

Mathematics and Indian Culture: Indus Valley Civilization to Modern Era

Sudhir Pujara¹, Arpana Garg²

¹Govt. College, Panipat

²I.B. (P.G.) College, Panipat

ABSTRACT

Mathematics has contributed largely in the development of culture and civilization of not only India but the whole World. Indian mathematicians, over a period of time, have made significant contributions in the growth of mathematics. The earliest confirmation of use of basic mathematics in the Indian subcontinent comes from the Indus Valley Civilization, which is believed to have begun around 2500 BC. The number zero can be considered as the most important discovery of the Indian society, which laid foundation to the development of decimal number system that we are using now days. The mathematics concealed in the Indian Vedas, Vedic Mathematics provided great insight into the subject and has great applications in almost every field of mathematics and science (especially astronomy). Sulbasutras of the later Vedic period laid down the foundations to the development of Geometry. Apart from that, Indians made significant contributions in the field of Number Theory, Algebra, Trigonometry, Infinite Series and Calculus.

HISTORY OF MATHEMATICS

Indus Valley Civilization

The remains of Indus Valley civilization, also called as Harappan Civilization (2500 – 1900 BC) were first of all found in 1921 in excavations at Harappa in the Punjab and a year later, at Mohenjodaro near the Indus River. In the early days of Harappan Civilization, the mathematics used was generally concerned with weights, and measuring scales. It has been discovered that Harappans used weights in the ratios of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500. The bricks used for construction in the Indus Valley civilization were of different sizes, but the dimensions of all the bricks were in the ratio 4:2:1 [1]. Quite interestingly, this ratio of 4:2:1 in the bricks dimension is found to be best for effective adherence between the bricks.

Excavations at Harappa and Mohenjodaro also indicate the knowledge of practical geometry in the Harappan culture. Carvings of this civilization had concentric and intersecting circles and triangles. Also, the discovered weights of this civilization were in definite geometrical shapes such as cone, cylinder and cuboid, etc. The well planned streets and architecture of the cities of this civilization also give a clear evidence of working knowledge of geometry of the Indus people [2]. In addition to this, small geometric patterns present on the seals and surface of pottery vessels of this civilization point to the knowledge of advanced geometry [3].

Vedic Period

The invasion of Indo-Aryans from the North caused the decline in the Harappan Civilization around 2000 BC. The next important development in mathematics in India lies in Vedas. The Vedas can be considered as the first literary record of Indian culture and mathematics. The Vedas and later Sulbasutras were primarily religious texts, but they contained quite considerable knowledge of mathematics and astronomy.

There are evidences of naming very large numbers as high as 10^{12} in Vedic texts. The Rishi Medhātithi, after preparing bricks for a Vedic ritual, prays to the Lord of fire, Agni [5]

Imā me Agna istakā dhenava
Santvekā ča desa ča satam ča
Sahasram čāyutam ča niyutam ča
Prayutam čārbudam ča nyarbudam ča
Samudrasča madhyam čāntasča
Parārdhasčaita me agna ishtakā
Dhenavasantvamutrāmushmimlloke .

