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Length Biased Quasi-Sujatha Distribution with Properties and Applications to Bladder Cancer Data

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ABSTRACT

In this paper we have introduced a new version of Quasi sujatha distribution known as weighted Quasi sujatha distribution. The weighted quasi sujatha distribution has three parameters. The different structural properties of the newly model have been studied. The maximum likelihood estimators of the parameters and the Fisher's information matrix have been discussed. Finally a real life data set has been analysed, where it is observed that weighted Quasi Sujatha distribution has a better fit compared to Quasi sujatha distribution.

Keywords: Weighted distribution, Quasi Sujatha distribution, Reliability Analysis, Order statistics, Maximum likelihood Estimation.

INTRODUCTION

Fisher (1934) introduced the concept of weighted distributions to model the ascertainment bias. This concept was later on developed by Rao (1965) in a unified manner while modelling the statistical data when the standard distributions were not appropriate to record these observations with equal probabilities. Warren (1975) was the first to apply the weighted distributions in connection with sampling wood cells. Patil and Rao (1978) introduced the concept of size biased sampling and weighted distributions by identifying some of the situations where the underlying models retain their form. The statistical interpretation of weighted and size biased distributions was originally identified by Buckland and Cox (1964) in the context of renewal theory. As a result, weighted models were formulated in such situations to record the observations according to some weighted function. Different authors have reviewed and studied the various weighted probability models and illustrated their applications in different fields. Weighted distributions were applied in various research areas related to reliability, biomedicine, ecology and branching processes. For survival data analysis, Jing (2010) introduced the weighted inverse Weibull distribution and beta-inverse Weibull distribution as a new lifetime models. Ayesha, (2017) discussed the Size Biased Lindley Distribution as a new life time distribution and discussed its various statistical properties. Shanker & Shukla (2018) discussed a generalized size- biased Poisson-Lindley distribution and Its Applications to model size distribution of freelyforming small group. Recently, Rather and Subramanian (2018) discussed the characterization and estimation of length biased weighted generalized uniform distribution. Quasi sujatha distribution is a newly proposed life-time model formulated by shanker (2016) for several medical applications and calculated its important mathematical and statistical properties as its hazard function, Bonferroni and Lorenz curves, stress strength reliability, stochastic ordering and mean residual life functions. The newly proposed two parameters life time distribution called as Quasi sujatha distribution has better flexibility in handling lifetime data as compared to sujatha, exponential and lindley distributions.

Length Biased Quasi Sujatha (LBQS) Distribution

The probability density function of quasi sujathadistribution is given by

$$f(x;\theta,\alpha)=\frac{\theta^2}{\alpha\theta+\theta+2}\left(\alpha+\theta x+\theta x^2\right)e^{-\theta x};\quad x>0,\theta>0,\alpha>0 \tag{1}$$

and the cumulative distribution function of the quasi sujathadistribution is given by

$$F(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta x(\theta x + \theta + 2)}{\alpha\theta + \theta + 2}\right] e^{-\theta x}; x > 0, \theta > 0, \alpha > 0 \qquad (2)$$

Assume X is a non-negative random variable with probability density function $f(x)$ .Let $w(x)$ be the non-negative weight function, then, the probability density function of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x) f(x)}{E(w(x))}, \qquad x > 0.$$

Where $w(x)$ be the non - negative weight function and $E(w(x)) = \int w(x) f(x) dx < \infty.$

In this Paper, we have considered the length biased version of quasi sujatha distribution as *w(x) = x* to obtain the length biased quasi sujatha model. The probability density function of length biased quasi sujatha distribution is given as

$$f_l(x;\theta,\alpha) = \frac{x f(x;\theta,\alpha)}{E(x)}$$

$$f_l(x;\theta,\alpha) = \frac{x\theta(\alpha + \theta x + \theta x^2) e^{-\theta x}}{(\alpha + 2\theta^4 + 6\theta^3)}, x > 0, \theta > 0, \alpha > 0 \qquad (3)$$

Where $E(x) = \dfrac{\alpha\theta^2 + 2\theta^4 + 6\theta^3}{\theta(\alpha\theta + \theta + 2)}$

