\rho_{nf}\nu_f}\right)^{1/2} \tag{5}$$

Where T_h and T_c are respectively, hot and cold temperature. σ_{nf} is electrical conductivity of the nanofluid.

The properties of nanofluid are obtained by Xuan and Roetzel [14] as:

$$\rho_{nf}=(1-\phi)\,\rho_f+\phi\rho_p \tag{6}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_P)_{nf}} \tag{7}$$

$$(\rho C_P)_{nf}=(1-\phi)(\rho C_P)_f+\phi(\rho C_P)_p \tag{8}$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_p \tag{9}$$

$$\sigma_{nf}=(1-\phi)\,\sigma_f+\phi\sigma_p \tag{10}$$

Where ϕ is the nanoparticle volume fraction. C_P and k are the specific heat at constant pressure and thermal conductivity, respectively. Here, subscript p indicates physical properties of nanoparticle.

The dynamic viscosity of the nanofluid is calculated using Brinkman's model [15] as follows:

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{11}$$

The thermal conductivity of the nanofluid is evaluated by the expression proposed by Maxwell-Garnett [16]:

$$\frac{k_{nf}}{k_f}=\left[\frac{(k_p+2k_f)-2\phi(k_f-k_p)}{(k_p+2k_f)+\phi(k_f-k_p)}\right] \tag{12}$$

The dimensionless boundary conditions are written as:

- on the left wall: $U=V=0,\quad \theta=1$ (13a)
- on the right wall: $U=V=0,\quad \theta=0$ (13b)
- on the top wall: $U=1,\ V=0,\quad \frac{\partial\theta}{\partial Y}=0$ (13c)
- on the bottom wall: $U=V=0,\quad \frac{\partial\theta}{\partial Y}=0$ (13d)

The stream function is calculated using : $U = \frac{\partial \psi}{\partial Y}$ and $V = -\frac{\partial \psi}{\partial X}$ (14)

In order to estimate the heat transfer enhancement in the cavity, the local Nusselt number on the left hot wall can be defined as:

$$Nu_Y = -\frac{k_{nf}}{k_f} \left. \frac{\partial \theta}{\partial X} \right|_{X=0} \tag{15}$$

Mean Nusselt number along the hot wall of the cavity is considered to evaluate the overall heat transfer rate and is defined as:

$$Nu_m = \int_0^1 Nu_Y \, dY \tag{16}$$

NUMERICAL METHOD AND VALIDATION

The non-dimensional governing equations (1) – (4), with the stated boundary conditions are solved numerically by the control volume method proposed by Patankar [17]. The power-law scheme is used for the convection and diffusion terms. In order to couple the velocity field and pressure in the momentum equations, the SIMPLE algorithm is adopted [17]. The solution of the resulting sets of algebraic equations was obtained using the sweeping method line by line with the Thomas algorithm. The convergence criterion is that the maximal residual is less than 10^{-6}.

In order to ensure that computational results are grid independent, Five different uniform grids is examined (see table 2). The grid independence tests are performed for $Ra=10^6$, Re=50, Ha=30 and $\phi=0.05$. Table 2 illustrates the results of independence studies for mean Nusselt number Num at the hot wall and the maximum stream function $|\psi|$max. As it can be observed from the table, an 101×101 uniform grid is sufficiently fine to ensure a grid independent solution. Based on these results, 101×101 grid is adopted for all computations.

Table 2: Result of the mean Nusselt number (Nu_m) and the maximum stream function ($|\psi|_{max}$)

Grid size (X×Y)	Nu_m	Error %	$\lvert\psi\rvert_{max}$	Error %
41×41	8.972	–	0.0526	–
61×61	8.762	2.340	0.0531	0.950
81×81	8.681	0.924	0.0533	0.376
101×101	8.636	0.518	0.0534	0.187
121×121	8.619	0.196	0.0535	0.187

The numerical model is validated by comparison between the predicted results with others for the natural convection in a differentially heated square cavity for the Rayleigh number ranging 10^4 to 10^6. The comparison is shown in table 3. The obtained results are good agreement with the existing solution. Another test has been performed for the lid-driven cavity problem of Nemati et al. [7]. Fig. 2 demonstrates the comparison results of vertical velocity on middle horizontal axis. The comparison results show good overall agreement. For validation of nanofluid behavior, the natural convection in an enclosure filled with water–Al_2O_3 nanofluid is simulated and mean Nusselt number is compared to the results of Oztop and Abu-Nada [4] as shown in fig. 3. The results obtained reveal a good agreement.

