International Journal of Enhanced Research in Science Technology & Engineering, ISSN: 2319-7463
Vol. 3 Issue 8, August-2014, pp: (206-216), Impact Factor: 1.252, Available online at: www.erpublications.com

Heat transfer characteristics of nano-fluid flow over a stationary flat plate under wall suction/blowing

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ABSTRACT: The flow properties and heat transfer characteristics of water base-copper nanofluid flow over a stationary flat plate under wall suction/blowing is considered. A similarity transformation based on the boundary layer approximation is employed to transform the Navier–Stokes equations and the energy equations into a set of nonlinear ordinary differential equations. A detailed study of the effects of the two parameters, nanoparticle volume fraction and wall suction, on the different physical properties of the flow is carried out. The variation of the velocity profile, the temperature profile, the hydraulic and thermal boundary layer thickness, the local skin friction coefficient and the local Nusselt number with the change of the nanoparticle volume fraction and wall suction is presented. The numerical results thus obtained show that the hydraulic boundary layer thickness decreases linearly as the wall suction factor increases. While it decreases nonlinearly as the nanoparticle volume fraction increases. The numerical results also show that the thermal boundary layer thickness is inversely proportional to the wall suction factor; meanwhile, it is directly proportional to the nanoparticle volume fraction. In addition, both local skin friction coefficient and Nusselt number increases, as a result of increase in either the nanoparticle volume fraction or the wall suction factor.

Key Words: similarity solution, Falkner-Skan, nanofluid, wall suction, boundary layer approximation.

Nomenclature

Problem parameters		**Greek symbols**	
C_p =	specific heat capacity, kJ/kg. K	α =	diffusivity, kg/m^3
$Cf_{Norm.}$ =	Normalized local skin friction Coefficient.	γ =	wall suction factor
f =	similarity stream function	δ =	boundary layer thickness
h =	convective heat transfer coefficent	η =	similarity variable,
k =	thermal conductivity, W/m. K	θ =	normalized temperature
$Nu_{Norm.}$ =	Normalized local Nusselt number	B =	wedge angle
Pr =	Prandtl Number	μ =	dynamic viscosity, kg/m. s
Re =	Reynolds number, $\rho U D_h/\mu$	ν =	kinematic viscosity, kg/m
T =	temperature,	ρ =	density, kg/m^3
U =	free stream velocity, m/s	τ =	shear stress
u =	instantinoues x-component velocity, m/s	ϕ =	nanoparticles volume fraction
v =	instantinoues y-component velocity, m/s	ψ =	stream function
		Subscripts	
		f =	fluid
		p =	particles
		nf =	nanofluid
		w =	wall

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1. INTRODUCTION

In many industrial applications cooling using an external fluid flow over different surface geometry, such as: flat surface, wedge surface and vertical surface, is very popular. Since 1992, there has been an enormous advancement of nanotechnology, which encouraged researchers to start using nanofluids in different industrial applications. Nanofluids as a concept was first introduced in 1995 by Choi, from Argon National Laboratory. Nanofluid is a fluid consists of solid (nanoparticles, nanotubes or nanofiber with size range from 10 to 500 nm) suspended on a base fluid with a certain solid volume fraction. The solid volume fraction typically varies from 0.1-0.4%. Lately, researchers are paying more attention on the effect of nanofluids on the well known classical problems of fluid mechanics. Yacob et al. [1], studied numerically the nanofluid flow past a wedge with wedge angle β.

They investigated the effect of nanoparticles type and the nanoparticles volume fraction on the flow and heat transfer characteristics for ($0 \leq \beta \leq 1$). They found out that the flow and heat transfer characteristics of the problem are more impacted with the nanoparticles volume fraction compared to the nanoparticles type. Yacob et al. [2], studied the flow and the heat transfer characteristics of a wedge surface with a prescribed wall temperature. Three types of nanofluid were used in their study. The three nanofluids used were water based with Al2O3, TiO2 and Cu nanoparticles. The results were compared with that of Riely et al. [3], and Ishak et al. [4], their results shows that the nanofluid using Cu nanoparticles shows more enhancement of the heat transfer over the other two types of nanoparticles.

Noor Alfazl [5], studied the similarity solution of Falkner-Skan flow exposed to simultaneous effects of stretching surface and wall suction or blowing. The study determined the critical parameters: wall slip velocity, the pressure gradient parameter, and the transpiration parameter, effects on the skin friction and the velocity profile. An approximation solution is found and it had been compared to exact solution. Hassani et al. [6], analytically studied the boundary layer problem of a nanofluid flow over a stretching surface. The study considered the nanoparticles effects of Brownian motion and thermophoresis on the problem formulation. They defined an analytical solution as function of Prandtl number, Brownian number, thermophoresis number, and Lewis number. The results addressed the effect of the above mention parameters on Nusselt number and Sherwood number. Their investigation was for the case of reduced Nusselt number and the case of reduced Sherwood number and their results were in good agreement with the results of Khan and Pop [7].