The corresponding cdf of length biased Quasi sujatha distribution is obtained as

$$F_l(x;\theta,\alpha) = \int_0 f_l(x;\theta,\alpha) dx$$

$$F_l(x;\theta,\alpha) = \int_0^x \frac{x\theta(\alpha + \theta x + \theta x^2) e^{-\theta x}}{(\alpha + 2\theta^4 + 6\theta^3)} dx$$

$$F_l(x;\theta,\alpha) = \frac{1}{(\alpha + 2\theta^4 + 6\theta^3)} \int_0^x x\theta(\alpha + \theta x + \theta x^2) e^{-\theta x} dx$$

After simplification, we obtain cdf of weighted Quasi Sujatha distribution as

$$F_l(x;\theta,\alpha) = \frac{1}{(\alpha\theta^4 + 2\theta^8 + 6\theta^7)} (\alpha\theta^3 \gamma(2,\theta x) + \theta^3 \gamma(3,\theta x) + \theta^2 \gamma(4,\theta x)), \quad x > 0, \theta > 0, \alpha > 0 \qquad (4)$$

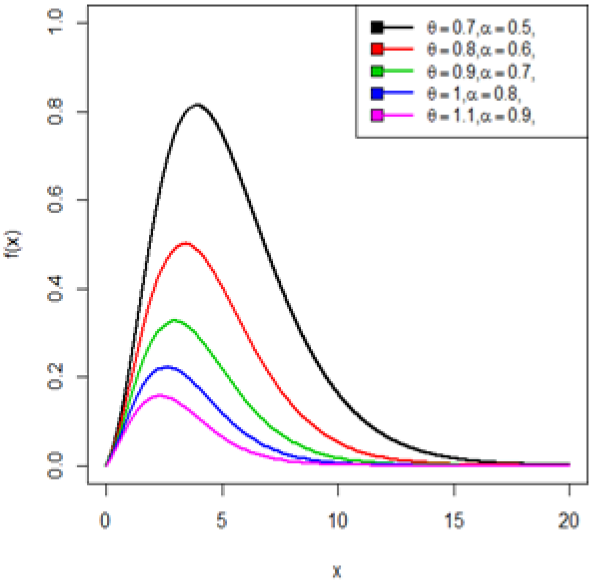

Fig.1:Pdf plot of LBQS distribution

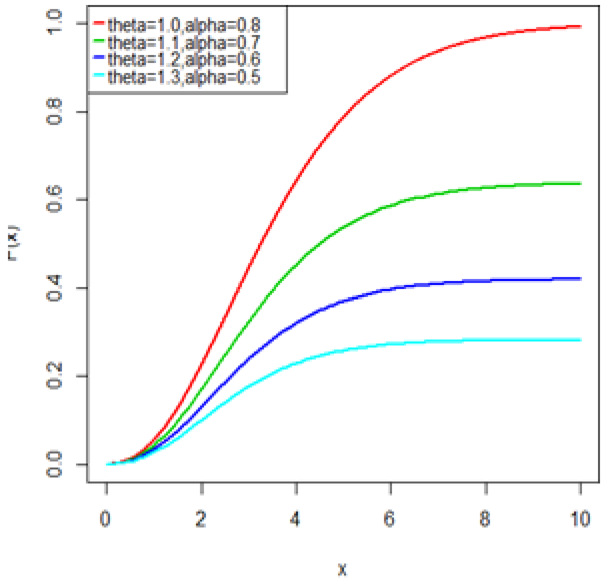

Fig.2 Cdf plot of LBQS distribution

Reliability, Hazard and Reverse Hazard Functions

In this section, we have obtained the Reliability, hazard rate, Reverse hazard rate functionand the Mills Ratio of the Proposed Weighted Quasi Sujathadistribution.

Reliability function R(x)

The reliability function is defined as the probability that a system survives beyond a specified time. It is also known as survival or survivor function.it can be computed as complement of the cumulative distribution function of the model. The reliability function or the survival function of length biased Quasi Sujatha distribution is computed as:

$$R(x;\theta,\alpha)=1-F_l(x;\theta,\alpha)$$

$$R(x;\theta,\alpha)=1-\frac{(\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x))}{(\alpha\theta^4+2\theta^8+6\theta^7)}$$

Hazard function

The hazard function is also known as hazard rate defined as the instantaneous failure rate orforce of mortality and is given as:

$$H.R=h(x;\theta,\alpha)=\frac{f_l(x;\theta,\alpha)}{R_W(x;\theta,\alpha)}$$

$$H.R=\frac{x3\theta^5(\alpha+\theta x+\theta x^2)}{(\alpha\theta^4+2\theta^8+6\theta^7-(\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x))}$$