Table 3: Comparison of results obtained in this study by previous works for different Rayleigh numbers

Ra	Present work	De Vahl Davis [18]	Fusegi et al. [19]	Barakos et al. [20]
10^4	2.217	2.243	2.302	2.245
10^5	4.475	4.519	4.646	4.510
10^6	8.727	8.799	9.012	8.806

REDULTS AND DISCUSSION

Two-dimensional mixed convection is studied for a water–silver nanofluid in a square lid-driven cavity for Re=50, $Ra=10^4$ –10^6, Ha=0–60 and solid volume fraction 0 to 0.05. Prandtl number is fixed at Pr=6.2. The results depicted in fig. 4 and fig. 5 demonstrate the influence of magnetic field (Ha=0, 30 and 60) and different Rayleigh numbers ($Ra=10^4$, 10^5 and 10^6) on the fluid flow and the temperature distribution in the enclosure. In this section the solid volume fraction is fixed at $\phi=0.05$. The flow field presented in the form of streamlines consists of a one-cell pattern with clockwise rotation for all values of the Rayleigh and Hartmann numbers. For $Ra=10^4$ the effect of lid-driven flow is more dominant. Increase in Rayleigh number enhances the buoyancy effect and so the intensity of flow; but the dominant effect of the force convection is still observed. As Hartmann number increases circulation strength is decreased. This is expected because the presence of magnetic fields generally retards the flow. The streamlines are affected by variations in the Hartmann number, they are elongated, and the core region of the eddy is very close to the top wall of the cavity. These effects are more noticeable at Ha=60. It is also observed that the application of the magnetic field clearly affects the isotherms patterns, in particular at low Rayleigh number. On increasing the values of Hartmann number 30 and 60, the isotherms are more straightened out. This means that the heat transfer regime is changed to almost conduction mode. This behavior gradually disappears as the value of the Rayleigh number increases.

Fig. 6 demonstrates the effect of the Hartmann number on the vertical velocity along the horizontal mid–span of the enclosure at different values of the Rayleigh number (Ra $=10^4$, 10^5, and 10^6) and for a solid volume fraction ϕ = 0.05. It is observed from this figure that for Ra $=10^4$ and 10^5 the effect of forced convection predominate the buoyancy effect. Upward flow and downward flow are not symmetric with respect to the center point of the cavity. As can be seen the applied magnetic field leads to render the flow almost stagnant. The figure also shows that the maximum vertical velocity increases when the Rayleigh number increases due to strong buoyant flows and decreases when the Hartmann number increases due to the influence of the magnetic field on the convective flows.

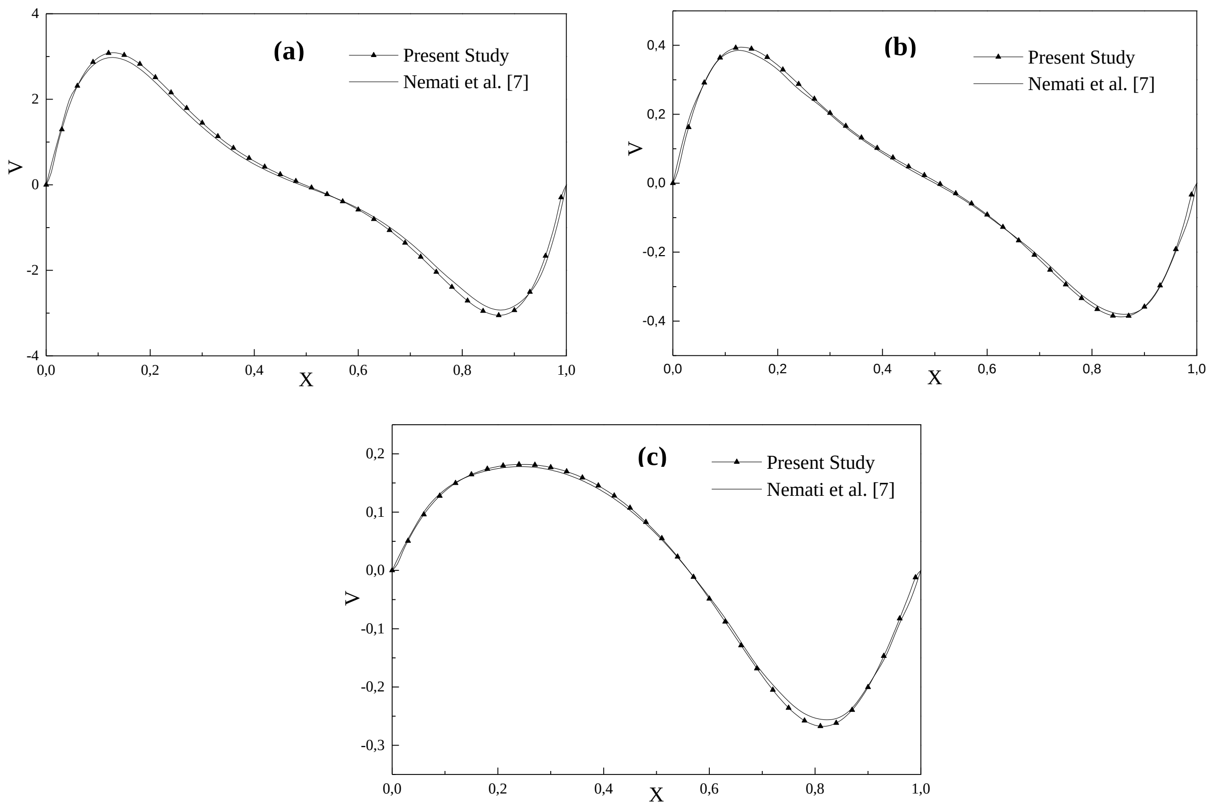


Figure 2. Comparison of the V-profile for Ra=10^4 and ϕ=0 (a) Re=1, (b) Re=10, (c) Re=100

