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Chaotic behaviour of multiparticle production in relativistic heavy ion collisions

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Abstract: An attempt has been made to understand the chaotic behaviour of multiparticle production in relativistic heavy ion collisions. For this goal first we have measured the distributions of Scaled Factorial Moments (SFMs) and found a scaling behaviour, which supported to chaoticity or spatial fluctuations in relativistic heavy-ion collisions at high energies. Finally, the values of entropy indices, μ_q are calculated which indicate the chaotic nature of multiparticle production system with a specific self-similar structure.

Keywords: Fractality and Chaoticity, Nuclear emulsions experiment and Global features in relativistic heavy ion collisions.

Introduction

The ultimate aim of relativistic heavy ion experiments at AGS, CERN SPS and relativistic heavy ion collider, RHIC, at Brookhaven National Laboratory is to provide an opportunity to investigate strongly interacting matter at energy densities unprecedented in a laboratory, which ultimately gives an evidence for the quark-gluon plasma (QGP) formation. The QGP is a state of matter in which quarks and gluons are no longer confined to volumes of hadronic dimensions. In deep inelastic scattering experiments, it has already been revealed that quarks at very short distances move freely, which is referred to as the asymptotic freedom. Quantum Chromodynamics (QCD) describes the strong interactions of quarks and gluons. A Variety of possible signatures for the existence of a deconfined state of matter in nucleus-nucleus (A-A) collision have been proposed theoretically and also studied experimentally by various workers [1], [2]. The experimental observation of large rapidity fluctuations [3] has provided interest and excitement about their nature and origin. Bialas and Peschanski [4] have suggested that a power law scaling behaviour of normalized SFMs $(<F_q> \propto M^{\alpha_q})$ on the bin size and described the phenomenon as "intermittency", a term coined from hydrodynamic turbulence [5]. The SFMs method cannot only predicts the existence of large non-statistical fluctuations but it could also investigate the pattern of fluctuations and their origin.

It is generally believed that through the heavy ion collisions at ultra-relativistic energies big systems with very high energy density [6] might be produced. In these systems novel phenomena, such as colour deconfinement [7], chiral-symmetry restoration [8], discrete-symmetry spontaneous-breaking [9], etc., are expected to be present and different events might be governed by different dynamics. With this goal in mind, the event-by-event (E-by-E) study of high-energy collisions has attracted more and more attention [10]. As it is already stated before that, the power law dependence of SFMs referred to as the intermittency [4], [5] has been extensively used to investigate fluctuations and chaos in multiparticle production in high-energy hadronic and heavy-ion nucleus-nucleus collisions [11], [12]. On the basis of E-by-E the values of scaled factorial moments, F_q^e, are envisaged to help disentangle some interesting and very much useful informations about the chaotic behaviour of multiparticle production. A few moments of F_q^e distribution, for example, the normalized moments $C_{p,q}$ are likely to serve the purpose. If $C_{p,q}$ shows a power law behaviour then such behaviour is referred to as erraticity [13], [14]. It may be stressed that erraticity analysis would like into account simultaneously the spatial as well as the E-by-E fluctuations beyond the intermittency. Studies involving erratic fluctuations in hadronic and heavy-ion collisions, carried out so far [15], [16] are not conclusive. It was, therefore, considered worthwhile to examine erraticity behaviour in relativistic nucleus-nucleus collisions. Attention is focused on the behaviour of erraticity exponents and erraticity spectrum, which are likely to provide maximum informations on self-similar fluctuations [13], [14]. Hence in the present work an exercise has been made to perform the study of (E-by-E) spatial fluctuations of relativistic shower particles produced in the collisions of ^{28}Si+Em at energy14.6A GeV in 1-D and 2-D phase spaces of X -variable. The findings are compared with the predictions of Ultra-relativistic Quantum Molecular Dynamics (UrQMD) model [17], [18].

Experimental Detail

In the present experiment, FUJI nuclear emulsion pellicles were irradiated horizontally with a beam of ^{28}Si nuclei at 14.6A GeV at Alternating Gradient Synchrophasotron (AGS) of Brookhaven National Laboratory (BNL), NewYork, USA have been used. The method of line scanning has been adopted to scan the stacks, which was carried out carefully using Japan made NIKON (LABOPHOT and Tc-BIOPHOT) high-resolution microscopes with 8 cm movable stage using 40X objectives and 10X eyepieces by two independent observers, so that the bias in the detection, counting and measurements can be minimized. The interactions due to beam tracks making an angle < 2° to the mean direction and lying in emulsion at depths > 35 μm from either surface of the pellicles were included in the final statistics. The other relevant details about the present experiments and target identifications may be seen in our previous work [19]-[22]. Using the technique of erraticity moments, $C_{p,q}$, this analysis has been taken out for three samples of total data of 951 events to understand the dependence of the erratic behaviour on the mean multiplicity of relativistic shower particles. For this purpose all the necessary mathematical tools regarding the erraticity moments, we will be explain in the next section.