Reverse hazard function

The reverse hazard function of LBQS distribution is given as

$$R.H.R=h^r(x;\theta,\alpha)=\frac{f_l(x;\theta,\alpha)}{F_l(x;\theta,\alpha)}$$

$$R.H.R=h^r(x;\theta,\alpha)=\frac{x3\theta^5(\alpha+\theta x+\theta x^2)e^{-\theta x}}{(\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x))}$$

Mills Ratio

$$\text{Mills Ratio} = \frac{1}{h^r(x;\theta,\alpha)} = \frac{(\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x))}{x3\theta^5(\alpha+\theta x+\theta x^2)e^{-\theta x}}$$

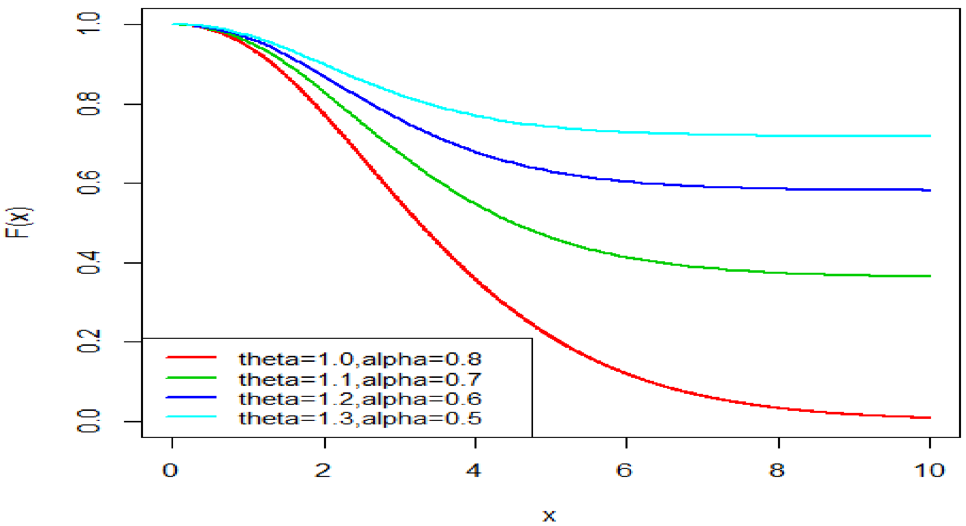

Fig.3 Reliabilty plot of LBQS distribution

Moments and Related Measures of LBQS Distribution

In this Portion, we obtain the different Statistical properties of length biased Quasi Sujatha distribution. Especially moments, Harmonic mean, moment generating function and characteristics function

Moments

Let X denotes the random variable of length biased Quasi Sujatha distribution with parameters θ and α then

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \frac{x^{r+1}\theta(\alpha+\theta x+\theta x^2)e^{-\theta x}}{(\alpha+2\theta^4+6\theta^3)}dx$$

$$= \frac{\theta}{(\alpha+2\theta^4+6\theta^3)}\int_0^\infty x^{r+1}(\alpha+\theta x+\theta x^2)e^{-\theta x}dx$$

$$= \frac{\theta}{(\alpha+2\theta^4+6\theta^3)}\left(\alpha\int_0^\infty x^{(r+2)-1}e^{-\theta x}dx+\theta\int_0^\infty x^{(r+3)-1}e^{-\theta x}dx+\theta\int_0^\infty x^{(r+4)-1}e^{-\theta x}dx\right)$$

$$E(X^r) = \mu_r' = \frac{\alpha\theta\Gamma(r+2)+\theta\Gamma(r+3)+\theta\Gamma(r+4)}{(\alpha\theta^{r+2}+2\theta^{r+6}+6\theta^{r+5})}$$

(5)

For r = 1,2 in equation (5)we get mean and second moments of length Quasi Sujathadistribution.