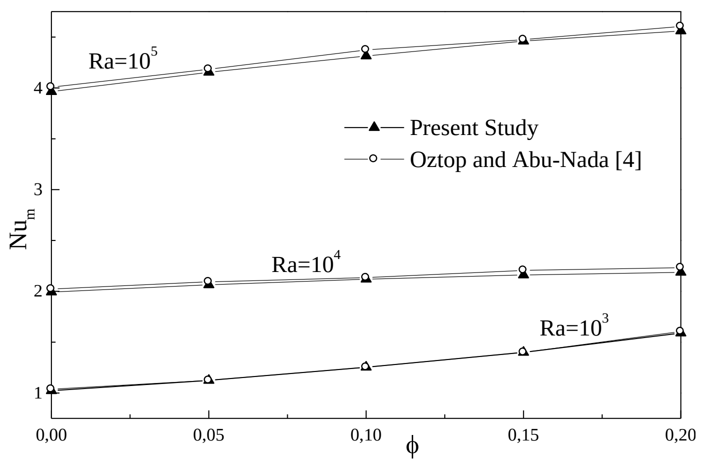


Figure 3. Comparison of the mean Nusselt number

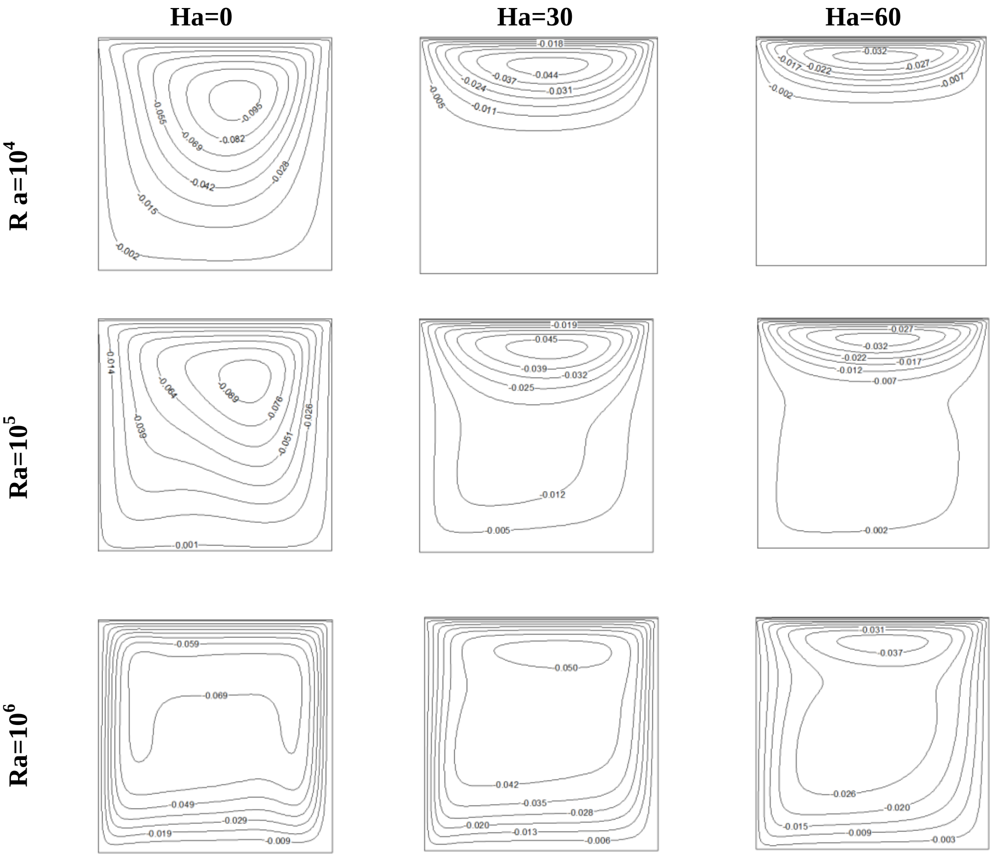
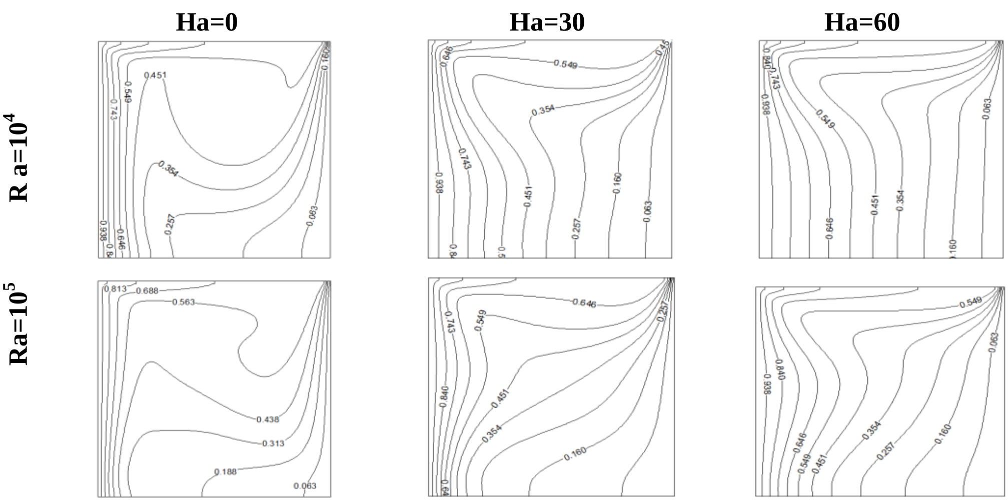