A. Classification of Tracks

All charged secondaries in these events were classified, in accordance with the emulsion terminology, into the following groups [23]:

(i) Black track producing particles (N_b):

Tracks with specific ionization $g^* > 10$ ($g^* = g/g_0$, where g_0 is the Plateau ionization of a relativistic singly charged particle and g is the ionization of the charged secondary) have been taken as black tracks. These correspond to protons of relative velocity $\beta < 0.3$ and range in emulsion $L < 3.0$ mm.

(ii) Grey track producing particles (N_g):

Tracks with specific ionization $1.4 \le g^* \le 10$ corresponding to protons with velocity in the interval $0.3 \le \beta \le 0.7$ and range $L \ge 3.0$ mm in nuclear emulsion are called grey tracks.

(iii) Shower tracks producing particles (N_s):

Tracks with specific ionization $g^* < 1.4$ corresponding to protons with relative velocity $\beta > 0.7$ are classified as shower tracks. These tracks are mostly due to relativistic pions with small admixture of charged K-mesons and fast protons.

In order to eliminate all the possible backgrounds due to γ overlap (where a γ from a π^0 decay converts into $e^+ e^-$ pair) close to shower tracks near vertex, special care was taken to exclude such $e^+ e^-$ pairs from the primary shower tracks while performing angular measurements. Usually all shower tracks in the forward direction were followed more than 100 - 200μm from the interaction vertex for angular measurement. The tracks due to $e^+ e^-$ pair can be easily recognized from the grain density measurement, which is initially much larger than the grain density of a single charged pions or proton track. It may also be mentioned that the tracks of an electron and positron when followed downstream in nuclear emulsion showed considerable amount of Coulomb scattering as compared to the energetic charged pions. Such $e^+ e^-$ pairs were eliminated from the data.

Mathematical Approach

In order to perform a meaningful analysis of chaoticity, normalized cumulative variables, $X(\eta)$ and $X(\phi)$ were used to reduce the effect of non-uniformity in single charged particle distributions. In terms of new scaled variables, $X(\eta)$ and $X(\phi)$, the single particle density distribution is always uniform in between X = 0 and 1 and both "vertical" and "horizontal" averaging of scaled factorial moments should produce the same result.
The cumulative variable in the phase space (say η) is defined as [24].

$$X(\eta) = \int_{\eta_{\min}}^{\eta} \rho(\eta')\,d\eta' \Big/ \int_{\eta_{\min}}^{\eta_{\max}} \rho(\eta')\,d\eta' \qquad (1)$$

where, $\rho(\eta)$ is the single particle pseudorapidity density of shower particles and $\eta_{\min}$ ($\eta_{\max}$) is the minimum (maximum) value of η. Similar relation as Eq. (1) was used to calculate $X(\phi)$. Though our entire analysis on scaled factorial moments will henceforth be performed taking $X_\eta(X_\phi)$ as the basic variable, we shall continue to call the corresponding space $\eta(\phi)$ -space.

Various experimental efforts have established the existence of the empirical phenomenon of “intermittency” in multiparticle production using normalized scaled factorial moments. On the basis of bin averaging the normalized scaled factorial moments of the order of q is defined in vertical form as:

$$F_q^V(\delta\eta) = \frac{1}{M^d}\sum_{m=1}^{M^d}\frac{< n_m^q >}{< n_m >^q} \quad (2)$$

and its horizontal form is defined as:

$$F_q^H(\delta\eta) = < \frac{1/M^d \sum_{m=1}^{M^d} n_m^q}{(< n > / M^d)^q} > \quad (3)$$

where, $n_m^q = n_m(n_m - 1).....(n_m - q + 1)$, and also bracket $< .... >$ of Eq. (3) indicates the average over all events in the whole data sample.

Using the normalized scaled factorial moments, $< F_q >$ an increasing trend in non-statistical self-similar fluctuations with decreasing bin size is representation of an intermittent behaviour, which leads to a power law expressed by:

$$\left.\begin{array}{l} F_q(\delta X) \propto \delta X^{-\alpha_q} \;\; (\delta X \to 0) \\ \text{or } F_q(\delta X) \propto \delta M \;\; (M \to 0) \end{array}\right\} \quad (4)$$

where, α_q is the intermittency exponents, and δX is bin size, which is defined as: $\delta X = \Delta / M$ or $\delta X = (X(y)_{max} - X(y)_{min})/M$.

This analysis in a single phase-space dimension in η and ϕ spaces respectively was extended to two dimensions $(\eta\phi)$ -space. In order to use above formulism in two dimensions, a rectangle in the $(\eta\phi)$-space was considered, which was divided into $M_{\eta\phi} = M_\eta \times M_\phi$ bins of each size $\delta\eta\delta\phi = (\Delta\eta / M_\eta)(\Delta\phi / M_\phi)$ with $M_\eta = M_\phi$, where the sum now extends over M^2 bins in Eqs. (3-4) and n_m is the number of particles in the m^{th} bin in the $(\eta\phi)$-space. The pseudorapidity interval, $\Delta\eta$, is divided into M bins of uniform width $\delta\eta = \Delta\eta = \{X(\eta_{\max}) - X(\eta_{\min})\}/M$.