$$E(X) = \mu_1' = \frac{2\alpha\theta + 6\theta + 24\theta}{(\alpha\theta^3 + 2\theta^7 + 6\theta^6)}$$

$$E(X^2) = \mu_2' = \frac{6\alpha\theta + 24\theta + 120\theta}{(\alpha\theta^4 + 2\theta^8 + 6\theta^7)}$$

$$\text{Variance } (\mu_2) = \mu_2' - (\mu_1')^2$$

$$\text{Variance } (\mu_2) = \frac{6\alpha\theta + 24\theta + 120\theta}{(\alpha\theta^4 + 2\theta^8 + 6\theta^7)} - \left(\frac{2\alpha\theta + 6\theta + 24\theta}{(\alpha\theta^3 + 2\theta^7 + 6\theta^6)}\right)^2$$

$$S.D(\sigma) = \sqrt{\left(\frac{6\alpha\theta + 24\theta + 120\theta}{(\alpha\theta^4 + 2\theta^8 + 6\theta^7)} - \frac{(2\alpha\theta + 6\theta + 24\theta)^2}{(\alpha\theta^3 + 2\theta^7 + 6\theta^6)^2}\right)}$$

Harmonic mean

The Harmonic mean is the reciprocal of the arithmetic mean of the reciprocals. Theharmonic mean for the proposed length biased Quasi Sujatha distribution is obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \frac{\theta(\alpha + \theta x + \theta x^2)e^{-\theta x}}{(\alpha + 2\theta^4 + 6\theta^3)} dx$$

$$= \frac{\theta}{(\alpha + 2\theta^4 + 6\theta^3)}\left(\alpha\int_0^\infty e^{-\theta x} dx + \theta\int_0^\infty x e^{-\theta x} dx + \theta\int_0^\infty x^2 e^{-\theta x} dx\right)$$

$$= \frac{\theta}{(\alpha + 2\theta^4 + 6\theta^3)}\left(\alpha\int_0^\infty e^{-\theta x} x^{2-2} dx + \theta\int_0^\infty e^{-\theta x} x^{2-1} dx + \theta\int_0^\infty e^{-\theta x} x^{3-1} dx\right)$$

$$\Rightarrow H.M = \frac{\theta}{(\alpha + 2\theta^4 + 6\theta^3)}(\alpha\gamma(2,\theta x) + \theta\gamma(2,\theta x) + \theta\gamma(3,\theta x))$$

Moment Generating Function and Characteristics Function

In this sub section we derive the moment generating function and the Characteristics function of LBQS distribution.The moment generating function is the expectation of a function of the random variable We begin with the well-known definition of the moment generating function given by

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f_l(x;\theta,\alpha,)dx$$

$$= \int_0^\infty \left[1 + tx + \frac{(tx)^2}{2!} + ...\right] f_W(x;\theta,\alpha)dx$$

$$= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_l(x;\theta,\alpha)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!} \mu_j'$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!} \left[\frac{\alpha\theta\,\Gamma(j+2) + \theta\,\Gamma(j+3) + \theta\,\Gamma(j+4)}{(\alpha\theta^{r+2} + 2\theta^{r+6} + 6\theta^{r+5})}\right]$$

$$M_X(t) = \frac{1}{(\alpha\theta^{r+2} + 2\theta^{r+6} + 6\theta^{r+5})} \sum_{j=0}^\infty \frac{t^j}{j!} (\alpha\theta\,\Gamma(j+2) + \theta\,\Gamma(j+3) + \theta\,\Gamma(j+4))$$

Characteristic function

In probability theory and statistics characteristic function is defined as the function of any real-valued random variable completely defines the probability distribution of a random variable.The characteristic function exists always even when there is no moment generating function.

$$\varphi_x(t) = M_X(it)$$

$$M_X(it) = \frac{1}{(\alpha\theta^{r+2} + 2\theta^{r+6} + 6\theta^{r+5})} \sum_{j=0}^\infty \frac{(it)^j}{j!} (\alpha\theta\,\Gamma(j+2) + \theta\,\Gamma(j+3) + \theta\,\Gamma(j+4))$$

Order Statistics

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denotes the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a Continuous distribution with cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)(F_X(x))^{r-1}(1-F_X(x))^{n-r}, \quad r = 1,2,3,\ldots\ldots\ldots, n$$

Using (3) and (4) the probability density function of the r^{th} order statistics of length biasedQuasi Sujatha distribution is given by

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{x\theta(\alpha+\theta x+\theta x^2)e^{-\theta x}}{(\alpha+2\theta^4+6\theta^3)}\right)$$
$$\times\left(\frac{\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x)}{(\alpha\theta^4+2\theta^8+6\theta^7)}\right)^{r-1}$$
$$\times\left(1-\frac{\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x)}{(\alpha\theta^4+2\theta^8+6\theta^7)}\right)^{n-r} \quad (6)$$