Figure 4. Streamlines for different Rayleigh and Hartmann numbers (ϕ = 0.05)

Fig. 7 illustrates the variation of local Nusselt number along the hot wall for various Hartmann number (Ha=0, 30 and 60) at different values of the Rayleigh number and solid volume fraction (ϕ=0.05). For all values of Rayleigh number, the local Nusselt number decreases as the Hartmann number increases. For Ra= 10^4, where the effect of lid-driven flow is more dominant, the Nusselt number augments gradually as Y increases and reaches the maximum value at Y=0.94 then decreases as Y increases to 1. The increase of the Rayleigh number augments the intensity of buoyancy and hence the intensity of natural convection within the cavity. For high Rayleigh number (Ra=10^6), the local Nusselt number begins with a high value at the bottom end and reduces monotonically to a low value towards the top end.

Fig. 8 shows the variation of mean Nusselt number with the Hartmann number for different Rayleigh numbers when the enclosure is filled with a water-Ag nanofluid (ϕ=0.05). It can be seen that the Nusselt number decreases with increasing the Hartmann number for all Rayleigh numbers. The figure shows also that, for a given Hartmann number, the Nusselt number increases as the Rayleigh number increases.



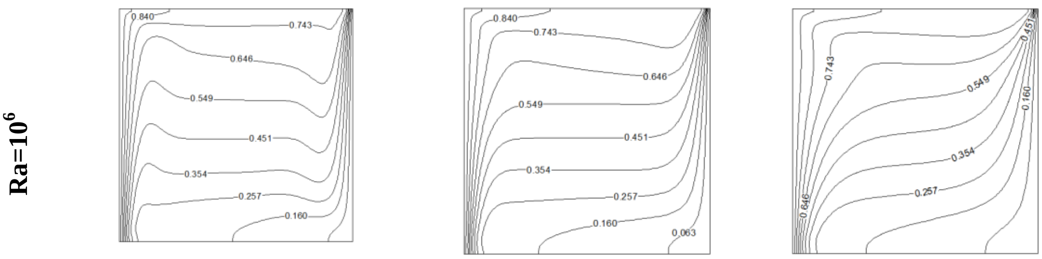


Figure 5. Isotherms for different Rayleigh and Hartmann numbers (ϕ = 0.05)