Grosan and Pop [8], studied numerically the similarity solution of water-base Cu nanofluid for a steady flow over an axisymmetric vertical cylinder. The effect of the following parameters: the particle volume fraction, the mixed convection and the curvature on the performance of the flow and heat transfer. The results of skin friction, Nusselt number, the velocity profile and the temperature profile were reported from the numerical analysis. The study shows that the solid particles volume fraction has a clear effect on the reduced Nusselt number and the reduced skin friction. Hamad and Ferdows [9], investigated the similarity solution of a two dimensional stagnation point flow. A stretching surface with a stationary porous media surface was considered under suction/blowing, heat generation/absorption and thermophoresis of the nanofluid flow.

The temperature, the concentration, wall heat flux, and wall mass flux were examined under different parameters such as: particles volume fraction, thermophoresis, Brownian motion, Lewis number, suction/injection, permeability source/sink, and Prandtl number. Makind and Aziz [10], investigated numerically the similarity solution of the boundary layer flow of nanofluid past a stretching sheet with convective boundary condition. The effects of convective boundary condition on boundary layer, heat transfer and particles fraction over a stretching surface in nanofluid are addressed. The study reported the effect of parameters on the fluid velocity, temperature, and particles concentration. They found out that under constant conditions, the reduced Nusselt number decreases while the reduced Sherwood number increases as Brownian motion and thermophoresis effects increases. Also, the Lewis number has minimum effect on temperature distribution.

In the formulation of the present work, the effect of the presence of the nanoparticles on the flow and heat transfer characteristics of the flow were treated by altering the thermo physical properties, where the classical governing equations of Navier-Stoke and energy equations of the base fluid were used by substituting the associated effective thermo physical properties of the nanofluid.The main objective of this paper is to study the effect of the nanoparticles volume fraction and the wall suction of stationary wedge with constant wall temperature on the flow and heat transfer characteristics. The numerical analysis is restricted on the special case $m = 0$ which represents the flow over a flat plate.

2. PROBLEM FORMULATION

A steady, laminar and incompressible two dimensional flow of a water base- copper nanofluid over a stationary flat plate is considered in this analysis. Using the boundary layer approximation and the nanofluid model proposed by Tiwari and Das

[13] and Choi [15], for the nanofluid, the basic governing equations derivation and the boundary conditions are explained in the following set of equations:

2.1 The Basic Governing Equations are:

The continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

The momentum equation in the flow stream direction with boundary layer approximation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_e\frac{\partial U_e}{\partial x}+\nu_{nf}\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

The inviscid solution of the free main velocity out of the boundary layer region is described by the following equation:

$$U_e(x)=U_o \qquad (3)$$

The boundary conditions are:

$$y=0 \quad \Longrightarrow u(0)=0, v(0)=v_w \qquad (4a)$$

$$y\to\infty \Longrightarrow u(\infty)=U_e(x) \qquad (4b)$$

The energy equation is given by:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \qquad (5)$$

And the boundary conditions are:

$$y=0 \quad \Longrightarrow T(0)=T_w \qquad (6a)$$

$$y\to\infty \Longrightarrow T(\infty)=T_\infty \qquad (6b)$$

where T_w is the surface temperature and it is assumed to be constant.

2.2 Nanofluid Model

The nanofluid model used in the study was introduced by combining Das et al [13] and Choi model[15].When nanofluid is used, the thermal physical properties will be changed to encounter the effect of the nano particles. For the present study the water based-copper nanofluid is used, the properties for the solid copper particle and base water fluid are listed in table 1
.

Material	k (W/m K)	Cp (J/kg K)	ρ (kg/m3)	α (m2/s)	ν (kg/m)
Copper	400	385	8933	1163.1×10^{-7}	
Water	0.613	4179	997.1	1.47×10^{-7}	0.9×10^{-6}

The thermo physical properties (the effective properties) of the nanofluid were calculated by the following model:

The effective nanofluid density:

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \qquad (7a)$$

The effective nanofluid specific heat:

$$C_{Pnf}=\frac{(1-\phi)C_{Pf}\,\rho_f+\phi C_{Ps}\,\rho_s}{\rho_{nf}} \qquad (7b)$$