The pdf of the ist order $X(1)$ of LBQS distribution is given by

$$f^{w}_{x(1)}(x) = \frac{nx\theta(\alpha+\theta x+\theta x^2)e^{-\theta x}}{(\alpha+2\theta^4+6\theta^3)}\left(1-\frac{\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x)}{(\alpha\theta^4+2\theta^8+6\theta^7)}\right)^{n-1}$$

and the pdf of the nth order $X(n)$ of LBQS distribution is given by

$$f^{w}_{x(n)}(x) = \frac{nx\theta(\alpha+\theta x+\theta x^2)e^{-\theta x}}{(\alpha+2\theta^4+6\theta^3)}\left(\frac{\alpha\theta^3\gamma(2,\theta x)+\theta^3\gamma(3,\theta x)+\theta^2\gamma(4,\theta x)}{(\alpha\theta^4+2\theta^8+6\theta^7)}\right)^{n-1}$$

Likelihood Ratio Test

Let $X_1, X_2, X_3 \ldots\ldots X_n$ be a random sample from the Quasi Sujatha distribution or lengthbiased Quasi Sujatha distribution. We test the hypothesis

$H_o : f(x) = f(x;\theta,\alpha)$ against $H_1 : f(x) = f_w(x;\theta,\alpha,c)$

For testing whether the random sample of size n comes from the Quasi Sujatha distributionor length biased Quasi Sujatha distribution, the following test statistic is used

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\frac{f_l(x;\theta,\alpha)}{f(x;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\left[\frac{x_i\theta(\alpha\theta+\theta+2)}{(\alpha\theta^2+2\theta^4+6\theta^3)}\right]$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\theta(\alpha\theta+\theta+2)}{(\alpha\theta^2+2\theta^4+6\theta^3)}\right)^n \prod_{i=1}^{n} x_i$$

We reject the null hypothesis if

$$\Delta = \left(\frac{\theta(\alpha\theta + \theta + 2)}{(\alpha\theta^2 + 2\theta^4 + 6\theta^3)} \right)^n \prod_{i=1}^{n} x_i > k$$

Equivalently, we reject the null hypothesis

$$\Delta^* = \prod_{i=1}^{n} x_i > k \left(\frac{(\alpha\theta^2 + 2\theta^4 + 6\theta^3)}{\theta(\alpha\theta + \theta + 2)} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i > k^*, \text{Where } k^* = k \left(\frac{(\alpha\theta^2 + 2\theta^4 + 6\theta^3)}{\theta(\alpha\theta + \theta + 2)} \right)^n$$

For a large sample of size *n*, *2log Δ* is distributed as chi-square distribution with one degree of freedom and also p- value is obtained from the chi-square distribution. Also, we reject the null hypothesis, when the probability value is given by

$p\left(\Delta^* > \alpha^*\right)$, Where $\alpha^* = \prod_{i=1}^{n} x_i$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i$ is the observed value of the statistic Δ^*.

Bonferroni and Lorenz Curves

The Bonferroni and the Lorenz curves have assumed relief not only in economics to study the distribution of income and wealth or income and poverty, but also being used in other fields like reliability, medicine, insurance and demography. The bonferroni and Lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1'} \int_0^q x f_l(x; \theta, \alpha) dx$$

and $L(p) = pB(p) = \frac{1}{\mu_1'} \int_0^q x f_l(x; \theta, \alpha) dx$

Where $\mu_1' = E(X) = \frac{2\alpha\theta + 6\theta + 24\theta}{(\alpha\theta^3 + 2\theta^7 + 6\theta^6)}$ and $q = F^{-1}(p)$

$$B(p) = \frac{(\alpha\theta^3 + 2\theta^7 + 6\theta^6)}{p(2\alpha\theta + 6\theta + 24\theta)} \int_0^q \frac{\theta x^2 (\alpha + \theta x + \theta x^2) e^{-\theta x}}{(\alpha + 2\theta^4 + 6\theta^3)} dx$$

$$B(p) = \frac{3\theta^3}{p(2\alpha\theta + 6\theta + 24\theta)} \int_0^q \theta x^2 (\alpha + \theta x + \theta x^2) e^{-\theta x} dx$$