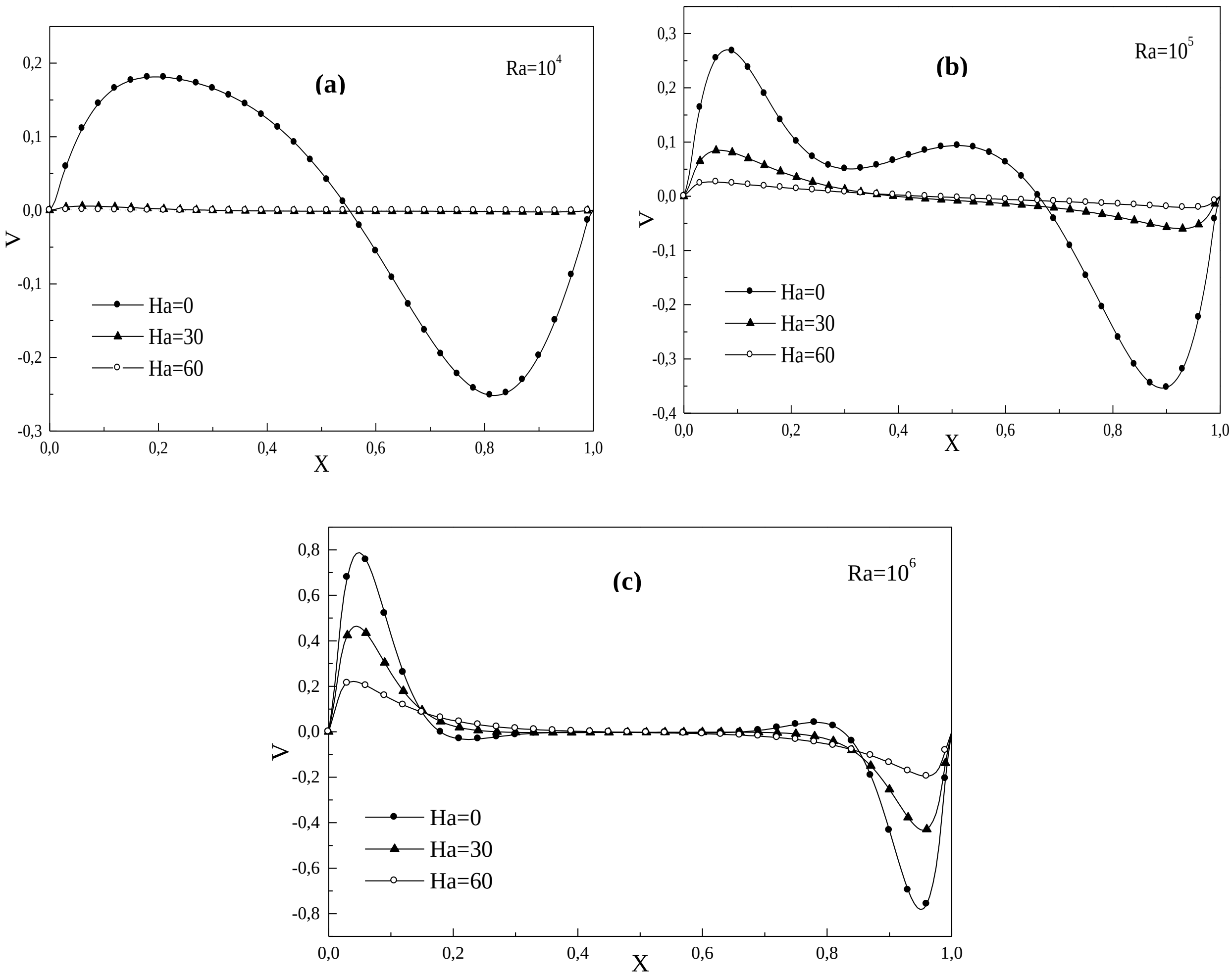
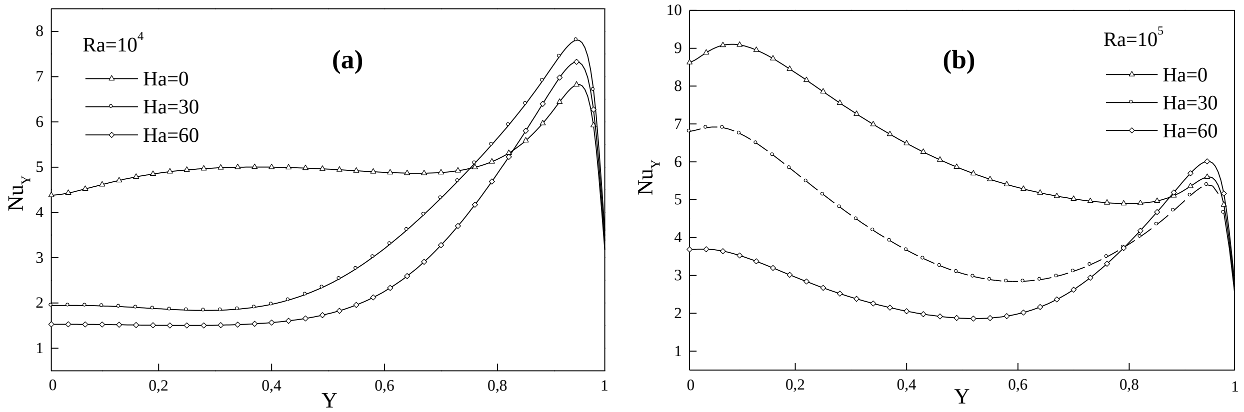


Figure 6. Vertical velocity at mid line of the cavity (*Y*=0.5) for ϕ=0.05 (a) Ra=10^4, (b) Ra= 10^5, (c) Ra=10^6



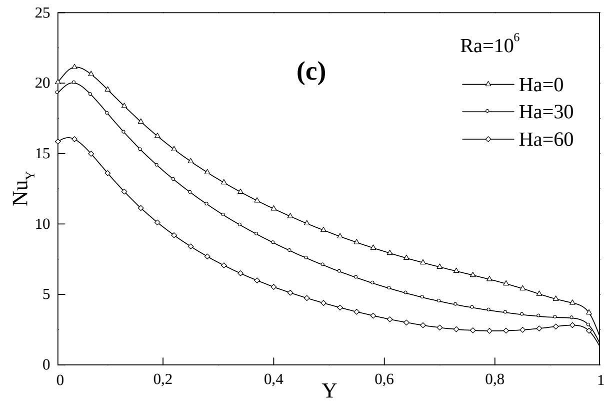


Figure 7. Local Nusselt number at various Hartmann number for ϕ=0.05 (a) Ra=104, (b) Ra=105, (c) Ra=106

The comparison study between the pure fluid (ϕ=0) and the nanofluid (ϕ=0.05) in terms of the streamlines and isotherms is presented in fig. 9 and fig. 10, respectively, for different Rayleigh and Hartmann numbers. In the absence of the magnetic field, the results show that the addition of nanoparticles results in an increase of the maximum stream function.