The nanofluid's viscosity is approximated by adopting the model develop by [14]. It is assumed that the effective nanofluid viscosity is a function of the base fluid viscosity and the spherical nanoparticle volume fraction and it is given by the following relation:

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \qquad (7c)$$

Accordingly, the effective nanofluid heat capacity is defined as:

$$(\rho C_p)_{nf} = (1-\phi)(\rho_f C_{pf})_f + \phi(\rho_s C_{ps})_s \tag{7d}$$

The present study adopted the renovated Maxwell model for the effective thermal conductivity developed by Choi (2003) [15]:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f + 2(k_s - k_f)(1+\ \omega)^3 \emptyset}{k_s + 2k_f - (k_s - k_f)(1+\omega)^3 \emptyset} \tag{7e}$$

Where ω is the ratio of the nanolayer thickness to the original particle radius that was considered 0.1.

2.3 Similarity Solution

The Similarity solution approach has been used by many authorsto study the fluid mechanics of various flow settings [1-12, 15 and 16]. The previous governing equations and boundary conditions can be transferred into a set of similarity equations using the following similarity variables:

$$\eta = \left[\frac{U_0}{2\nu_f\, x}\right]^{1/2} y \tag{8a}$$

The continuity equation can be integrated by introducing the stream function ψ such that

$$\psi = [2\nu_f\, x\, U_0]^{\frac{1}{2}} f(\eta) \tag{8b}$$

Then the velocity field components are:

$$u = \frac{d\psi}{dy}, \qquad v = -\frac{d\psi}{dx}, \tag{8c}$$

The non-dimensional temperature is defined as:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{8d}$$

Then using the above similarity variables and the nanofluid model, the overall similarity model for the momentum equation and the energy equation will be as follows:

The momentum equation:

$$\frac{1}{(1-\emptyset)^{2.5}\left(1-\emptyset+\emptyset\frac{\rho_s}{\rho_f}\right)} f''' + f f'' = 0 \tag{9}$$

The initial and boundary conditions will be given by:

$$\eta = 0 \Longrightarrow f'(0) = 0, f(0) = \gamma, \quad \eta \to \infty \Longrightarrow f'(\infty) = 1 \tag{10}$$

Where γ is the wall suction factor which is considered constant.
The similarity equation for the energy equation is given by:

$$\frac{\alpha_{nf}}{\nu_f}\theta'' + f\theta' = 0 \tag{11}$$

Whereα_{nf} is the nanofluid thermal diffusivity and

$$\frac{\alpha_{nf}}{\nu_f} = \frac{1}{Pr}\frac{\frac{k_{nf}}{k_f}}{\left[1-\phi+\phi\frac{(\rho_s C_{ps})_s}{(\rho_f C_{pf})_f}\right]}$$

The initial and boundary conditions for the thermal equation will be:

$$\eta = 0 \Longrightarrow \theta(0) = 1 \tag{12a}$$

$$\eta \to \infty \Longrightarrow \theta(\infty) = 0 \tag{12b}$$

3. RESULTS AND DISCUSSION

The above system of ordinary differential equations are solved numerically using the software Mathematica. It covers a range of wall suction factors of γ= 0, 0.1, 0.2 and 0.3. The nanoparticle volume fraction values considered are: φ= 0, 0.05, 0.1, 0.15, 0.2 and 0.25. Figure 1 presents the dimensionless velocity profiles for constant wall suction factor and variable nanoparticles volume fraction as a function of the similarity variable (η). The results show the same trend for all the studied cases. One common thing to notice is that the dimensionless velocity profile is getting steeper as we increase the normalized wall suction factor and the nanoparticle volume fraction, this behavior can be confirmed by checking the hydraulics boundary layer thickness results presented in Figure 2. It shows that the hydraulic boundary layer thickness, the solid lines represents the correlation results generated using equation (17), while symbols represent the numerical simulation results. The result suggests that the hydraulic boundary layer thickness decreases nonlinearly with increasing the nanoparticle volume fraction φ, while it decreases linearly with increasing the wall suction factor γ.
In order to study the effect of the variable parameters on the skin friction coefficient and the Nusselts number, the local Skin friction coefficient is defined as:

$$C_{fx} = \frac{\tau_w}{\rho_f U_2^2}, \qquad \text{where} \tau_w = \mu_{nf}\left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{13a}$$

Using the similarity variables, the local skin friction coefficient will be

$$C_{fNorm.} = C_{fx}\left[\frac{(m+1)}{2\,Re_x}\right]^{-1/2} = -\frac{1}{(1-\phi)^{2.5}} f''(0) \tag{13b}$$

