Channel Estimation for Orthogonal Frequency Division Multiplexing

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ABSTRACT

The Channel estimation is one of the fundamental issues of OFDM system design. Techniques that are used for channel estimation of OFDM system are least-square (LS) and minimum-mean-square-error (MMSE) techniques. The LS Estimator algorithm is very simple, but it may be subject to noise enhancement, so the performance is worse. The MMSE Estimator gives better performance than LS and MSE is minimized. In this thesis, DFT-based channel estimation technique is used to increase the performance of LS and MMSE Estimator using different interpolation techniques like Linear interpolation, Spline interpolation. When higher order interpolation is used then it improves the performance but complexity also increases. For best system SNR should be high. Channel Estimation in SISO and MIMO-OFDM using BPSK modulation can also be calculated.

Keywords: OFDM, MIMO, LS, MMSE, DFT.

INTRODUCTION

OFDM is a multicarrier transport technology for high data communication systems. Because of using the orthogonal carrier technology without interference and no guard band between single carriers, OFDM system requires less bandwidth than FDM. OFDM originates from FDM. Orthogonality is affected by inter-symbol interference (ISI), and ISI is caused by leakage of symbols into another due to multipath interference. To fight with ISI, a guard time is introduced before the OFDM symbol. Guard time is selected longer than impulse or multipath delay so as not to cause interference of multipath components of one symbol with the next symbol. ISI is reduced by Cyclic Prefix (CP). Orthogonality is also affected by Inter-carrier Interference (ICI), which is crosstalk between subcarriers, since now the multipath component of one subcarrier can disturb the other. ICI is prevented by cyclically extended the guard interval.

Channel Estimation:- In an OFDM system, the transmitter modulates the message bit sequence into PSK/QAM symbols, IFFT converts them into time-domain signals, and sends them through a (wireless) channel. The received signal is usually distorted by the channel characteristics. To recover the transmitted bits, the channel effect must be estimated and compensated in the receiver. The orthogonality allows each subcarrier component of the received signal to be expressed as the product of the Transmitted signal and channel frequency response at the subcarrier. The transmitted signal can be recovered by estimating the channel responses at each subcarrier. The channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver, and employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. Data signal as well as training signal can be used for channel estimation.

OFDM system channel estimation method can be divided into two ways, pilot-based channel estimation and blind channel estimation. The pilot channel estimation methods are based on the pilot channel and pilot symbol. Blind channel estimation is focusing on the correlation between pilot channel and pilot symbol. Blind channel estimation is focusing on the correlation between the data sent and received, without knowing the information of the transmitted data.

Least Square (LS) Channel Estimation

The least-square (LS) channel estimation method finds the channel estimate in such a way that the following cost function is minimized:

\[ J(\theta) = \sum (Y - \theta)(Y - X) \]

\[ = \sum (Y - \theta)(Y - (Y - X)) \]

\[ = \sum (Y - \theta)Y - \sum (Y - \theta)X \]

\[ = Y - \theta Y + \theta X \]
By setting the derivative of the function with respect to to zero

\[ ( ) = -2( )* + 2( )* = 0 \]

which gives the solution to the LS channel estimation as

\[ ( ) = \]

Let us denote each component of the LS channel estimate by \( [k] \), \( k = 0,1,2,\ldots,N-1 \). Since \( X \) is assumed to be diagonal due to the ICI-free condition, the LS channel estimate can be written for each subcarrier as

\[ [k] = \frac{1}{|\cdot|}, \quad k = 0,1,2,\ldots,N-1 \]

The mean-square error (MSE) of this LS channel estimate is given as

\[ = E ( - ) ( - ) \]

The above equation of MSE is inversely proportional to the SNR, which implies that it may be subject to noise enhancement, especially when the channel is in a deep null. Due to its simplicity the LS method has been widely used for channel estimation. So, without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but obviously it suffers from a high MSE. LS method is utilized to get initial channel estimates at the pilot subcarriers, which are then further improved via different methods.

**Minimum Mean Square Error (MMSE) Channel Estimation**

The minimum mean-square error is widely used in the OFDM channel estimation since it is optimum in terms of mean square error (MSE) in the presence of AWGN. The MMSE estimator employs the second-order statistics of the channel conditions to minimize the MSE. The estimation error vector \( e \) is given as

\[ e = H - \]

Where \( H \) is the actual channel estimation and \( \hat{H} \) is the raw channel estimation.