Recently, Cao and Hwa [13] first introduced to measure the spatial pattern of particles in an event using normalized factorial moments associated with it. In contrast to the horizontally averaged vertical moments, F_q^V and vertically averaged horizontal moments, F_q^H of the qth order, they define event factorial moments as:

$$F_q^{(e)} = \left[\frac{1}{M}\sum_{m=1}^{M} n_m(n_m - 1).......(n_m - q + 1)\right] \times \left(\frac{1}{M}\sum_{m=1}^{M} n_m\right)^{-q} \quad (5)$$

where, M is the partition number in phase space, n_m is the number of shower tracks producing particles falling into the mth bin and q = 2.3.4…. is the order of the moment.

The event factorial moments, F_q^e, fluctuates from event-to-event, and the degree of fluctuation can be estimated from the probability distribution $P(F_q^e)$ over all events. One can obtain a distribution $P(F_q^e)$ for the whole sample of events. In the given situation, a normalized factorial moment of a single event is defined as:

$$\phi_q(M) = \frac{F_q^e(M)}{< F_q^e(M) >} \quad (6)$$

and

$$< F_q^e(M) > = \frac{1}{N_{ev}}\sum_{e=1}^{N_{ev}} F_q^e(M) \quad (7)$$

where, N_{ev} is the number of events in a sample and $F_q^e(M)$ represents the event factorial moment describing the spatial pattern of an event. It is important to mention that the SFMs introduced to study the intermittency or fractality in multiparticle production is only an estimate of the mean of the distribution $P(F_q^e)$. It should be realized that the simple mean procedure, apart from its clear advantages, suppresses a lot of important information about the fluctuations of spatial patterns of final state of multiparticle production. In particular, some interesting effects present only in a part of sample of events produced in high-energy collisions, may be lost. A possible example of this kind is the quark-gluon plasma. In order to quantify the degree of the fluctuations, a new normalized moment related to the chaotic nature of the system is defined as [13], [14], [16], [25]:

$$C_{p,q}(M) = < \phi_q^p(M) > \quad = \frac{1}{N_{ev}} \sum_{e=1}^{N_{ev}} \phi_q^p(M) \qquad (8)$$

where, p is any positive real number, it should not be negative, $F_q^e(M)$ may vanish for some events, if p is negative. If $C_{p,q}(M)$ exhibits a power law dependence on the number of bins M as:

$$C_{p,q}(M) \;\propto\; (M)^{\psi_q(p)}, \qquad M \;\to \infty \qquad (9)$$

Then, the phenomenon is referred to as erraticity of non-statistical fluctuations and $\psi_q(p)$ is called the erraticity exponent and is obtained from the slope of graph plotted between $C_{p,q}(M)$ vs. $\ln M$. The information contained in the scaling function $C_{p,q}(M)$ can be alternatively displayed through the entropy index, μ_q, which is given by [26], [27]:

$$\mu_q = \frac{d}{dp}\psi_q(p)\Big|_{p=1} \qquad (10)$$

The derivative of $\psi_q(p)$ at $p=1$ also describes the width of the fluctuation. It has been shown by Z. Cao et al., [26], [27] that the entropy index, μ_q, can be used as a measure of chaoticity in the systems, where only the spatial patterns could be observed and the presence of chaos in the system could be experienced for positive value of μ_q ($\mu_q > 0$). The new parameter which is related to μ_q, defined in the event space and is also known to the entropy as given:

$$S'_q = \ln(N_{ev} M^{-\mu_q}) \qquad (11)$$

where, N_{ev} is the number of events. Eq. (11) tells us that on increasing the value of entropy index, μ_q, i.e., the event-by-event fluctuations of the scaled factorial moments, the values of S'_q will decrease. For better understanding of this postulate, Hwa [13], [16], [26], [27] gave an illustrative example. One can consider two extreme cases: (a) if F_q^e is the same for every event, then $S'_q = \ln N_{ev}$; (b) if only one event has $F_q^e \neq 0$, and $F_q^e = 0$ in all others, then $S'_q = 0$. Thus, case (b) is more ordered in the event space than (a), that is, it is more disordered to spread out an observable (F_q^e in this case) over all events than to confine it to a few events having non-zero values (analogous to the increase of entropy of an expanding gas). Thus, S'_q decreases when there is more events with $F_q^e = 0$, signifying more order in the event space. From Eq. (10), it is now obvious thus μ_q is a measure of that decrease which in turn implies more fluctuation in F_q^e.