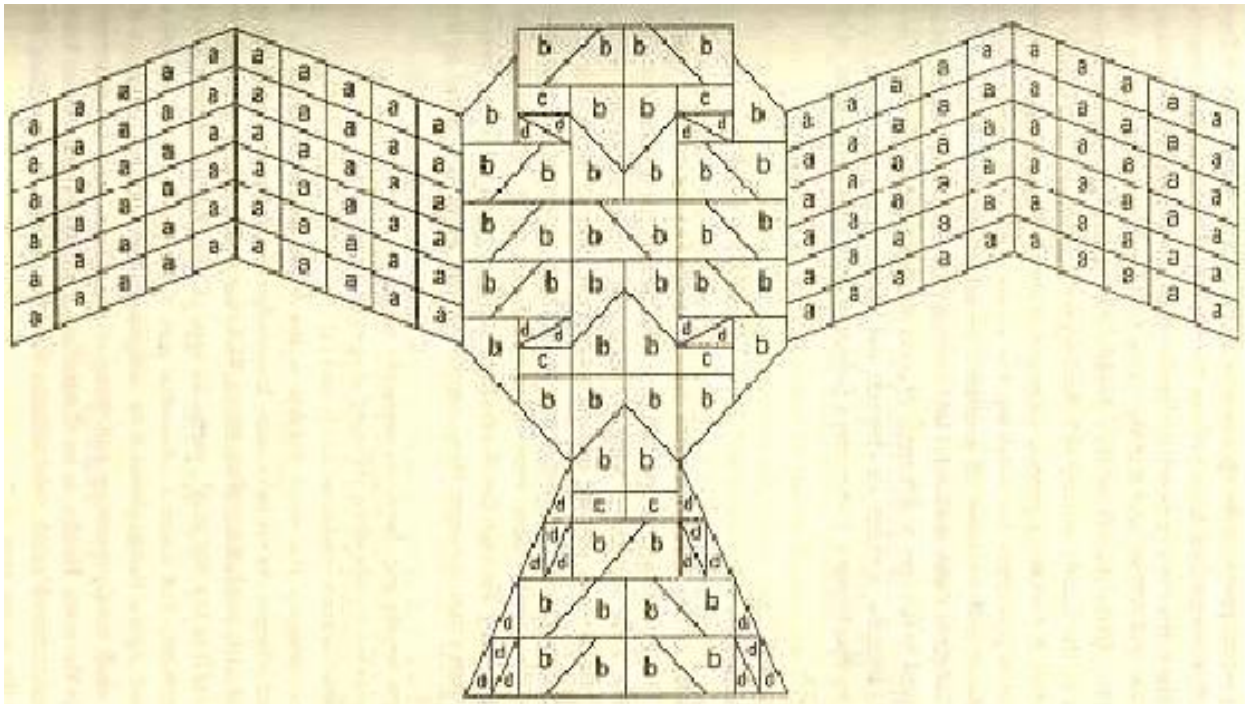
The words in this mantra meant as follows:

eka - 1 - one - 10^0
dasa - 10 - ten - 10^1
satam - 100 - hundred - 10^2
sahasram - 1000 - thousand - 10^3
ayutam - 10000 - ten thousand - 10^4
niyutam - 100000 - one lakh - 10^5
prayutam - 1000000 - ten lakh - 10^6 - million
arbudam - 10000000 - one crore - 10^7 - ten million
nyarbudam - 100000000 - ten crore - 10^8 - hundred million
samudram - 1000000000 - hundred crore - 10^9 - billion
madhyam - 10000000000 - thousand crore - 10^{10} - ten billion
antam - 100000000000 - ten thousand crore - 10^{11} - hundred billion
parardham - 1000000000000 - one lakh crore - 10^{12} - trillion

This number system was known to each rishi of the vedic time and it provides great insight into the developed number system of that time. In addition, Arithmetic operators such as addition, subtraction, multiplication and division, existence of zero and prime numbers can also be seen in the vedic texts.

It is clear from the Yajurveda that, vedic people knew the almost accurate duration of year, month and day which they calculated on the basis of positions of sun. They also knew the concept of 360 degree angle in a circle. The Sulabsutras used primarily for construction of altars contained enormous amount of geometrical knowledge.

An altar in the shape of a bird named Falcon is shown below:



The mathematics in the vedic period really evolved a lot. Sulbasutras actually led to the development of many rules of geometry such as : Construction of various geometrical figures such as squares, rectangles, circles and trapezia. Squaring the Circle and Circling the square, Pythagorean triplets, estimations for π and irrational numbers such as $\sqrt{2}$.

Jaina Mathematics

After the decline of vedic period around 400 BC, Jaina religion became the most dominant religion in the Indian subcontinent and Jaina Mathematics developed at this time. Mathematics in the Vedic time was concerned mainly with the spiritual works, and it was during the time of Jaina religion that mathematics started to flourish as a discipline.

