# Trilokesh Ultimate Sum Method 

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## Abstract: In this manuscript, the authors have evaluated a method to find the ultimate sum of the digits of any number (a) raised to power (n) i.e. $a^{n}$ without any actual calculation.

Keywords: ultimate sum, method, digit, power.

## INTRODUCTION

This method evaluates the ultimate sum of all the digits of a number up to a single digit. For eg., ultimate sum of 56789.

$$
\begin{aligned}
& \quad=5+6+7+8+9 \\
& =35 \\
& \text { Again, } 3+5=8
\end{aligned}
$$

So, the ultimate sum of $56789=8$
This method is helpful in finding the ultimate sum of digits of any number (a) raised to power $n$ i.e. $\mathrm{a}^{\mathrm{n}}$ without any actual calculation.
For e.g., let us take $23^{4}$. Now we have to calculate the ultimate sum of its digits without actual calculation it will come out be 4

$$
\begin{gathered}
\text { As, } 23^{4}=279841 \\
\text { Now, } 2+7+9+8+4+1=31 \\
\text { and } 3+1=4
\end{gathered}
$$

As we can see the above method is quite long and deals with lots of calculations. And the difficulties increase as the value of the number and power increases. But you will amazed to know that by using Trilokesh method you can calculate it within a fragment of second without doing such a large calculations.

Let us see how can we determine it without actual calculation.
Let us consider any number a ${ }^{\text {n }}$

- Firstly sum up the digits of a until it get converted into a single digit e.g. $(2396)^{124}$

Here, $\mathrm{a}=2396$
Now sum of its digits will be $2+3+9+6=20$
Again,

$$
2+0=2
$$

- Now we have a table which will help us in further steps

| sum of digits'a' | 1 | 2 | 3 | 4 | 5 | 6 | value of ' $n$ ' |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |
| 2 | 2 | 4 | 8 | 7 | 5 | 1 | corresponding |
| 3 | 3 | 9 | 9 | 9 | 9 | 9 | values of |
| 4 | 4 | 7 | 1 |  |  |  | ultimate sum |
| 5 | 5 | 7 | 8 | 4 | 2 | 1 |  |
| 6 | 6 | 9 | 9 | 9 | 9 | 9 |  |
| 7 | 7 | 4 | 1 |  |  |  |  |
| 8 | 8 | 1 |  |  |  |  |  |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 |  |
|  |  |  |  |  |  |  |  |

- Now if the sum of the digits of $a$ is 1 and 9 , then the ultimate sum of $a^{n}$ will always be 1 and 9 respectively.
- If the sum of the digits of a is 3 and 6 ,then the ultimate sum of $a^{n}$ will depend on $n$, if $n=1$ then the ultimate sum will be 3 and 6 respectively otherwise it will be always 9 .
- If the sum of the digits of a is $2,4,5,7,8$, then the ultimate sum of $\mathrm{a}^{\mathrm{n}}$ will depend on n . If n has value such a number for which the value of ultimate sum is given in the table then we take as it is e.g.: $(23)^{4}, \quad a=23, n=4$

The ultimate sum of digits of $a=2+3=5$.
Now we have value corresponding to $\mathrm{n}=4$ for the value 5 i.e. 4 in the table. Then the ultimate sum of $(23)^{4}$ is 4 .

- But in case we do not have value of ultimate sum for any value $n$ then we have to divide the power by the total number of terms for which its value is given and then we take the remainder as n. For e.g.-now let us take $(23)^{8}$ $a=23, n=8$

Sum of the digits $a=2+3=5$
Now we do not have value for $\mathrm{n}=8$ corresponding to 5 .
Now we will check out for how many values of $n$ we have corresponding values of ultimate sum.
In case of 5 there are 6 values therefore we will divide the power i.e. 8 by 6 and the remainder comes out 2 .
Now we will take as $=2$ and we will check the value of ultimate sum for 2 from table i.e. 7 . Therefore the ultimate sum of $(23)^{8}$ will be 7 .

Note: if remainder comes out zero then we will take the last value of ultimate sum in the table corresponding to the value of sum of digits of 'a', e.g. in case of 2 it is 1 .

Let us verify,

$$
(23)^{8}=78310985281
$$

Now,

$$
7+8+3+1+0+9+8+5+2+8+1=52 \text { and } 5+2=7
$$

## Hence verified

Now we can follow this procedure for any number $\mathrm{a}^{\mathrm{n}}$

## Applications of Ultimate Sum Method

It is helpful in many proofs.
$>$ For eg. Prove that the ultimate sum of digits of square of any natural is either $1,4,7$ or 9 .
Proof:

* As we know that ultimate sum method defines the sum of digits of a digit raised to power ' $n$ '. All the natural numbers are included in the table as if the no. is greater than 9 then we firstly find ultimate sum of the digits of the number. For ex. if we take 457 then at very first we convert it into a single digit by adding it's digits. i.e.

$$
\begin{aligned}
4+5+7 & =16 \\
1+6 & =7
\end{aligned}
$$

* This proves that all natural numbers are included in this table.
* It means sum of digits of squares of all natural numbers are included in 1 to 9 in the table. So we will check the value of power2 in all columns from 1 to 9 .

| sum of digits'a' | 1 | 2 | 3 | 4 | 5 | 6 | value of ' $n$ ' |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |
| 2 | 2 | 4 | 8 | 7 | 5 | 1 | corresponding |
| 3 | 3 | 9 | 9 | 9 | 9 | 9 | values of |
| 4 | 4 | 7 | 1 |  |  |  | ultimate sum |
| 5 | 5 | 7 | 8 | 4 | 2 | 1 |  |
| 6 | 6 | 9 | 9 | 9 | 9 | 9 |  |
| 7 | 7 | 4 | 1 |  |  |  |  |
| 8 | 8 | 1 |  |  |  |  |  |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 |  |
|  | 9 |  |  |  |  |  |  |

* In the above table we can see that values of power2 are only $1,4,7$ and 9 .

Hence, the ultimate sum of digits of square any natural number is either $1,4,7$ or 9 has been proved.
Similarly we can do it for any power. For e.g., Prove that sum of digits of cube of any natural number is either 1,8 or 9 .