The local Nusselt Number is defined as

$$Nu_x = \frac{hx}{k_f}, \qquad \text{where} h = \frac{k_{nf}\left[\frac{\partial T}{\partial y}\right]_{y=0}}{(T_w - T_\infty)} \frac{x}{k_f} \tag{14a}$$

Then the normalized local Nusselt number will be

$$Nu_{Norm.} = Nu_x\left[Re_x \frac{(m+1)}{2}\right]^{-1/2} = -\frac{k_{nf}}{k_f}\theta''(0) \tag{14b}$$

Figure 3 shows the variation of the normalized local skin friction coefficient, C_{Norm}, as a function of the nanoparticles volume fraction φ and the wall suction factor γ. The solid lines represents the correlation results generated using equation (15), while symbols represent the numerical simulation results. Figure (3-a) suggests that for a constant wall suction factor γ, the normalized local skin friction coefficient C_{Norm} increases linearly as the nanoparticles volume fraction φ increases. A similar trend is observed for the variable wall suction factor γ at a constant nanoparticle volume fraction φ. This result is expected especially as the nanoparticles introduced to the flow, more friction is anticipated due to the generated friction between the nanoparticles and the surface, and due to the nanoparticles interactions. The reported results are in agreements with other researcher's results such as [1], [2] and [8]. Increasing the normalized wall suction factor γ as expected has a negative effect on the flow, where it increases the friction as a result of having a more steep velocity profile and in turn a higher velocity gradient developed as a consequence of increasing the shear stress, this is clearly demonstrated in the results shown in Fig. 1 and 3.

Figure (4) shows the dimensionless temperature profiles θ(η) for different values of the nanoparticles volume fraction φ while the wall suction factor γ is kept constant. A common trend is found in all cases; when the nanoparticles volume fraction φ increases, the profile gets steeper, and as a result the thermal boundary layer thickness increases, as shown in Figure (4-a). Also, for the case of a constant nanoparticle volume fraction φ, the thermal boundary layer decreases as we increase the wall suction factor γ. For controlling applications, the effect the two independent variables: the nanoparticle volume fraction φ and the wall suction factorγ on the normalized local skin friction C_{fNorm} and the local Nusselt number Nu_{Norm} will be studied. It is found that the normalized local skin friction C_{fNorm} is linearly proportional to the two variables φ and γ. Equation (15) represents the linear regression equation of C_{fNorm} as a function of the two variables φ and γ.

$$C_{fNorm} = 0.44354 + 0.14521\gamma + 2.84391\emptyset \tag{15}$$

The predicted values are in good agreement with the data obtained as indicated in Figure 5.
The same analysis reveals that the Nusselt number Nu_{Norm} is also linearly proportional to the two variables φ and γ. The regression equation is presented in Equation (16).

$$Nu_{Norm} = 0.87933 + 0.40343\gamma + 2.279\emptyset \quad (16)$$

The effect of the two independent variables: the nanoparticle volume fraction φ and the wall suction factor γ on the hydraulic boundary layer thickness δ_h is presented in Figure 5. It is found that the boundary layer thickness is inversely proportional to $\frac{1}{\varphi^3}$ and it is linearly proportional to γ. The correlation equation is given by Equation (17).

$$\delta_h = 1.113 + \frac{1.7843}{0.7563 + 3.025\phi - 9.7324\phi^2 + 6.8524\phi^3} - 0.1752\gamma \quad (17)$$

The same relation is found between the thermal boundary layer and the two quantities φ and γ. The correlation equation is presented by Equation (18).

$$\delta_t = -0.5866 + \frac{101.67}{170.04 - 1.396\phi - 6.14\phi^2 + 0.7224\phi^3} - 0.00393\gamma \quad (18)$$

4. CONCLUSIONS

The characteristics of nanofluid flow over a stationary flat plate with suction and blowing is considered in this study. The similarity transformation is used to transform the Navier-Stokes equations into a coupled nonlinear boundary value problems. The set of equations is then solved numerically using the software Mathematics. The effect of the wall suction factor (γ) and the nanoparticle volume fraction (ϕ) on the various fluid features such the boundary layer thickness, the thermal boundary layer thickness is studies. A nonlinear regression technique is carried out in order to find a set of correlation which address the main dependent variable: hydraulic boundary layer thickness, thermal boundary layer thickness, normalized local skin friction coefficient and normalized Nusselt number, as function of two independent variable: the wall suction factor (γ) and the nanoparticle volume fraction (ϕ).