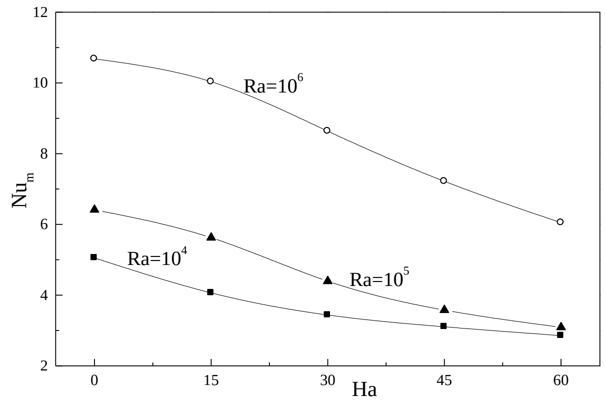
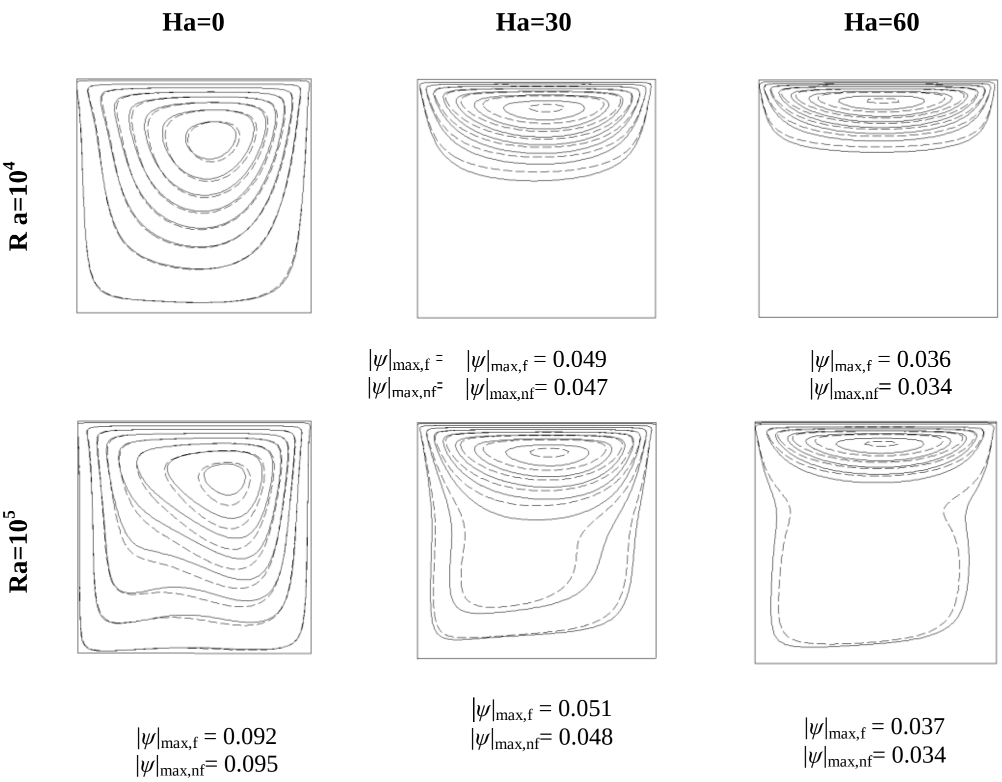