Analysis and Results

A. Frequency Distribution of Single Event Factorial Moments

The frequency distributions of single event normalized scaled factorial moments, F_2^e has been shown in Fig. 1 (a-c) in η, ϕ and $\eta\phi$ - phase spaces respectively. The above calculation has been performed for the number of bins M = 2-30 in the interactions of ^{28}Si nuclei with nuclear emulsion at 14.6A GeV along with UrQMD prediction. The entire range of values of single event factorial moments for a particular partition number M has been divided into a number of smaller groups, and the frequency distributions are obtained. Though majority of the values of F_2^e are confined within a limited range, large values of F_2^e are also encountered in significant numbers in each case. It tells us that, these fluctuations in event space can be quantified in terms of the erraticity moments and can be related to the chaotic nature of multiparticle production phenomena and/or its dynamics.

B. Dependence of $C_{p,q}(M)$ on $\ln M$

The erraticity moments, $C_{p,q}(M)$, have been calculated with the knowledge of relation (6) for order of moments q = 2-4, and for p = 0.5, 0.9, 1.0, 1.2, 1.4 and 1.6 for the present experimental data of nucleus-nucleus collisions. The findings in the forms of the pictorial graphs have been plotted between the natural log of normalized erraticity moments $\ln C_{p,q}$ as a function of $\ln M$ in Figs. 2 (a-c) to 4 (a-c) for η, ϕ and $\eta\phi$ - phase spaces respectively at energy 14.6A GeV. For the sake of comparison purpose the plots of corresponding UrQMD predictions are also shown in the same figures. From these graphs one may conclude the following:

It is evident that the erraticity parameters can all be derived from the variation pattern of the erraticity moments in the neighbourhood of $p = 1$, the analysis has been performed and the plots are shown only for that regime. In general, a non-linear dependence of $\ln C_{p,q}(M)$ with $\ln M$ can be observed, a feature that is more prominent for moments with $p < 1$ than for moments with $p > 1$. For higher values of order of moments and for $p > 1$, saturation effects in the values of $C_{p,q}(M)$, could be seen from Figs. 2 (a-c) to 4 (a-c) in the higher M region. This feature can be attributed to a finite number of particles in an event, because with increasing bins lesser number of events contributes to the higher order of q. A few kinks are seen in these plots, which are probably due to large E-by-E fluctuations in a particular bin. For each order of moments, q, the type of errors are standard statistical, which are due to E-by-E fluctuations of the SFMs associated with experimental data points and are shown only for the maximum and minimum values of p. The simulated data using UrQMD prediction show the same pattern as experimental data. The dependence of $\ln C_{p,q}$ as a function of $\ln M$ for UrQMD is high and low similar to that of the experiment, but the magnitudes of erraticity moments are always significantly less in comparison to the experimental values.

C. Nature of Erraticity Exponent μ_q

The spatial fluctuations on E-by-E multiplicities are more prominent than the fluctuations on the bin-by-bin multiplicity. So the linear dependence of erratic moments, $\ln C_{p,q}(M)$ on $\ln M$ has been assumed in spite of the non-linearity observed from a graphical representation of the present experimental data. By making a linear fitting in Figs. 2 to 4 for p = 0.9 and 1.2, the values of erraticity exponents, $\psi_q(p)$ have been obtained for q = 2-4. With the knowledge of $\psi_q(p)$, the values of entropy index, μ_q has been calculated for the total data in η, ϕ and $\eta\phi$ phase spaces along with UrQMD prediction. These values are depicted in Table 1. The values of $\mu_q(\phi)$ in ϕ-space are consistently higher than its value in η-space. It is also observed that the entropy index is not independent of the phase space variable. The values of $\mu_q(\eta\phi)$ in $\eta\phi$-space are even higher than its value in η and ϕ-space. The values of μ_q in all spaces using the UrQMD predictions are much less than experimental values. This indicates that the erraticity effect is more effective in $\eta\phi$-space rather than in η or ϕ phase space. The observation of experimental results clearly supports a stronger chaoticity in $\eta\phi$ -space.

With the help of the slopes of Figs. 2 (a-c) to 4 (a-c) and according to the Eq. (9), the erraticity exponent, $\psi_q(p)$ for p = 0.9 and 1.1have been obtained and shown in Table 1. To measure the degree of event-by-event fluctuation in the analysis of event factorial moments, $F_q^e(M)$, for q = 2-4, the values of entropy index, μ_q, are calculated with the knowledge of Eq. (10) and are also depicted in Table 1. It is evident from the table that μ_q increases with q for present data and UrQMD predictions in η, ϕ phase spaces, whereas; in $\eta\phi$ space the difference in the values of μ_q are more. These values of entropy indices, μ_q, for q = 2-4 are in good agreements with the results reported by other workers [28]-[30].

The values of entropy indices, μ_q, have been plotted as a function of order of moments, q, in Fig. 5 for total experimental data along with the UrQMD data. It is inferred from the figure that the values of μ_q, increase with the order of q for total data and UrQMD data in η, ϕ and $\eta\phi$ spaces. It also follows that the pattern of variations of μ_q with q observed experimentally are nicely reproduced by UrQMD data in η and ϕ spaces, whereas in $\eta\phi$-space the difference between two values are more. Since higher values of μ_q corresponds to smaller entropy and show more chaotic behaviour [31]. It may be concluded that the present experimental data clearly exhibits the chaoticity in multiparticle production in nucleus-nucleus collisions at high energies. Similar results are reported by other workers [28]-[32].