The overall founding of this study can be summarized by the major effect of the nanoparticles volume fraction on the various physical properties of the flow. It is found that increasing the nanoparticles volume fraction is leading to the increase, linearly, in the thermal boundary layer thickness, the normalized local skin friction and the normalized local Nusselt number. An inverse nonlinear effect is depicted for the hydraulic boundary layer thickness. Increasing the wall suction factor decreases the hydraulic and the thermal boundary layer thickness, while, it increases the normalized local skin friction and the normalized local Nusselt number.

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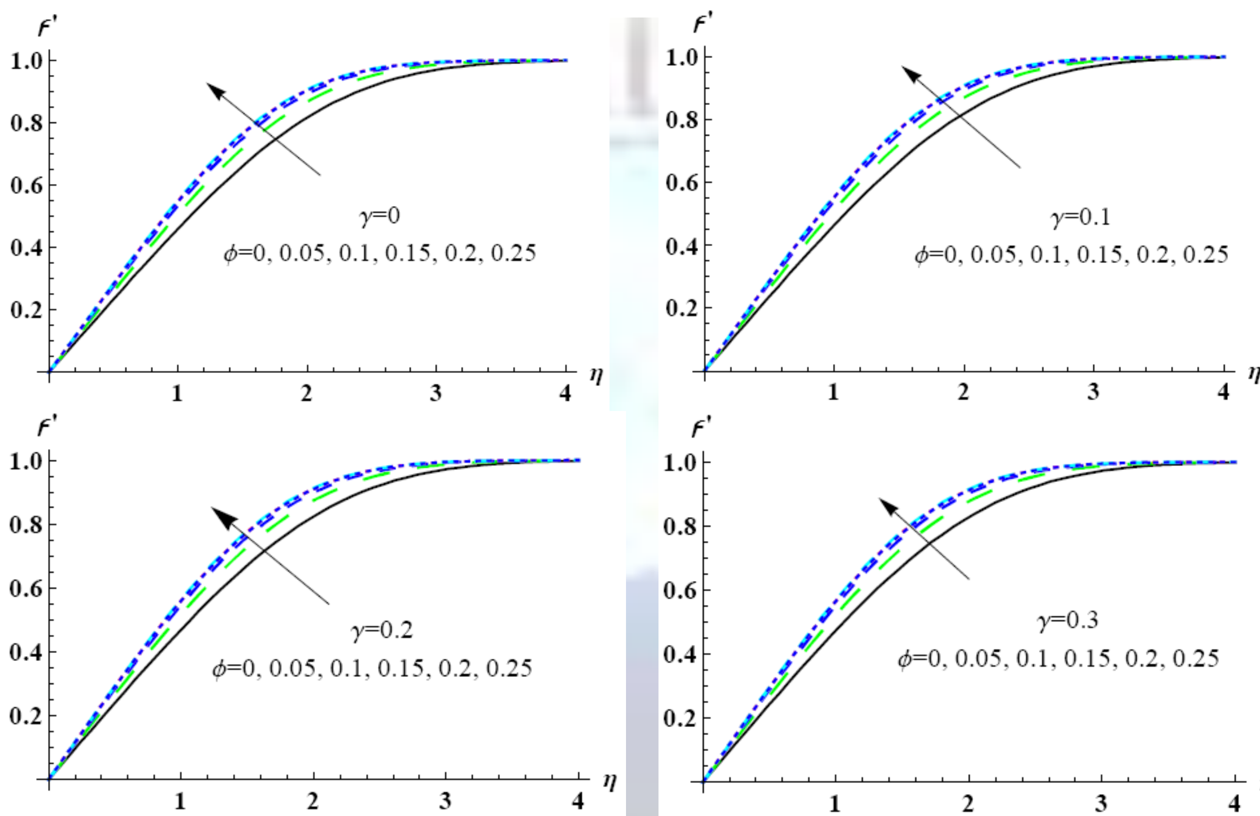
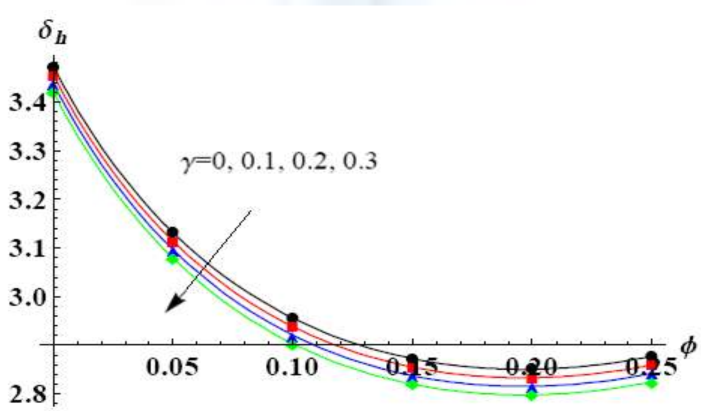


Figure 1. Effect of variable particle volume fraction on the dimensionless velocity Profile, for different wall section values.



