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Systematic Calculation of Energy Levels with a Rotational Model for Even-Even $^{164\text{-}180}$Hf Isotopes

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Abstract: It is proposed that the even -even energy levels in the rotational bands of heavy deformed nuclei can be reasonably characterized by a discrete approximation of the Lagrange interpolation (fit three points with an second degree polynomial) of the even -even energy as a function of angular momentum I. Calculations can reasonably describe the $B(E2\ ;\ 2^+ \rightarrow 0^+)$ of studied 2^+ state in $^{164\text{-}180}$Hf isotopes, and also to understand the nuclear structure of first excited state. The quantities are predicted, which could be useful for future studies.

Keywords: Even-Even Nuclei; Rotational Model and Transition Probability.

1. Introduction

The even-even nuclei in the region $120 \leq A \leq 200$ show the typical spectrum of rotation of a deformed axially symmetric structure, vise, and a band of energy levels of character $0^+, 2^+, 4^+, \ldots$ having energies closely given by [1]:

$$E(I) = \frac{\hbar^2}{2\vartheta} I(I+1) \ldots (1)$$

From the observation of these energy levels, the effective moment of inertia ϑ is at once known. The actualamount of distortion of the nucleus cannot be deduced from ϑ, however, without more detailed knowledge of the nuclear motion. Experimentally, the nuclear quadrupole deformation is derived from measurements of the pure electric quadrupole transition rate between adjacent levels in the rotational band, usually between the 0^+groundstate and the 2^+ first excited state. This rate may be measured either by measuring the cross section for Coulomb excitation of the 2^+state from the 0^+ ground state, or by measuring the half life, $t_{\frac{1}{2}}$ of the reversegamma transition $(2^+ \rightarrow 0^+)$ in a radioactive source [2].

2.1 Lagrange Interpolation

To interpolate a function f, we define the values of interpolation as follows [3]:

$$y_i = f(x_i) \ldots (2)$$

To determine the unique polynomial of degree n, P_n this verifies:

$$P_n(x_i) = f(x_i) \ldots (3)$$

where x_i are points of interpolation, the polynomial which meets this equality is Lagrange interpolation polynomial:

$$P_n(x) = \sum_{k=0}^{n} l_k(x) f(x_k) \ldots (4)$$

where l_k are polynomials of degree n that form a basis of P_n:

$$l_k(x) = \prod_{i=0, i\neq k}^{n} \frac{x - x_i}{x_k - x_i} \ldots (5)$$

2.2 Transition Probability

The reduced transition probability B(E2) is obtained from the experimental half life $t_{\frac{1}{2}}$ and the reduced transition probability as [4]:

$$B(E2; I_f \rightarrow I_i) = 0.05667 \left[t_{\frac{1}{2}} (1+\alpha) E_\gamma{}^5 \right]^{-1} \ldots\ldots (6)$$

Where, E_γ is experimental energy of γ-ray, I_i and I_f are the spin of the initial and final states, respectively, α is the total conversion coefficient. The intrinsic quadrupole moment Q_0 and the deformation parameter β have been obtained using the following relations [5]:

$$B(E2; I_f \rightarrow I_i) = \frac{15}{32\pi} \frac{I_f(I_f-1)}{(2I_f-1)(2I_f+1)} Q_0{}^2 \ldots\ldots (7)$$

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R_0{}^2 \beta \ldots\ldots (8)$$

where Z is the atomic number and $R_0 = 1.2\, A^{\frac{1}{3}}$ fm, and A is the mass number.

3. Discussion

Before evaluating the theoretical results, we consider how well the rotational model applies to them. In fig.1, we have plotted the energy of the first excited state, E_2 and the ratio of the energy of the second excitedstate to that of the first excited state, $R_4 = \frac{E_4}{E_2}$ as functions of A for the nuclei of this study. Form the lower part of the fig.1; we may thus accept all the nuclei except the ^{164}Hf isotope as "rotational", on the criterion [6], and form the upper part of the same figure also shows that the values of R_4 for all the nuclei except the ^{164}Hf isotope are near the rotational value $3.0 \leq R_4 \leq 3.33$, as deduced from equation (1), for the ^{164}Hf isotope, the values of R_4 lie closer to the value 2.787, characteristic of transitional region.

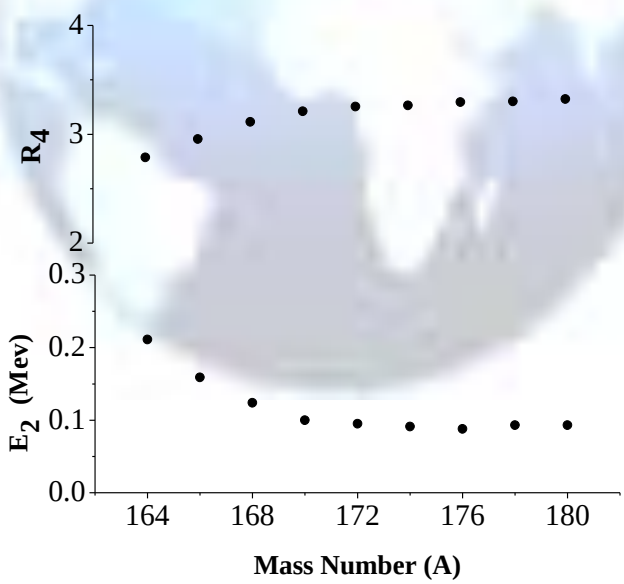

Figure 1: (Lower part): The energy E_2, of the first excited states of each of the even-even $^{164\text{-}180}$Hf isotopes and (Upper part): The ratio, $R_4$of the energies of the second and first excited states, plotted as a function of A.