Conclusions and Final Remark

Some significant results have been obtained from the analysis of event-by-event fluctuations of produced charged particles in heavy ion collisions at 14.6A GeV. One can draw the following conclusions on the basis of present work:
Our experimental results exhibit the power law behaviour of normalized moments, $C_{p,q}$, which indicates the erratic fluctuations. The variation of μ_q with q agrees with the predictions of UrQMD model in η and ϕ-spaces (1D) and also $\eta\phi$-space (2D). This behaviour indicates chaoticity in the multiparticle production system. It is demonstrated that like multifractal spectral through the multifractal moments (G_q-moments), erraticity spectrum may also be constructed, which will help to extract maximum information on self-similar fluctuations in nucleus-nucleus collisions at high and ultra-high energies. Erraticity may also give useful information regarding the entropy and chaotic nature of particle in heavy ion collisions. It is believed that these fluctuations may be a weak signal of QGP formation in such experiment. Further, evidence of these fluctuations has also been observed in low energy nuclear collisions, whereas the formation of QGP is not expected. Even in target fragmentation process, where the QGP phase transition is most unlikely, some physicists have reported evidence of dynamical fluctuations in earlier work. So far, QGP phase transition cannot be the only reason for the fluctuations observed in present experimental data. It may be possible that the observed fluctuations may have more remarkable explanation.

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Table 1: Values of the erraticity exponents, $\psi_q(p)$ and entropy index, μ_q, in the collisions of ^{28}Si+Em at energy 14.6A GeV along with UrQMD prediction

Phase space/Data	P	$\psi_q(p)$	μ_q	Ref.
q = 2				
η-Experimental	0.9 1.2	-0.063±0.010 0.119±0.011	0.607±0.007	Present work
η-UrQMD	0.9 1.2	-0.056±0.013 0.122±0.011	0.593±0.009	Present work
ϕ-Experimental	0.9 1.2	-0.118±0.019 0.131±0.019	0.830±0.007	Present work
ϕ-UrQMD	0.9 1.2	-0.107±0.017 0.115±0.020	0.740±0.013	Present work
ηϕ-Experimental	0.9 1.2	-0.206±0.037 0.439±0.037	2.150±0.026	Present work
ηϕ-UrQMD	0.9 1.2	-0.163±0.033 0.429±0.038	1.973±0.025	Present work
q = 3				
η-Experimental	0.9 1.2	-0.060±0.006 0.124±0.002	0.613±0.003	Present work
η-UrQMD	0.9 1.2	-0.059±0.005 1.200±0.002	0.597±0.003	Present work
ϕ-Experimental	0.9 1.2	-0.116±0.011 0.139±0.004	0.850±0.006	Present work
ϕ-UrQMD	0.9 1.2	-0.107±0.011 0.125±0.004	0.773±0.006	Present work
ηϕ-Experimental	0.9 1.2	-0.212±0.023 0.456±0.008	2.227±0.012	Present work
ηϕ-UrQMD	0.9 1.2	-0.404±0.021 0.203±0.009	2.023±0.011	Present work
q = 4				
η-Experimental	0.9 1.2	-0.086±0.012 0.128±0.001	0.713±0.006	Present work
η-UrQMD	0.9 1.2	-0.086±0.012 0.115±0.004	0.670±0.006	Present work
ϕ-Experimental	0.9 1.2	-0.172±0.023 0.259±0.008	1.437±0.012	Present work
ϕ-UrQMD	0.9 1.2	-0.173±0.023 0.241±0.008	1.380±0.012	Present work
ηϕ-Experimental	0.9 1.2	-0.246±0.037 0.455±0.037	2.337±0.026	Present work
ηϕ-UrQMD	0.9 1.2	-0.212±0.037 0.421±0.038	2.110±0.027	Present work