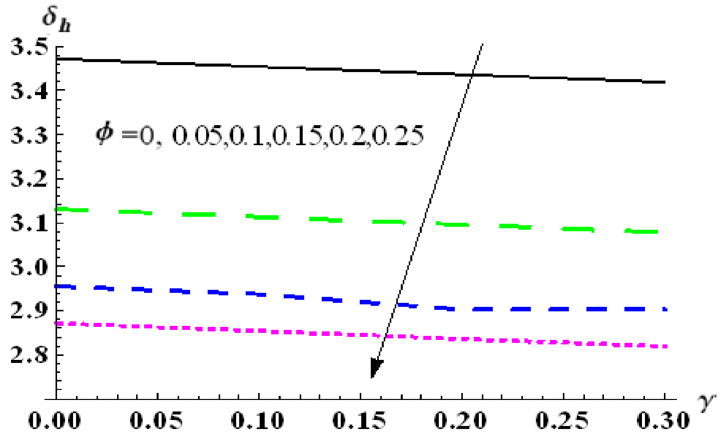


Figure 2. Effect of the hydraulic boundary layer thickness due to (a) the variable wall suction factor as function of the nanoparticle volume fraction (b) the variable nanoparticle volume fraction as function of the normalized wall suction factor.

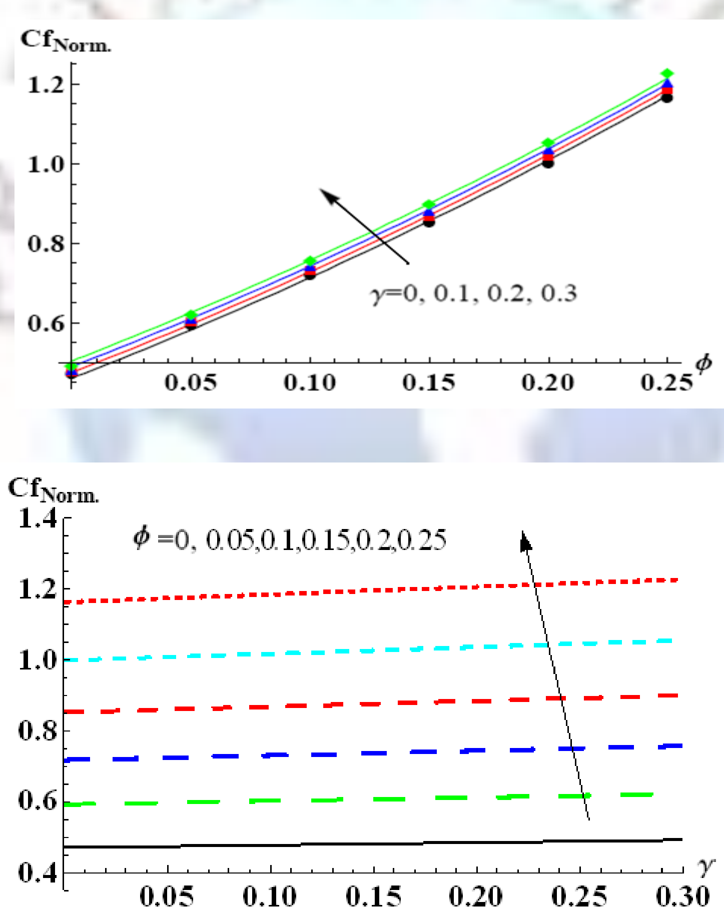


Figure 3. Effect of the normalized local skin friction coefficient due to (a) the variable wall suction factor as function of the nanoparticle volume fraction (b) the variable nanoparticle volume fraction as function of the normalized wall suction factor.

