A thermo-mechanical analysis on fiber reinforced composite by Numerical Method

Dr. Dinesh Shringi¹, Raj Kumar², Piyush Sharma³

Abstract: Thermo-mechanical behavior of a Fiber Reinforced composite is a function of resin type, fiber type and architecture, fiber volume fraction, direction of heat flow, and service temperature. The thermal and mechanical responses in Fiber Reinforced composite (FRC) play a critical role in their performance, accurate thermal behavior measurements of FRC are essential. The objective of proposed work is to find out the thermo-mechanical behavior of natural fiber reinforced composite and compare it with the available theoretical models for verification. The results obtained in this proposed work are useful for prediction of thermal conductivity and heat flux for 1 fiber reinforced composites along the transverse direction of composite with respect to different Fiber volume fractions. In this work thermal conductivity calculation for cylindrical fiber with cubical matrix of unit length for 20%, 30%, 40%, 50% and 60% fiber volume fraction for single piece fiber and 4 piece fiber arrangement is generated and compared with the theoretical models. With rise in volume fraction the thermal conductivity rises. The results of this research would show that thermal conductivity and heat flux increase as fiber volume fraction increases. The results indicate that the proposed method is efficient and accurate in analyzing the thermo-mechanical behavior of fiber reinforced composites.

Index Terms: FRC, fiber orientation, fiber volume fraction, finite element simulation, heat flux, Rule of Mixture Model, thermal conductivity.

1. Introduction

The first reinforced polymeric based materials appear to have been used by the people of Babylonia around 4000-2000 B.C. The materials consisted of reinforced bitumen or pitch. Around 3000 B.C. evidences from various sources indicate that in Egypt and Mesopotamia, types of river-boat were constructed from bundles of papyrus reed embedded in a matrix of bitumen. The beginning of composite materials may have been the bricks fashioned by the ancient Egyptians from mud and straw (1500 BC) laminated metals in the forging of swords (1800 AD). The ancient brick-making process can still be seen on Egyptian tomb paintings in the Metropolitan Museum of Art. By 500 B.C., the Greeks were building ships with three banks of oars called triremes. They possessed keels that were much longer than could have been accomplished by using a single length of timber. Thus, it can be seen that the origin of composite technology goes back into antiquity[1].

Commercialization of the composites could be traced to early century when the cellulose fibers were used to reinforce phenolics, urea and melamine resins. Composites in the world of today have wide range of applications, wherever high strength-to-weight ratio remains and important consideration for use. Its principal use is found in automotive, marine and construction industries. In majority of cases, requiring high performance in the automotive and aerospace industries, the discontinuous phase or filler is in the form of a fiber. In most cases, composite matrices are the thermosets having carbon and ceramics for high temperature applications. Thermosets (epoxy, polysulfones) and thermoplastics (polyetherether ketone, polyimide) due to high strength and performance are pioneer for research and industrial applications [2].

In the 20th century, modern composites were used in 1930s, where glass fibers reinforced resins. Boats and aircrafts were built out of these glass composites, commonly called fiber glass. Since the 1970s [4], the application of composites has widely increased due to development of new fibers such as carbon, boron and aramids, and new composite systems with matrices made of metals and ceramics. In late 1980s, the development of biocomposites had been intensified and polymer matrices reinforced with natural fibers had gained more attention among Nano-composites in the latest advances have high aspect ratio and improved electrical, mechanical and thermal properties that could be fabricated for various purposes[3]. Research and development in the field of composite materials has been on the rise in the recent history of material science, with applications being explored both in academia and industry worldwide. Researchers have proposed that the portion of plant bio-resources for chemical and material needs will increase to 50% by 2050. Global consumption of oil will increase by 57% over the next 15 years. Researches in biotechnology are projected to make bio-polymers cost-effective with their petroleum based counterparts.
Composite materials are multiphase materials obtained through the artificial combination of different materials in order to attain properties that the individual components by themselves cannot attain. They are not multiphase materials in which the different phases are formed naturally by reactions, phase transformations, or other phenomena. An example is carbon fiber reinforced polymer. Composite materials should be distinguished from alloys, which can comprise two more components but are formed naturally through processes such as casting. Composite materials can be tailored for various properties by appropriately choosing their components, proportions, distributions, and morphologies, degrees of crystallinity, crystallographic textures, as well as the structure and composition of the interface between components [5]. Due to this strong tailorability, composite materials can be designed to satisfy the needs of technologies relating to the aerospace, automobile, electronics, construction, energy, biomedical and other industries. As a result, composite materials constitute most commercial engineering materials.