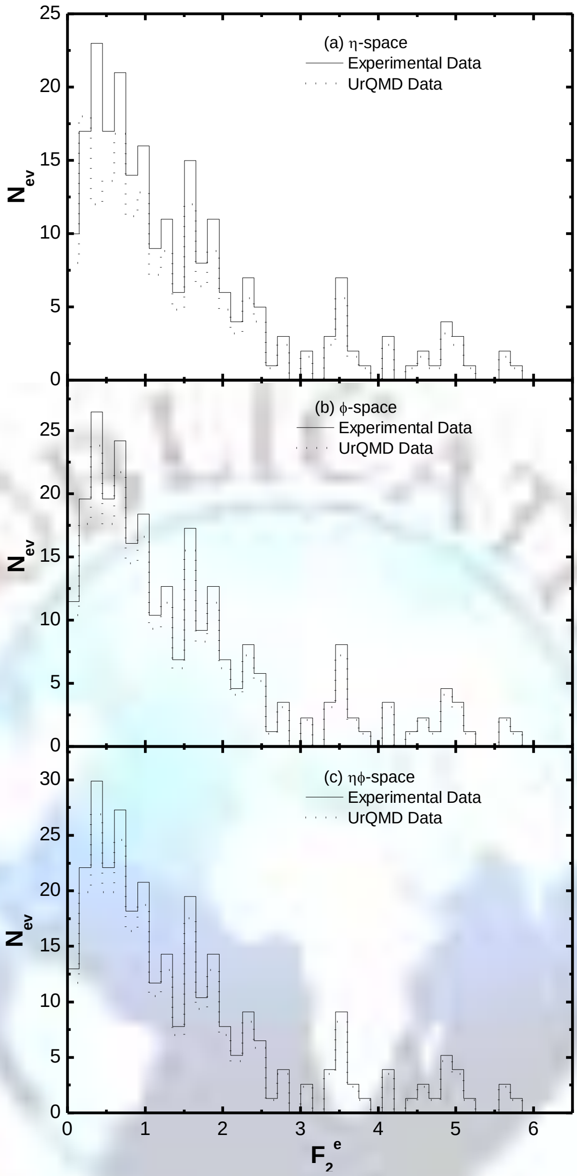

Figure. 1(a-c): Frequency distribution of single event factorial moments for M = 2-30 and q = 2 in the collisions of ^{28}Si+Em at energy 14.6A GeV.

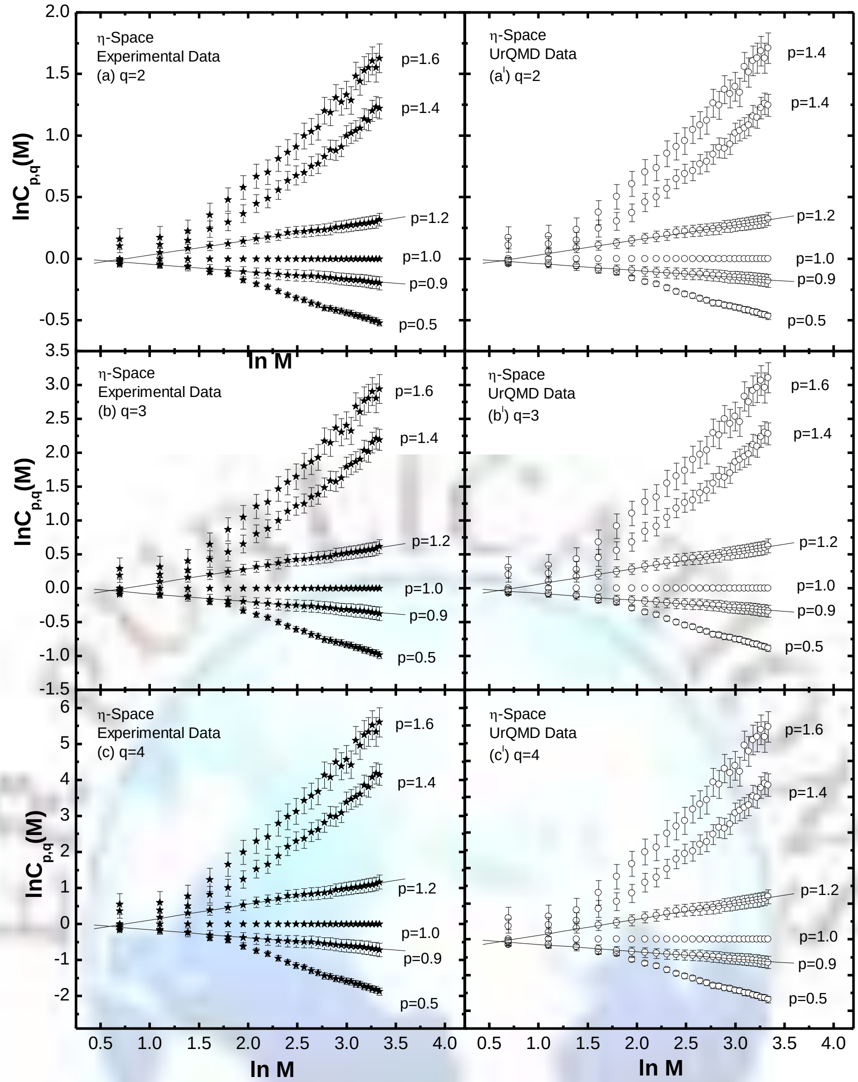

Figure. 2(a-c): Variations of $\ln C_{p,q}(M)$ as function of ln M in η-space (1D) in the collisions of ^{28}Si+Em at energy 14.6A GeV.