Jaina mathematicians were very fascinated about the large numbers and they divided the large numbers into three different categories: enumerable, innumerable and infinite. Here is a very large number from a Jaina book of cosmology :

$2^{588} = 1013\ 065324\ 433836\ 171511\ 818326\ 096474\ 890383\ 898005\ 918563\ 696288\ 002277\ 756507\ 034036\ 354527\ 929615\ 978746\ 851512\ 277392\ 062160\ 962106\ 733983\ 191180\ 520452\ 956027\ 069051\ 297354\ 415786\ 421338\ 721071\ 661056.$

They used this number to tell age of the universe in years.[6]

Theory of numbers, theory of indices, algebraic equations, set theory, permutations and combinations were other major developments during this period. The period of Jaina mathematicians (300 BC - 400 AD) is considered as a bridge between ancient Indian mathematics and the classical period (Period of Aryabhata I).

CONTRIBUTION OF INDIAN MATHEMATICIANS

Aryabhata

The classical period of Indian mathematics started around 500 AD with the marvelous work of Aryabhata. This is stated as the beginning of a new era for mathematics and astronomy. Aryabhata's works in the field of astronomy were astonishing. He was perhaps the first to say that the earth is spherical and not flat. He introduced trigonometry and also calculated the distance between the earth and the sun using it.

He was such a great mathematician and astronomer that India's first satellite launched in 1975 was named Aryabhata in his honour.

Brahmagupta

During the 7th Century, India produced a famous mathematician and astronomer named Brahmagupta. This genius did amazing innovations in the field of mathematics. Brahmagupta was probably the first mathematician who treated zero as a number in his book Brahmasphutasiddhanta. Before this, Babylonians used zero as simply a placeholder whereas Greeks and Romans used it as a symbol for lack of quantity. Brahmagupta also gave the rules for using zero as a number.

He stated "*When zero is added to a number or subtracted from a number, the number remains unchanged. A number multiplied by zero becomes zero.*"[7]

Brahmagupta also gave the rules for arithmetic with positive and negative numbers. Prior to Brahmagupta's work, the sum like $4 - 5$ (four minus five) was considered meaningless. Brahmagupta realized this as a negative number and he referred to negative numbers as "debt" and positive numbers as "fortune". His other major contributions were: Finding the area of a cyclic quadrilateral (Brahmagupta's Formula) and solving quadratic equations with two unknowns.

Bhaskara I

He was a contemporary of Brahmagupta, and his major contribution to the field of mathematics is extensive commentary of Aryabhata's work. He also gave the positional number system for the representation of numbers with a circle for zero. Earlier the numbers were written in words, such as moon for 1, eyes for 2, and so on.

Bhaskaracharya

Bhaskaracharya was a mathematical genius of 12th century and his work "Siddhanta Shiromani" is a masterpiece of all time. His work "Siddhanta Shiromani" was divided into four parts: Lilavati, Bijaganita, Grahaganita and Goladhyaya in which he

dealt with four different disciplines of mathematics namely arithmetic, algebra, astronomical mathematics and spherical trigonometry.

He has a lot of major contributions to the field of mathematics. A few of them are: He was the first to state that Division by zero gives infinity. He gave solution of quadratic, cubic and quartic indeterminate forms. He gave a proof of Pythagoras theorem by calculating the same area in two different ways. He introduced derivative, differential coefficients and hence differential calculus. He also introduced spherical trigonometry and a number of other trigonometric results.

Srinivasa Ramanujan

Ramanujan (1887–1920) was truly a mathematical genius and without any formal education of higher mathematics, he gave nearly 3900 results on its own. The mathematical abilities of this genius were first of all discovered by English mathematician G.H. Hardy in 1913, when Ramanujan sent him a letter of about ten pages containing statements of theorems on number theory, continued fractions, infinite series and improper integrals. Prof. Hardy called Ramanujan to England in 1914 and this started an astonishing journey of this young mathematician in collaboration with Hardy.

Ramanujan's main contributions were: Ramanujan's number, Ramanujan's sum, the Ramanujan conjecture, Ramanujan theta function, Ramanujan prime, partition formulae and mock theta function. He was one of the youngest fellows in the history of the Royal Society, a premier institution of science in the United Kingdom. He died in 1920, just at the age of 32, but left behind a legacy of mathematics which inspired and is still inspiring a lot of young mathematicians. His birth anniversary 22nd December is celebrated as National Mathematics Day every year.