The structure of a composite is commonly such that one of the components is the matrix while the other components are fillers bound by the matrix, which is often called the binder. For example, in carbon fiber reinforced polymer, which is important for lightweight structures, the polymer is the matrix, while the carbon fiber is the filler [6]. The components in a composite can take the form of layers. An example is laminate flooring that consists of layers of polymer, paper and fiberboard that are joined together during fabrication. The potential pay-off for composites materials is so high that they have become one of the fastest developing research and development areas of materials science. Rapid advancement in the science of fibers, matrix materials, processing, interface structures and bonding have taken place in the recent years [7]. The technological developments in composite materials are responsible for partially meeting the global industrial demand for materials with improved performance capabilities.

2. Thermal Conductivity of Composites

The theory of thermal conductivity was proposed by Fourier in 1822. According to Fourier, the fundamental heat conduction equation can be stated as “For a homogeneous solid, the local heat flux is proportional to the negative local temperature gradient”. For one dimensional steady state heat transfer, this statement can be represented by Equation 1:

\[ q = -K \frac{dT}{dx} \]  

\[ \text{...... (1)} \]

Where,

- \( q \) is the heat flux,
- \( K \) is the thermal conductivity of the material, which is a positive 2nd order tensor quantity,
- Represents change in temperature across the thickness and negative sign indicates the temperature reduction from hotter surface to cooler surface.

![Fig-1: Thermal Conductivity for steady state condition](image)

According to Equation 1, conductivity can be given as (under the assumption, that heat is not lost in its plane)

\[ k_e = \frac{L_y}{T} \sum_i k_i x_i \left[ \frac{\partial T_i}{\partial y} \right] \]  

\[ \text{...... (2)} \]

Where

- \( K \) is the thermal conductivity (W/m-K),
- q is the Heat Flux (W),
- A is the cross sectional area of the specimen (m$^2$),
- $\Delta T$ is the Temperature difference (K),
- $\Delta x$ is the overall distance (m).

The thermal conductivity of a material can be defined as a rate at which heat is transferred by conduction through a given unit area of a given material, when the temperature gradient is normal to the cross sectional area[8]. The thermal conductivity of a composite material depends on the fiber, resin materials, fiber volume fraction, orientation of the fiber, direction of heat flow and operating temperature.

**Factors Affecting Thermal Conductivity of Composite Materials**

The knowledge of thermal conductivity of composites is needed for accurate design. Data about thermal conductivity of resin facilitates to reduce stresses related to shrinkage of composites during cure and mismatch in thermal expansion coefficients [9]. Before conducting experiments to determine thermal conductivity of various composites, knowledge about effect of different parameters influencing thermal conductivity is essential.

**Fibers**

Fiber is the reinforcing phase of a composite material. Thermal conductivity of a composite depends upon the thermal conductive nature of the fiber and matrix. Commonly used fibers for composites include Glass, Carbon, and Aramid etc[10]. The microstructure of any fiber plays vital role in carrying heat. The glass fiber has an amorphous structure. It consists of SiO$_2$ molecules and forms a three dimensional silica polyhedral network along the length of the fiber. It behaves nearly isotropic, resulting in nearly same conductivity properties in any direction of the fiber[11]. Carbon fibers are manufactured using precursor materials like rayon, petroleum or coal tar pitches and polyacrylonitrile(PAN). In PAN based carbon fibers, during the graphitization stage the linear structure of carbon atoms transforms into a planar structure called as basal planes and are oriented or stacked along the axis of the fiber. These basal planes are closely packed and are responsible for the high modulus and higher electrical and thermal conductivities along the axis of the fiber[12]. Natural fiber is filled with cellulose material, which acts as an insulator, thus a natural fiber composite shows much lesser thermal conductivity when compared to a glass fiber reinforced polymer (GFRP) composite.

**Resins**

Polymer matrices can be either thermoplastic or thermoset. Thermoplastic materials are formed by addition polymerization. Thermoplastics soften or fuse when heated, harden and become rigid after cooling. Unlike thermosets, thermoplastics can be modified or reused upon the need. Thermoplastics have longer shelf life and higher fracture toughness than thermoset resins [13]. Thermoplastic resins have high viscosity and less creep resistance when compared to thermosets Epoxy resin has excellent adhesion property compared to other resins. In addition to that it has low shrinkage upon curing, good chemical resistance, excellent mechanical properties [14].