Figure 8. Variation of mean Nusselt number with Hartmann number for ϕ=0.05



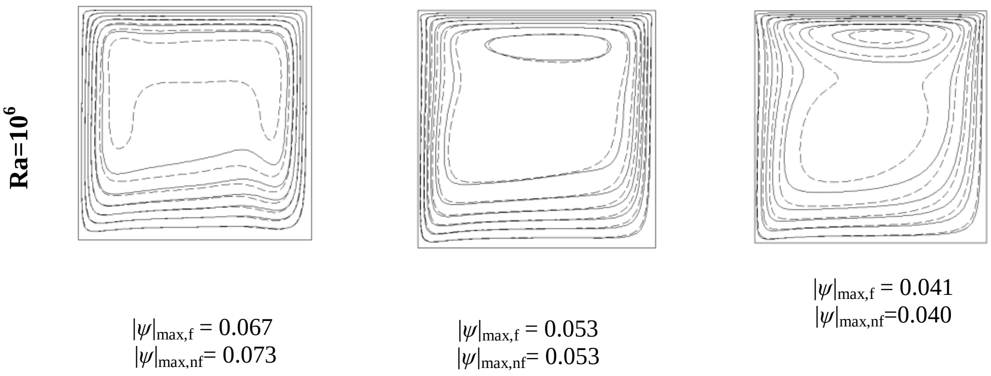
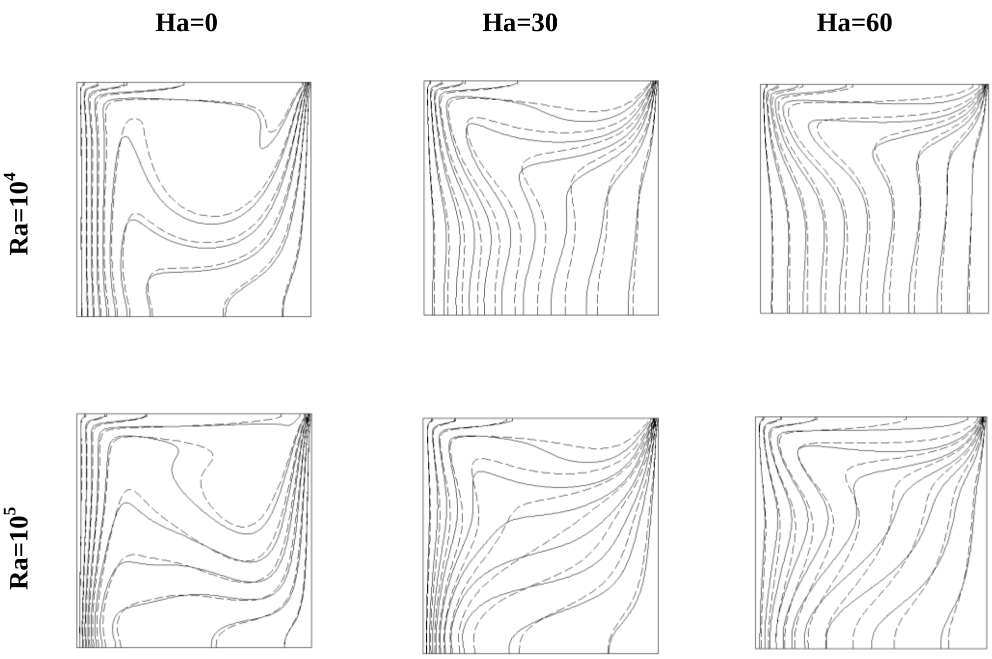


Figure 9. Streamlines for different Rayleigh and Hartmann numbers, pure water (−) and nanofluid (- - - -) with ϕ=0.05

It is also observed that for Ra=10^4 (forced convection dominated regime), the solid volume fraction does not have considerable effect on the flow pattern. However, as seen from the isotherms, the addition of nanoparticles has more effect to increase the heat penetration; because the conduction heat transfer has more effective role at the lower Rayleigh number. But the effect of conduction heat transfer decreases with the increase in Rayleigh number, so the solid volume fraction has a smaller effect on the thermal distribution as seen at Ra=10^6. On the other hand the addition of nanoparticles when the magnetic field is present, leads to a decrease in the maximum stream function as shown in fig. 9.

Fig. 11 presents the variation in the mean Nusselt number ratio ($Nu_m/Nu_{m,\phi=0}$) with the solid volume fraction at different values of Hartmann numbers. From this figure, it can be found that, for Ra=10^4 where conduction dominate the heat transfer mechanism, the mean Nusselt number ratio increases as the solid volume fraction increases. When the solid volume fraction increases from 0% to 5%, the Nusselt number increases by about 8%, 4.4% and 2.6% for Ha=0, Ha=30 and Ha=60, respectively. The results, which are presented for Ra=10^5 show that in the presence of the magnetic field, the mean Nusselt number ratio decreases as the solid volume fraction increases. This is because the magnetic field has a negative effect on buoyancy force and decreases the flow motion. It is observed that as solid volume fraction increases from 0% to 5%, the Nusselt number increases by about 6.7% for Ha=0 and decreases around 2.4% and 3.3% for Ha=30 and Ha=60, respectively. For Ra =10^6, the mean Nusselt number ratio increases as the solid volume fraction increases when magnetic field is not applied. At the lower Hartmann number Ha=30, the mean Nusselt number ratio is not sensitive to the increase of the solid volume fraction. But at the higher Hartmann number Ha = 60, the increase in the volume fraction leads to a decrease in the mean Nusselt number ratio.