MSE of channel estimation is given as

\[ E\{ || \} = E ( - ) \]

\[ = E ( - ) \]

Where \( E \{ \} \) is the expectation.

Since the channel and AWGN are not correlated, then above equation can be written as

\[ + = Y \]

The auto-correlation matrixes of \( H, Y \) can be denoted by \( \) respectively. The cross-correlation matrix between \( H \) and \( Y \) is . Let \( \) be the noise-variance, since the channel and AWGN are not correlated, we could get

\[ = E \{ H \} \]

\[ = E \{ 1 \} \]

If 1 are known to the receiver, channel

\[ = X \]

impulse response could be calculated by MMSE estimator as below

\[ + = \]

( of MMSE(estimator))
The performance +
than LS estimator, especially under the lower.

And MMSE estimator could gain 10-15dB more performance than LS.

However, because of the required matrix inversions, the computation is very complex when the number of subcarriers of OFDM system increases. Drawbacks of the MMSE estimator can be the high computational complexity & it requires one to know the correlation of the channel and the operating SNR in order to minimize the MSE between the transmitted and received signals.

DFT-Based Channel Estimation

The DFT-based channel estimation technique has been derived to improve the performance of LS or MMSE channel estimation by eliminating the effect of noise outside the maximum channel delay. The estimate of channel gain at subcarrier is [ ], obtained by either LS or MMSE channel estimation method. The IDFT of the channel estimate

\[ \text{IDFT} \{ \} \]

Where z[n] is the noise component in time domain. Ignoring the coefficient \( h[ ] \) that contain the noise only, define the coefficient for the maximum channel delay L as

\[ h[ ] = h[ ] + \{ h[ ] \}, \quad n=0,1,\ldots,N-1 \]

And transform the remaining L elements back to the frequency domain as follows

\[ Q\{ h[ ] \} \]

SIMULATION RESULT

For OFDM system the BER should be minimum and SNR should be maximum. When BER decreases then SNR value increases. SNR value should be high.
LS estimator has low complexity and high mean-square error, to minimized MSE, MMSE estimator is used. MMSE estimator has high complexity as compared to LS estimator. LS and MMSE estimator gives low performance, so to increase the performance of LS and MMSE estimator DFT-based channel estimation is used.

Fig.2 DFT-Based Channel Estimation

When MSE changes then is also varying. When the channel MSE increases then decreases.

Fig.3 Channel MSE vs. EsN0 dB for MIMO-OFDM Transmission System

Fig.4 NRMSE vs. SNR for MIMO-OFDM channel estimation
This graph shows the relation between normalized MSE and SNR for MIMO-OFDM channel estimation. When NRMSE is varying then SNR value also varies.

**Fig. 5 SNR vs. MSE for MMSE and LS Estimator for SISO using 8QAM**

Above figure compares MMSE and LS estimator of SISO. SNR varies with MSE. For LS Estimator, when MSE increases then SNR decreases.

**Fig. 6 MSE vs. SNR for MIMO-OFDM for Nt=Nr =3**

It shows the relation between MSE of LSE Channel Estimation and SNR for MIMO-OFDM has 3 transmitting and 3 receiving antennas.

**Fig. 7BER vs. SNR for MIMO-OFDM for Nt = Nr=3**
It shows the relation between BER and SNR for MIMO-OFDM and having 3 transmitting & 3 receiving antennas. Signal-noise ratio (SNR) increases when Bit error rate (BER) decreases.

CONCLUSION

LS estimator yields the worst performance but with the simplest complexity. LS estimator has high mean-square error (MSE). Based on LS algorithm, MMSE is analyzed and it shows better performance compared with LS but with more computational complexity. Linear Interpolation and Spline Interpolation are also discussed. The simulation results show that the performance becomes better as the increasing order of polynomial for interpolation. But the complexity also increases. So, DFT based channel estimation is used to increase the performance of LS estimator and MMSE estimator. For SISO and MIMO, MMSE estimator gives better performance.

FUTURE SCOPE

The channel estimation is an area which required a lot of attenuation and improper channel estimation degrades the performance of system. It is assumed that the channel is estimated perfectly. To improve the performance LS based MMSE i.e. LMMSE is used, it also reduces the complexity. Other type of pilot, block type and comb type pilots are extremely used for different fading environment. Block type pilot is used for slow fading and Comb type is used for fast fading. Other channel estimation techniques can also be used to improve the performance. MIMO-OFDM with a number of transmitting and receiving antennas is also designed with proper channel estimation. SNR should be large for best system

REFERENCES