### 3. Theoretical Approach For Thermal Conductivity Predictions

The theoretical approach brings more generalized equation for a two dimensional steady state heat flow. Various theoretical approaches are used to yield the thermal conductivity of a composite material so that the heat flow in anisotropic composite material in any direction can be estimated[15].

#### 3.1. The Rule of Mixture Method (ROM)

For homogeneous fibers of thermal conductivity $K_f$ embedded in a resin matrix of thermal conductivity $K_m$, the thermal conductivity $K_p$ parallel to the axis of the fiber is given by Barbero [16]

$$K_p = K_f V_f + (1-V_f) K_m \quad .... (3)$$

Where,

- $V_f$ is the fiber volume fraction

The thermal conductivity in transverse direction ($K_t$) can be given as

$$1/K_t = V_f/K_f + (1-V_f)/K_m \quad ......(4)$$

#### 3.2. Caruso Model

A general finite element analysis method was proposed by Caruso et al [17] to predict thermal conductivity of a composite.
This method is based on integrating Advanced Finite Element Methods with simplified micromechanics equations.

The longitudinal and transverse conductivity equations were given as

$$K_L = V_f * K_{fl} + V_m * K_{ml} \quad \ldots \ldots (5)$$

$$K_T = 1 - \sqrt{V_f} * \frac{K_m}{1 - \left(1 - \frac{K_{fl}}{K_{ml}}\right) * \sqrt{V_f}} \quad \ldots \ldots (6)$$

Where,
- $K_L$ is the thermal conductivity in longitudinal direction,
- $K_T$ is the thermal conductivity in transverse direction,
- $V_f$ is the fiber volume fraction,
- $V_m$ is the matrix volume fraction,
- $K_{fl}$ and $K_{ml}$ are thermal conductivity in longitudinal direction of fiber and matrix, respectively.

### 3.3. Cylinder Assemblage Model (Cy-As)

Cylinder Assemblage model was developed by Hashin[18] who considered a transversely isotropic fiber reinforced cylinder in which the phases are transversely isotropic with material axes of symmetry in cylindrical axis direction. The transverse thermal conductivity was given as:

$$K_T = K_{TM} * \frac{K_{TM} V_M + K_{TF} (1 + V_f)}{K_{TF} V_M + K_{TM} (1 + V_f)} \quad \ldots \ldots (7)$$

Where,
- $K_T$ is the transverse thermal conductivity,
- $K_{TM}$ is the thermal conductivity of the matrix in transverse direction
- $V_M$ is the matrix volume fraction,
- $V_f$ is the fiber volume fraction, and
- $K_{TF}$ is the thermal conductivity of fiber in transverse direction.

The above equation was derived using the expressions for finding elastic module and thermal expansion coefficients of unidirectional fiber composites consisting of transversely isotropic phases.

### 3.4. Shear Loading Analogy (Sh-Ld)

A shear loading analogy method was proposed by Springer and Tsai [19] to estimate thermal conductivity of a composite. Hence, a numerical approach was presented based on analogy between the response of the unidirectional composite to shear loading and to heat transfer along with a thermal model. This model was developed under some assumptions regarding the placement of fibers and packing patterns like elliptical, cylindrical and square shapes. In particular, the matrix and fibers were assumed as parallel and series as in electrical circuits depending on the heat flow direction i.e., longitudinal or transverse, respectively.

The longitudinal conductivity was given as

$$K_{11} = V_f K_f + V_m K_m \quad \ldots \ldots (8)$$

and the transverse thermal conductivity was given as

$$K_{22} = K_m + \left[1 - 2 \frac{V_f}{\pi}\right]$$

$$+ \frac{1}{B} \left[\sqrt{1 - \frac{B^2 V_f}{\pi} \tan^{-1} \left(\frac{B^2 V_f}{\pi}\right)} - \frac{4}{\sqrt{1 - \frac{B^2 V_f}{\pi}}} \tan^{-1} \left(\frac{1 + \frac{B^2 V_f}{\pi}}{B^2 V_f}\right) \right]$$

Where,
- $K_{11}$ and $K_{22}$ are the longitudinal and transverse thermal conductivities respectively,
• \( V_f \) and \( V_m \) are the fiber volume fraction and matrix volume fraction respectively,
• \( K_m \) and \( K_f \) are the thermal conductivities of matrix and fiber respectively.
• \( B=2(K_m/K_f-1) \)
The shear loading analogy approaches under predicted thermal conductivity by about 10% depending on fiber volume fraction.