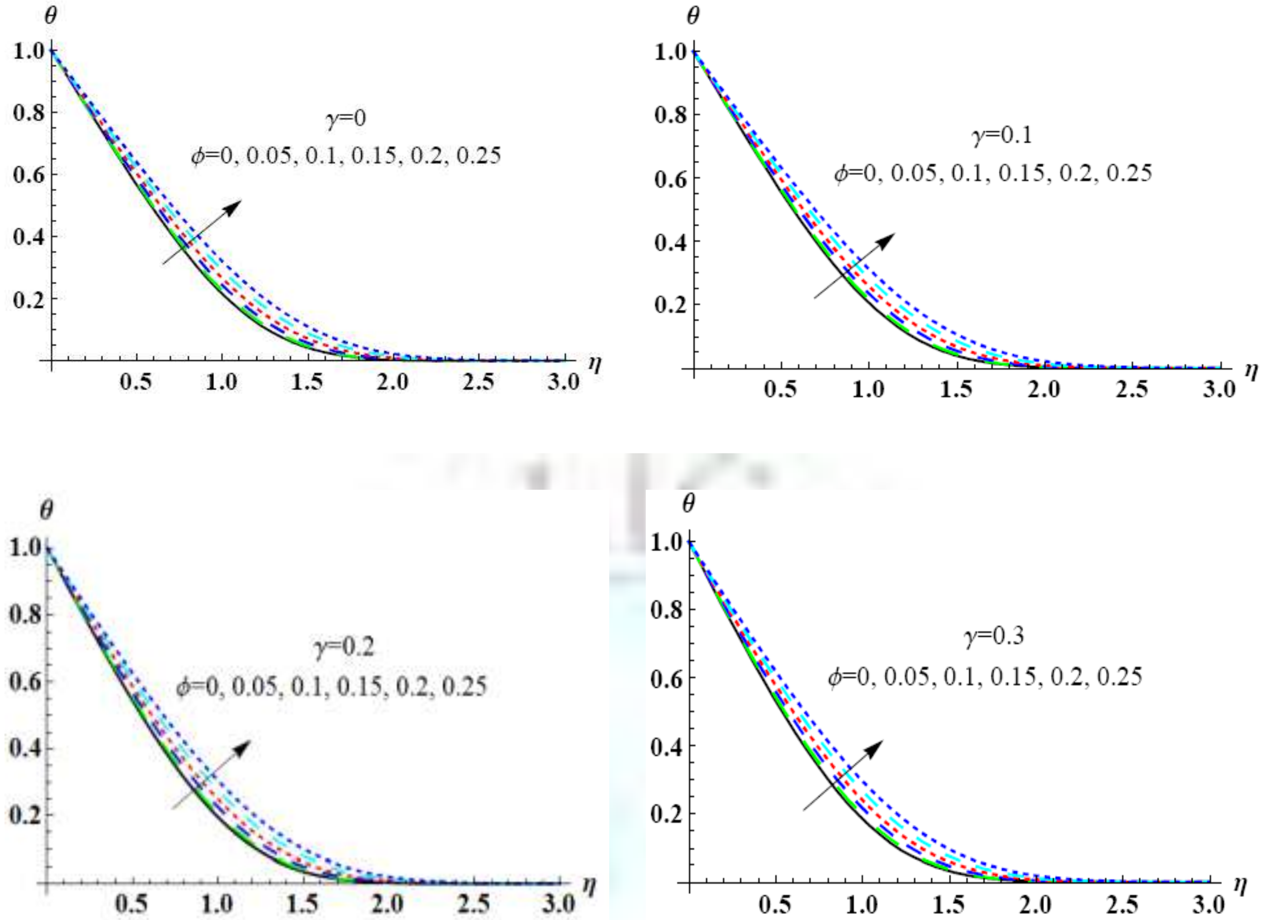
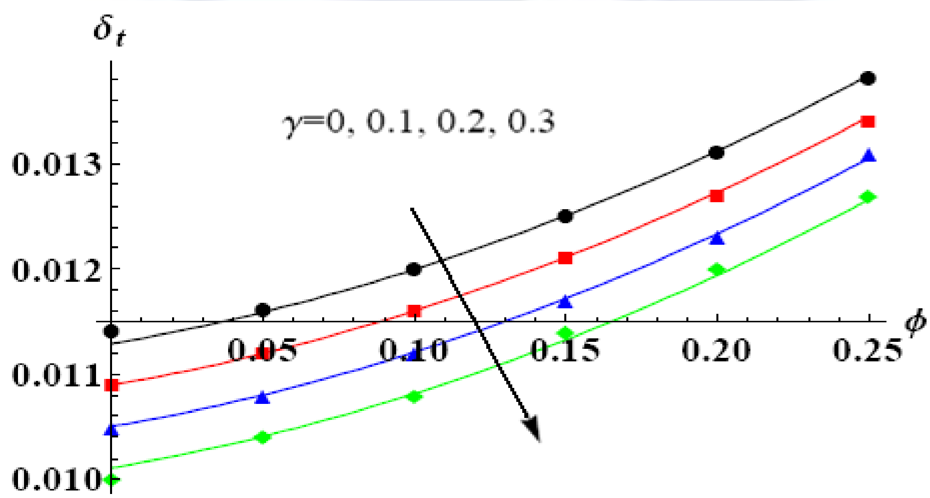
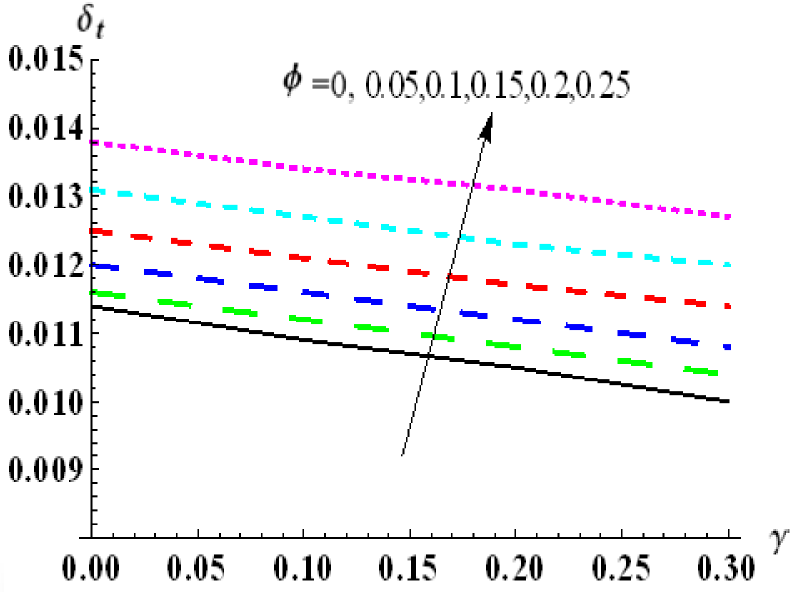