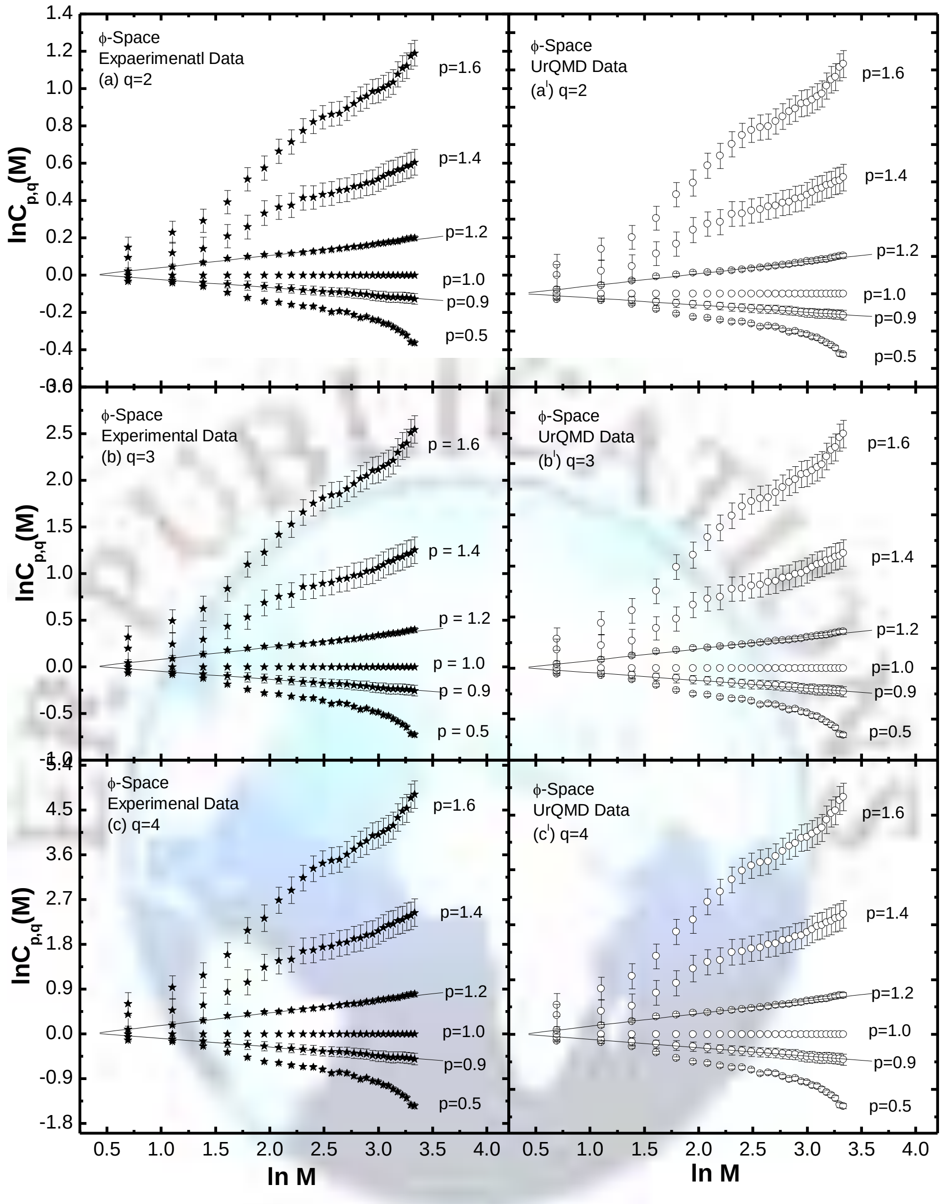

Figure. 3(a-c): Variations of $\ln C_{p,q}(M)$ as function of ln M in ϕ-space (1D) in the collisions of ^{28}Si+Em at energy 14.6A GeV.