$$B(p)=\frac{3\theta^{3}}{p(2\alpha\theta+6\theta+24\theta)}\left(\alpha\theta\int_{0}^{q}e^{-\theta x}x^{3-1}dx+\theta^{2}\int_{0}^{q}e^{-\theta x}x^{4-1}dx+\theta^{2}\int_{0}^{q}e^{-\theta x}x^{5-1}dx\right)$$

$$B(p)=\frac{3\theta^{3}}{p(2\alpha\theta+6\theta+24\theta)}\left(\alpha\theta\gamma(3,\theta q)+\theta^{2}\gamma(4,\theta q)+\theta^{2}\gamma(5,\theta q)\right)$$

$$L(p)=pB(p)=\frac{3\theta^{3}}{(2\alpha\theta+6\theta+24\theta)}\left(\alpha\theta\gamma(3,\theta q)+\theta^{2}\gamma(4,\theta q)+\theta^{2}\gamma(5,\theta q)\right)$$

Maximum Likelihood Method and Fisher's Information Matrix

Maximum likelihood estimation has been the most widely used method for estimating the parameters of the weighted Quasi Sujatha distribution. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from the weighted quasi Sujatha distribution, then the corresponding likelihood function is given as

$$L(x;\theta,\alpha)=\prod_{i=1}^{n} f_l(x;\theta,\alpha)$$

$$L(x;\theta,\alpha)=\prod_{i=1}^{n}\left[\frac{x_i\theta(\alpha+\theta x_i+\theta x_i^{2})e^{-\theta x_i}}{(\alpha+2\theta^{4}+6\theta^{3})}\right]$$

$$L(x;\theta,\alpha)=\frac{\theta^{n}}{(\alpha+2\theta^{4}+6\theta^{3})^{n}}\prod_{i=1}^{n}\left[x_i(\alpha+\theta x_i+\theta x_i^{2})e^{-\theta x_i}\right]$$

The log likelihood function is given by:

$$\log L=n\log\theta-n\log(\alpha+2\theta^{4}+6\theta^{3})+\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\log(\alpha+\theta x_i+\theta x_i^{2})-\theta\sum_{i=1}^{n}x_i \quad (7)$$

Differentiate the log likelihood equation (7) with respect to parameters. We obtain the normal equation

$$\frac{\partial\log L}{\partial\theta}=\frac{n}{\theta}-n\left(\frac{8\theta^{3}+18\theta^{2}}{(\alpha+2\theta^{4}+6\theta^{3})}\right)+\sum_{i=1}^{n}\frac{(x_i+x_i^{2})}{(\alpha+\theta x_i+\theta x_i^{2})}-\sum_{i=1}^{n}x_i=0$$

$$\frac{\partial\log L}{\partial\alpha}=-\sum^{n}\left(\frac{1}{(\alpha+2\theta^{4}+6\theta^{3})}\right)+\sum^{n}\left(\frac{1}{\ldots}\right)$$

$= 0$

$\partial\alpha \quad {}_{i=1} (\alpha + 2\theta \quad {}^{2} \quad {}_{i} \quad {}_{l}$

Where ψ (.) is the digamma function.

Because of the complicated form of likelihood equations, algebraically it is very difficult tosolve the system of nonlinear equations. Therefore we use R and wolfram mathematics for estimatingthe required parameters.
To obtain confidence interval we use the asymptotic normality results. We have that, if $\lambda = (\hat{\theta}, \hat{\alpha})$ denotes the MLE of $\lambda = (\theta, \alpha)$, we can state the result as follows

$$\sqrt{n}(\hat{\eta} - \eta) \rightarrow N_2\,(0, I^{-1}(\eta))$$

Where $I^{-1}(\eta)$ is the Fisher information Matrix

$$I(\eta) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial\theta\partial\alpha}\right) \\ E\left(\frac{\partial^2 \log L}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) \end{pmatrix}$$

$$I_{11} = E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right) = -\frac{n}{\theta^2} - n\left(\frac{(24\theta^2 + 36\theta)(\alpha + 2\theta^4 + 6\theta^3) - (8\theta^3 + 18\theta^2)(8\theta^3 + 18\theta^2)}{(\alpha + 2\theta^4 + 6\theta^3)^2}\right)$$

$$-\frac{\sum_{i=1}^{n}(x_i^2 + x_i^2)}{(\alpha + \theta x_i + \theta x_i^2)^2}$$

$$I_{22} = E\left(\frac{\partial^2 \log L}{\partial\alpha^2}\right) = \sum_{i=1}^{n}\left(\left(\frac{1}{(\alpha + 2\theta^4 + 6\theta^3)^2}\right)\right) - \sum_{i=1}^{n}\left(\frac{1}{(\alpha + \theta x_i + \theta x_i^2)^2}\right)$$