Figure 4. Effect of the variable particle volume fraction on dimensionlesstemperature profile, for different wall section values.

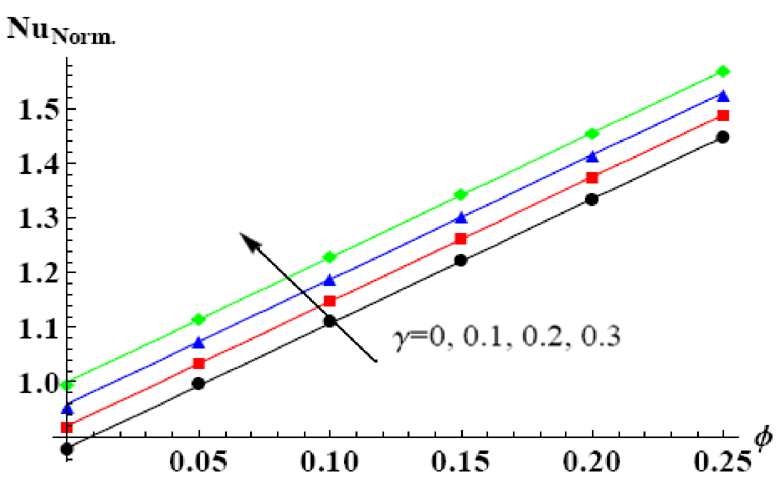


(a)

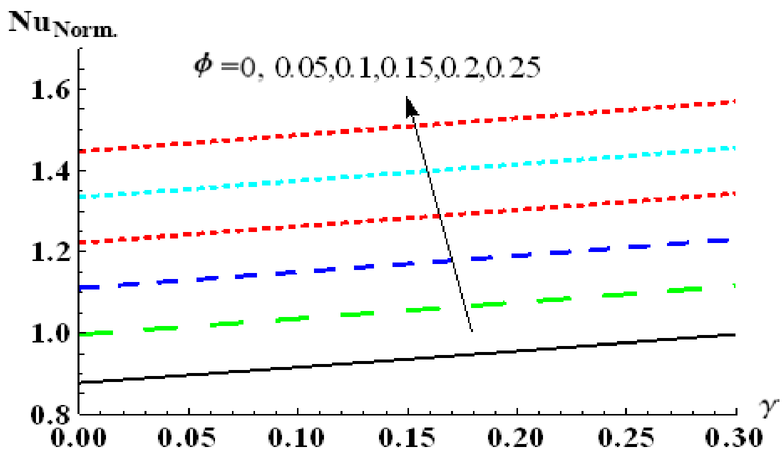


(b)

Figure 5. The variation of the thermal boundary layer thickness as a function of(a) the nanoparticle volume fraction ϕ for different values of wall suction γ , (b) the normalized wall suction factorγ for different values of the nanoparticle volume fractionϕ.



(a)



(b)

Figure (6) The variation of the Nusselt number Nu_{Norm} as a function of
(a) the nanoparticle volume fraction ϕ for different values of wall suction γ ,
(b) the normalized wall suction factorγ for or different values of the nanoparticle volume fractionϕ.