3.5. Bounding Solution Approach (BG-Sol)
In order to determine effective thermal conductivity value of a composite with unknown phase geometry parameter, Lim [20] developed a boundary solution method. In this method a lower and upper bounds were concluded for the effective thermal conductivity of a composite material.

\[
\frac{K_m}{K_f} = 1 - V_f^{1-R} \left[ \frac{1}{1-V_f^{R}} \left( \frac{1}{1-K_f} \frac{k_f}{k_m} \right) \right] \quad \cdots \cdots (10)
\]

\[
\frac{K_m}{K_f} = 1 - V_f^{R} \left[ \frac{1}{1-V_f^{1-R}} \left( \frac{1}{1-k_f} \frac{k_m}{k_f} \right) \right] \quad \cdots \cdots (11)
\]

The bounds were determined by considering a representative volume element and considering its geometry in three directions \( X, Y, Z \). This bounding solution technique has considered various geometrical assumptions in the reinforcements like unidirectional both in longitudinal and transverse, particulate, in plane and out of plane laminae etc. The bounds were given as

Where, are the lower and upper bounds of the thermal conductivity in a composite, is the volume fraction of fiber and, are the thermal conductivity of matrix and fiber respectively. \( R \) corresponds the reinforcement parameter such that \( R =1, 2/3, 0 \) for unidirectional fiber, particulate and laminate reinforcements respectively for thermal conductivity in parallel to reinforcement axis of symmetry and \( R=1/2, 2/3 \) and \( 1 \) for unidirectional fiber, particulate and laminate reinforcements respectively, for transverse to the reinforcement axis of symmetry.

4. Modeling of Composites

For a realistic computer simulation of a physical problem by the FEM an accurate modeling of the geometry, the material, and the loads involved is necessary. On the other hand, the FEM model must be numerically manageable and optimized in order to avoid excessive computer time and storage space requirements. The purpose of the finite element package was utilized to model the fiber reinforced polymer composite in three dimensions as SOLID.

Model Development

Three-dimensional models have been developed for Fiber-reinforced composites. For 3-D model, shape of the matrix material has been chosen as cube, whereas the filler material (Fiber) have been chosen as of Cylindrical shape. The dimensions of the cube matrix material are fixed and have been taken as unity. Heat fluxes found from the analyses have been used to find out the effective thermal conductivities of composite materials. For 3-D models, different concentration ratios, \( \Phi=20\%, 30\% \), \( 40\%, 50\% \) and \( 60\% \) have been taken so as to figure out how concentration ratio is effective in composite thermal conductivity. Moreover, quantity of fillers has been changed as keeping the concentration ratio same, in order to see the differences between heat flux and temperature distributions and to see the effect on thermal conductivity. Analyses have started first with 1 piece fiber within the matrix material and furthermore 4 pieces Fiber have been applied into matrix material 3-D thermal analysis has been started initially with 10% concentration ratio. The temperatures on boundary surfaces, have been kept constant throughout the analysis and given as \( T1=300 \, ^\circ \text{K} \) and \( T2=350 \, ^\circ \text{K} \). The fibers were assumed to be not in contact with each other and to be uniformly distributed into matrix material. The remaining side surfaces of the cube that are parallel to the heat flow were assumed to be Adiabatic.

Procedure in Modeling

There are major and sub important steps in model,
(a) Preprocessing
(b) Solution stage
(c) Post processing.

![Fig -2: Temperature distribution - 1 piece-filler model for Φ=20%](image1.png)

![Fig -3: Total heat flux distribution - 1 piece-filler model for Φ=20%](image2.png)

5. Calculation of Effective Thermal Conductivity

The following formulas are necessary to figure out the thermal conductivities.

\[
k_e = \frac{L_y \sum k_i x_i \partial T_i}{L_x T_y} \quad (12)
\]

Where,

\[\text{and } \frac{k_f}{k_m} \text{ (whether inside fiber or matrix)}\]

the term is equal to the area under the curve.