The first step in treating the experimental transition energy is to calculate the values of E in units of Mev, where the values of energy levels were obtained by interpolation in the first three experimental energy levels [3] as shown in figure 2.

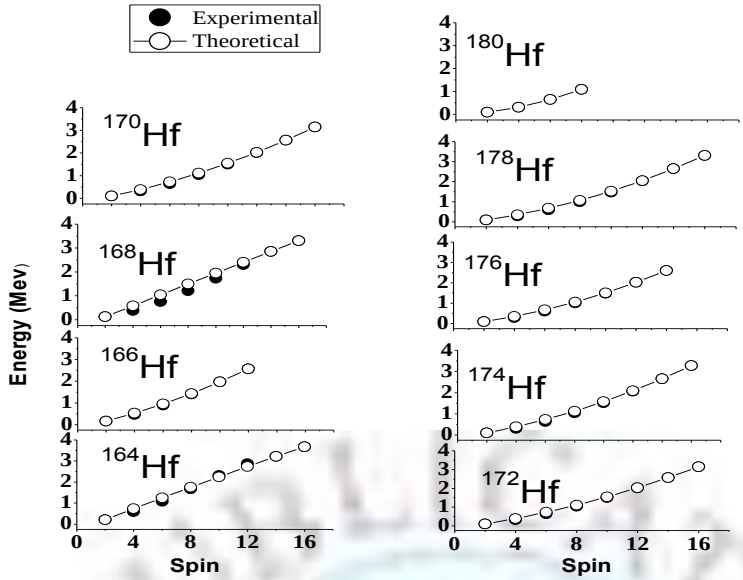

Figure 2: The experimental and theoretical ground state energy levels for even-even $^{164\text{-}180}$Hf isotopes.

In case of figure 2, the ground state energy levels of even-even Hf isotopes; from $A = 164 - 180$; are calculated and compared to the experimental results [7-15]. From this figure one notices that, the theoretical calculations of $^{164\text{-}166,170\text{-}180}$Hf give the best agreement with the experimental data rather than ^{168}Hf.

Table 1 lists the nuclei measured (col. 1), the values of E_γ(col. 2), the measured half life (col. 3), the values of $(1+\alpha)$ used in applying (6) (col. 4), and the experimental values of $B(E2\,;\,2^+ \rightarrow 0^+)$ (col. 5). Column 6 gives the resulting values for $B(E2\,;\,2^+ \rightarrow 0^+)$ for $^{164\text{-}180}$Hf, the discrepancies between their values and ours are in average 11%, quite consistent with the combined errors of measurement.

Column 7 of table 1 gives the values of the intrinsic quadrupole moment Q_0, derived from the $B(E2\,;\,2^+ \rightarrow 0^+)$ values by equation (7). Finally column 8 gives the values of β calculated from Q_0 by means of equation (8). At this point it is necessary to choose a value for R_0, the average nuclear radius. We have used $R_0 = 1.2\,A^{\frac{1}{3}}$ fm, so as to agree with other workers in this field.

Table 1: Adopted experimental transition energy (E_γ), half life ($t_{\frac{1}{2}}$), reduced transition probability $B(E2)$, intrinsic quadrupole moment (Q_0) and deformation parameter (β) values for 2^+ state in even-even $^{164\text{-}180}$Hf isotopes.

Nuclei	E_γ (Mev)	$t_{\frac{1}{2}}$ (psec)	$1+\alpha$	$B_{Exper.}(E2\,;\,2^+ \rightarrow 0^+)$ (e^2b^2)	$B_{Theor.}(E2\,;\,2^+ \rightarrow 0^+)$ (e^2b^2)	Q_0 (b^2)	β
^{164}Hf	0.211	300	0.903	0.41	0.40	4.488	0.057
^{166}Hf	0.159	497	1.064	0.70	0.46	4.833	0.062
^{168}Hf	0.124	890	1.026	0.84	1.05	7.249	0.092
^{170}Hf	0.100	1230	1.017	1.00	0.93	6.835	0.087
^{172}Hf	0.095	1550	--	0.82	--	--	--
^{174}Hf	0.091	1660	--	0.36	--	--	--
^{176}Hf	0.088	1390	--	1.09	--	--	--
^{178}Hf	0.093	1460	1.029	0.97	0.85	6.536	0.083
^{180}Hf	0.093	1520	1.049	0.93	0.91	6.766	0.086

Conclusions

We developed energy levels pattern in rare earth nuclei and actinides with long rotational bands ($I \geq 10^+$). The model energy expression (Lagrange interpolation polynomial) reproduces successfully the energy levels pattern in all considered nuclei up to $I^+ = 12^+$. It is shown that the $B(E2)$ can be interpreted reasonably as the result of the interaction of the band with the ground band in the framework of a collective model. The general behavior of the $B(E2)$ effect in rotational regions is studied in terms of the ground state band.

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