VEDIC MATHEMATICS

Around the year 1918, Sri Bharati Krsna Tirthaji (1884-1960) rediscovered the ancient Indian Mathematics from the Vedas, which is known as the Vedic Mathematics. There are sixteen sutras and thirteen sub-sutras in Vedic mathematics which give us the rules that can be applied to almost every field of modern mathematics. The sixteen main sutras in the translated form are:

- (1) By one more than the one before
- (2) All from 9, and the last from 10
- (3) Vertically and crosswise
- (4) Transpose and apply
- (5) If the assemblage is the same it is zero
- (6) If one is in ratio the other is zero
- (7) By addition and by subtraction
- (8) By the completion or non-completion
- (9) Differential calculus
- (10) By the deficiency
- (11) Specific and general
- (12) The remainders by the last digit
- (13) The ultimate and twice the penultimate
- (14) By one less than the one before
- (15) Product of the sum
- (16) All the multipliers

MATHEMATICS IN MODERN ERA

Algebraic Coding theory

Algebraic coding theory is a recent branch of mathematics that is based on codes. The basic concepts used in coding theory are from algebra. Coding theory is used for detection and correction of errors in a communication system when a message is sent through some noisy channel. Thus, it has wide applications in Network Communication and Satellite Communication. It is also used in USB channels, disks and other physical media that are prone to errors. Moreover, coding theory is used in Biological Systems too.

Differential Equations

Differential equations are almost used in every discipline of science and technology. They are used in medicine, economics, geography, physics, chemistry and engineering. Some of the uses of differential equations are:

- (1) for modelling cancer growth or the spread of disease
- (2) for describing the movement of electricity
- (3) for modelling chemical reactions
- (4) for weather forecasting
- (5) to find optimum investment strategies
- (6) to describe the motion of waves, pendulums or chaotic systems

Fractals

It is another recent branch of mathematics which has grown very fast since its origin in the 1970's. It has wide applications in the fields of arts, design and science, etc. Generally the points are taken to be of zero dimension, lines are of one dimension, circles and squares, etc. are of two dimension and spheres and cubes, etc. are of three dimension.

But, Mandelbrot stated that "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." [8] Mandelbrot coined the term fractals for such shapes. Dimension is also associated with these shapes, known as fractal dimension, but it is not necessarily an integer.

Fractals can be seen almost everywhere in nature. They are in rivers, mountains, coast lines, sea shells, ferns, trees, leaves, sea stars, sea urchins, broccoli, pineapple, clouds and lightning, etc. Some of the uses of fractals are:

- (1) In biology and medicine, the fractals can be used for detection of Cancer, Heart problems and Brain disorders.
- (2) In computer graphics industry, the fractals are used to create attractive images and stunning structures.
- (3) In textile industry, the fractals are used to create beautiful designs for textile designing.

REFERENCES

- [1] www.bbc.co.uk/schools/primaryhistory/indus_valley/technology_and_jobs/
- [2] T. A. Sarasvati Amma, *Geometry in Ancient and Medieval India* (1979, revised ed, 1999), Motilal Banarasidass, Delhi
- [3] *In Square Circle: Geometric Knowledge of the Indus Civilization*, Sitabhra Sinha, Nisha Yadav and Mayank Vahia, Dec. 2011.
- [4] Rao, S.R. 1985. Lothal, *Archaeological Survey of India*, New Delhi.
- [5] <http://mathlearners.com/history-of-mathematics/1500-bc-400-ad/indian-vedic-period/>
- [6] <https://hubpages.com/education/Invention-of-Zero-and-Jain-Mathematics>.
- [7] http://www.storyofmathematics.com/indian_brahmagupta.html.
- [8] Mandelbrot B., *The Fractal Geometry of Nature*, Macmillan, 1982.
- [9] Tirthaji, S.B.K., *Vedic Mathematics*, Motilal Banarsidass Publishers Private Limited, Delhi