Therefore the formula becomes:

\[
Q = \sum k_i x_i \partial T_i \quad (13)
\]

\[
k_e = \frac{L_y}{L_x T_y} * \frac{1}{Q} \quad (14)
\]

Since we took \(L_y\) and \(L_x\) as unity (1 meter), and \(\Delta T\) as 50\(^\circ\)K, by putting these values into Equation (14), we get the following formula:

\[
k_e = \frac{Q}{T} \quad (15)
\]

The only unknown for calculation of effective thermal conductivity is total heat amount, \(Q\), (the area under the heat flux curve) through surface. Heat flux values in y direction along x direction are shown in Figure below. The area lying under this curve gives us the total heat amount passing through the surface. By using integration methods the area under the curve could be calculated. Two columns matrix was constructed depending on data from heat flux vs. distance graph as shown in table 1.

Afterwards, Simpson’s rule was applied for integration.

\[
\int_a^b f(x)dx \approx \frac{b-a}{6} [f(a) + 4f(a+b) / 2 + f(b)] \quad (16)
\]
At the end, effective thermal conductivity of the composites was calculated.
All numerical studies were conducted in this way.
Effective thermal conductivities of each developed model have been extracted from this calculation method Table 2.3.

6. Results

**TABLE 1: Result of thermal conductivity for 1 Piece fiber for different Volume fraction**

<table>
<thead>
<tr>
<th>V.F.</th>
<th>FEM Model</th>
<th>ROM Model</th>
<th>Caruso Model</th>
<th>Cy-As Model</th>
<th>Sh-Ld Model</th>
<th>Bg-Sol Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.1284</td>
<td>1.2354</td>
<td>1.2198</td>
<td>1.2354</td>
<td>1.4785</td>
<td>1.7464</td>
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<tr>
<td>30</td>
<td>1.3248</td>
<td>1.3652</td>
<td>1.2999</td>
<td>1.3652</td>
<td>1.7356</td>
<td>1.9000</td>
</tr>
<tr>
<td>40</td>
<td>1.8949</td>
<td>1.6848</td>
<td>1.6342</td>
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<td>1.8963</td>
<td>1.9354</td>
</tr>
<tr>
<td>50</td>
<td>2.4756</td>
<td>1.8194</td>
<td>1.7384</td>
<td>1.8194</td>
<td>2.0295</td>
<td>2.1453</td>
</tr>
<tr>
<td>60</td>
<td>2.9495</td>
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<td>2.6767</td>
<td>2.5434</td>
<td>2.3958</td>
<td>2.7342</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Distance</th>
<th>Heat Flux(W/m²)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>20.841</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>20.873</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
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<td>4</td>
<td>0.15</td>
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</tr>
<tr>
<td>5</td>
<td>0.2</td>
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<tr>
<td>6</td>
<td>0.25</td>
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</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>21.937</td>
</tr>
<tr>
<td>8</td>
<td>0.35</td>
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<td>9</td>
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<tr>
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<td>20.873</td>
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<tr>
<td>21</td>
<td>1</td>
<td>20.839</td>
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</tbody>
</table>

**Table 2: Distance V/S Heat Flux On Surface**

**TABLE 3: Result Of Thermal Conductivity For 4 Piece Fiber For Different Volume Fraction**

<table>
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<tr>
<th>V.F.</th>
<th>FEM Model</th>
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<td>2.7342</td>
</tr>
</tbody>
</table>
Analytical or numerical models help to predict the properties of a material without conducting any experiments. Most of the literature on composite materials (anisotropic) dealt with mechanical properties and very few models were developed to predict thermal conductive properties in different directions. Mechanical properties can be verified with the previously published analytical models; however, found theoretical and limited experimental methods are available. Some theoretical models available are of no good use for the validation of experimental results, as they require various properties of fibers and matrix, which are difficult to acquire for practical purpose. Most of the models do not consider for thermal resistance, interaction between fiber and matrix, and fiber orientation. A numerical approach was used to determine the effective thermal conductivity of fiber and particle filled composite materials. Numerical study was conducted with different variables. In the frame of numerical analysis, concentration ratio and thermal conductivity ratios of filler and matrix on effective thermal conductivity of composites were separately investigated and result tables and result graphs were figured. In addition to numerical analysis, theoretical models that have been prepared to predict the effective thermal conductivity of composites were examined. As a consequence, it is found that for results obtained from Theoretical Model displayed close relations with numerical results.

![Graph 1: Thermal Conductivity for different Models for 1 Piece fiber for different Volume fraction](image1.png)

![Graph 2: Thermal Conductivity for different Models for 4 Piece fiber for different Volume fraction](image2.png)

**References**


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