Ra=10^6

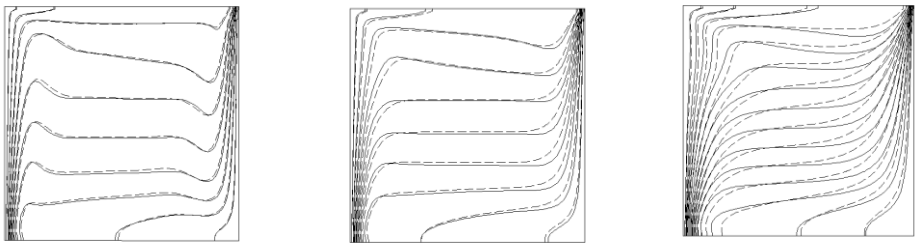


Fig. 10. Isotherms for different Rayleigh and Hartmann numbers, pure water (−) and nanofluid (- - - -) with ϕ=0.05

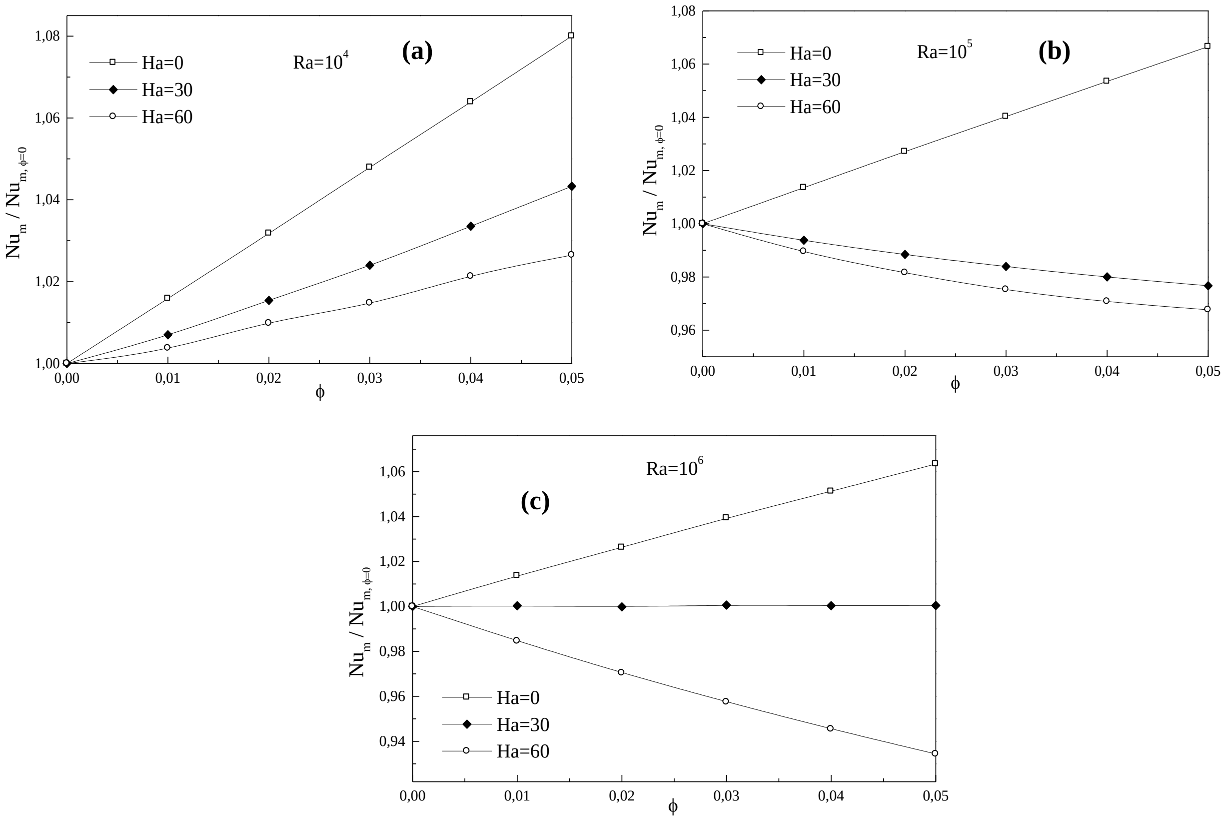


Figure 11. Variation of mean Nusselt number ratio with solid volume fraction at various Hartmann and Rayleigh numbers (a) Ra=104, (b) Ra=105, (c) Re=106

CONCLUSION

In this paper, a numerical investigation has been presented to study the mixed convection in a lid-driven cavity filled with water–silver nanofluid in the presence of magnetic field. The natural convection has been induced by subjecting the left vertical wall to the higher temperature than the right one .Whereas the top and bottom walls are insulated. The effects of Rayleigh number, Hartmann number and the solid volume fraction of nanoparticles on the fluid flow and heat transfer behavior are investigated. The study leads to the following conclusions:

- Presence of the magnetic field results in the reduction of the circulation strength and affects the isotherms patterns especially at the lower Rayleigh number.
- The heat transfer rate increases with an increase in the Rayleigh number but it decreases with an increase in the Hartmann number.
- The addition of nanoparticles augments the maximum stream function when the magnetic field is absent but the opposite behavior occurs when the magnetic field is applied.
- For all values of Hartmann number and Ra=10^4, the increase in the solid volume fraction augments the heat transfer rate. However, there are deterioration in heat transfer rate with increasing volume fraction for Ra >10^4 and Ha > 0.

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