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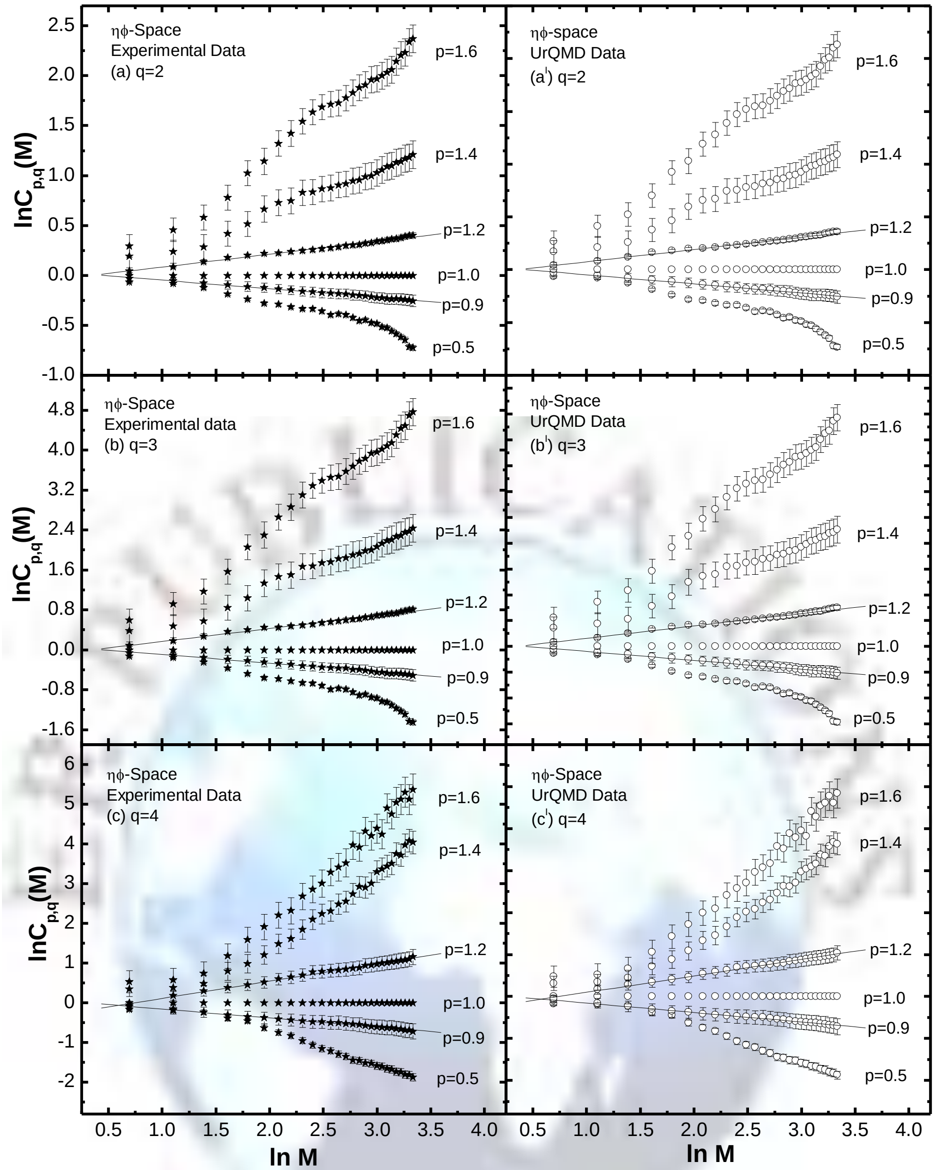

Figure. 4(a-c): Variations of $\ln C_{p,q}(M)$ as function of ln M in ηϕ-space (2D) in the collisions of ^{28}Si+Em at energy 14.6A GeV.

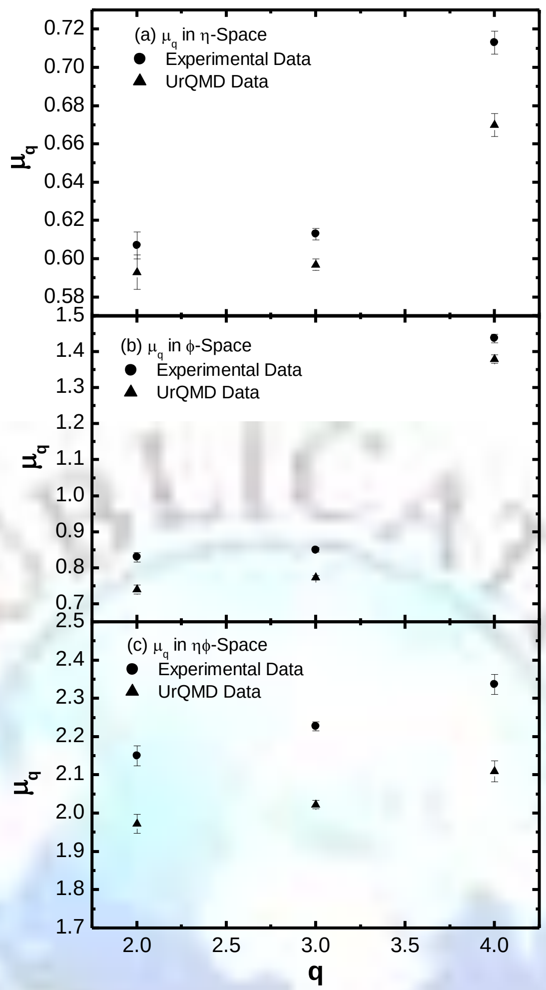

Figure. 5(a-c): Variation of entropy index, μ_q, as a function of q for the collisions of ^{28}Si+Em at energy 14.6A GeV in η, ϕ and ηϕ phase spaces respectively.

Biographies

Dr. M. Ayaz Ahmad completed Ph.D. in Experimental High Energy Physics in 2010 and M. Phil. (Physics) in 2005 from the Physics Department, Aligarh Muslim University, Aligarh, India, under the supervision of Prof. Shafiq Ahmad. I worked as a Guest Lecturer for B. Sc. Laboratory Classes in the same Department, Aligarh Muslim University, Aligarh w.e.f. 10th Oct. 2002 to 10th Oct. 2008 and also as a Lecturer at Senior Secondary College (Boy's) of Aligarh Muslim University, Aligarh w.e.f. 11th Oct. 2008 to 15th Dec. 2010.

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