Also,

$$\left(\frac{\partial^2 \log L}{}\right) \quad \left(\frac{\partial^2 \log L}{}\right) \quad \left((8\theta^3 + 18\theta^2) \right)$$

$I_{12} = I_{21} = E|$ Where $\psi(.)', \psi(.)''$ and $\psi'''(.)$ is the first, secondand third order derivatives of digamma function.

Since η being unknown, we estimate $I^{-1}(\eta)$ confidenceintervals for θ, α and c.by $I^{-1}(\hat{\eta})$ and this can be used to obtain asymptotic

Applications

In this section, we use a real-life data set to show that the length biased quasi sujatha distribution can be a better model than the quasi Sujatha, lindley and exponential distribution. We cosider a data set represents the survival times (in months) of patients suffering from melanoma studied by susarla and Vanryzin (1978). The data set is given below as

Data set: The following data represent the Survival times (in months) of patients of melanoma studied by Susarla and Vanryzin (1978) .The data set is given below in table 1 as

Table. 1 Data regarding Survival times (in months) studied by Susarla and Vanryzin

3.25	3.50	4.75	4.75	5.00	5.25
5.75	5.75	6.25	6.50	6.50	6.75
6.75	7.78	8.00	8.50	8.50	9.25
9.50	9.50	10.00	11.50	12.50	13.25
13.50	14.25	14.50	14.75	15.00	16.25
16.25	16.50	17.50	21.75	22.50	24.50
25.50	25.75	27.50	29.50	31.00	32.50
34.00	34.50	35.25	58.50		

In order to compare the length biased biased quasi Sujatha distribution with quasi sujatha, quasi shanker and new quasi lindley distribution, we are using the Criterion like AIC (Akaike information criterion), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion) .The better distribution corresponds to lesser AIC, AICC and BIC values.

$\text{AIC} = 2k - 2\log L$ $\quad$ $\text{AICC} = AIC + \frac{2k(k+1)}{n-k-1}$ and $\quad$ $\text{BIC} = k\log n - 2\log L$

where k is the number of parameters in the statistical model, n is the sample size and -2log$L$ is the maximized value of the log-likelihood function under the considered model. From table 2, it has been observed that the Weighted quasi Sujathadistribution have the lesser AIC, AICC, -2log$L$ and BIC values as compared to quasi Sujatha distribution. Hence we canconclude that the weighted quasi sujathadistribution leads to a better fit than quasi sujatha distribution.

Table.2 Performance of distribution

Distribution	MLE	SE	-2logL	AIC	BIC	AICC
Length biased Quasi Sujatha	$\hat{\alpha} = 0.0010$ $\hat{\theta} = 0.1915$	$\hat{\alpha} = 0.0012$ $\hat{\theta} = 0.0141$	221.7815	225.7815	229.4388	226.0605

Quasi sujatha	$\hat{\alpha} = 0.0010$ $\hat{\theta} = 0.1861$	$\hat{\alpha} = 0.0131$ $\hat{\theta} = 0.1312$	333.476	337.476	341.133	337.7550
Exponential	$\hat{\theta} = 0.0638$	$\hat{\theta} = 0.0094$	345.0919	347.0919	348.9205	347.180
lindley	$\hat{\theta} = 0.1208$	$\hat{\theta} = 0.0126$	333.699	335.699	337.5279	335.789

CONCLUSION

In the present study, we have introduced a new generalization of the Quasi sujatha distribution namely as Weighted Quasi Sujatha distribution with three parameters. The subject distribution is generated by using the weighted technique and the parameters have been obtained by using maximum likelihood estimator. Some mathematical properties along with reliability measures are discussed. The new distribution with its applications in real life-time data has been demonstrated. Finally the results are compared over Sujatha distribution and have been found that weighted Quasi Sujatha distribution provides better fit than quasi Sujatha distribution.

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