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Comparison of LQR and Pole Placement Controllers for Stabilizing Inverted Pendulum System

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Abstract: This paper presents comparison of the time specification performance between two controllers for an inverted pendulum system. The objective is to determine the control strategy that delivers better performance with respect to pendulum's angle and cart's position. The inverted pendulum represents a challenging control problem, which continually moves toward an uncontrolled state. The problem is to balance a pole on a mobile platform that can move in only two directions, to the left or to the right. A Linear-Quadratic-Regulator (LQR) and a pole placement technique for controlling the linearized system of inverted pendulum model are presented and compared. Simulation studies conducted in MATLAB environment show that both the controllers are capable of controlling the multi output inverted pendulum system successfully. The result shows that pole placement technique gives better response compared to LQR control strategies.

Keywords: Linear Quadratic Regulator (LQR), Inverted Pendulum System, pole placement technique.

مقارنة المنظم التربيعي الخطي وتحكم وضعية القطب لاستقرارية نظام البندول المقلوب

الملخص

هذا البحث يمثل مقارنة سلوك محدد زمني بين مسيطرين لنظام بندول معكوس. الهدف هو تحديد ستراتيجية السيطرة التي توصل الى أداء افضل نسبة الى زاوية البندول وعربة الموقف. البندول المعكوس يمثل مشكلة التحكم الصعبة والتي تتحرك باستمرار نحو حالة غير المنضبط. المشكلة هي لتحقيق التوازن بين قطب على منصة متحركة التي يمكن أن تتحرك في اتجاهين فقط، إلى اليسار أو إلى اليمين يستخدم المنظم الخطي المتعامد وتقنية منظم تخصيص الأقطاب للسيطرة على نموذج بندول معكوس تم تمثيلها ومقارنتها. الدراسات التمثيلية في بيئة الماتلاب بينت بان كلا المسيطرات يستطيع أن يسيطر على عدة اخراجات للبندول المعكوس بنجاح. النتيجة تبين بأن تقنية منظم تخصيص الأقطاب تعطي استجابة أفضل من ستراتيجية سيطرة المنظم الخطي المتعامد .

I. Introduction

An Inverted Pendulum System (IPS) is one of the most well known equipment in the field of control systems theory. It is inexpensive and can be easily built and installed in laboratories for control education purposes or for research applications. The inverted pendulum system is a nonlinear problem, which has been considered by many researchers, most of which have used linearization theory in their control schemes. In general, the control of this system by classical methods is a difficult task This is mainly because this is a nonlinear problem with two degrees of freedom (i.e. the angle of the inverted pendulum and the position of the cart), and only one control input [3].Inverted Pendulum is a very good model for the attitude control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship etc. To solve such problem with non-linear time variant system, there are alternatives such as real time computer simulation of these equations or linearization. However, it also has its own deficiency due to its principles, highly non-linear and open loop unstable system; causing the pendulum to fall over quickly whenever the system is simulated due to the failure of standard linear techniques to model the non-linear dynamics of the system [1].The common control approaches such as the linear quadratic regulator (LQR) control and pole placement technique. To overcome the problem of this system requires a good knowledge of the system and accurate tuning to obtain good performance [2]. This paper presents investigations of performance comparison between modem control (LQR) and pole placement technique for an inverted pendulum system. Performance of both control strategies with respect to pendulum's angle and cart's position is examined. Comparative assessment of both control schemes to the system performance is presented and discussed.

II. Problem Statement

a- To model the Inverted Pendulum system and linearizing the model for the operating range.
b- To design LQR and a pole placement controller for the linearized system under consideration.
c- To make comparison between the proposed controllers.

III. System Description

This section provides the modeling of the inverted pendulum system, as a basis of a simulation environment for development and evaluation of both control schemes. The system consists of an inverted pole with mass, m, hinged by an angle Θ from vertical axis on a cart with mass, M, which is free to move in the x direction as shown in Figure.1. A force, F is required to push the cart horizontally. In order to obtain the dynamic model of the system, the following assumptions have been made:[3]

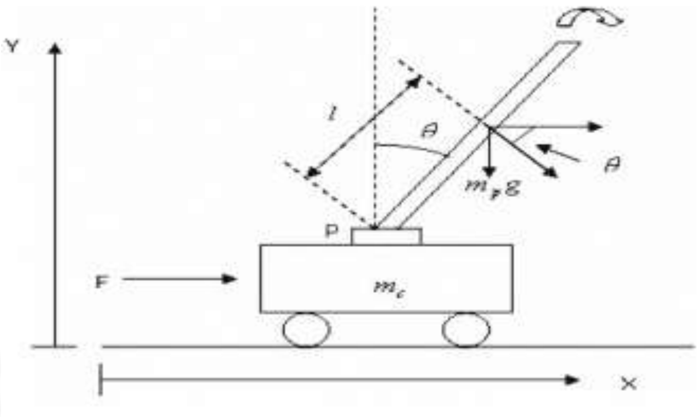

Figure 1. Inverted pendulum system

a) The system starts in a state of equilibrium meaning that the initial conditions are therefore assumed to be zero.
b) The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.
c)A step input is applied. The parameters of the system are given in Table 1.

Table 1. Parameters of The System

Symbol	Parameter	Value	Unit
M	Mass of the cart	0.5	kg
m	Mass of the pendulum	0.2	kg
B	Friction of the cart	0.1	N/m/s
L	Length of the pendulum	0.3	m
I	Inertia of the pendulum	0.006	Kgm2
g	Gravity	9.8	m/s2

Figure. 2 shows the free body diagram of inverted pendulum system. From the free body diagram, the following dynamic equation of the system is determined:[3]

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F \qquad ----1$$

$$(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta \qquad ------2$$

Both the controllers can only work with linear functions so this set of equations should be linearized about Θ = π. After linearization, above two equations of motion reduce to the following, (where u represents the input):

$$(I+ml^2)\ddot{\phi}-mgl\phi=ml\ddot{x} \qquad ------3$$

$$(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u \qquad ------4$$

The dynamic equations (3) and (4) can be represented in state space form as stated below:

$$\begin{bmatrix}\dot{x}(t)\\ \ddot{x}(t)\\ \dot{\phi}(t)\\ \ddot{\phi}(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.1818 & 2.6727 & 0\\ 0 & 0 & 0 & 1\\ 0 & -0.4545 & 31.1818 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\\ \phi(t)\\ \dot{\phi}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1.8182\\ 0\\ 4.5455\end{bmatrix}u(t) \qquad 5$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \phi(t) \\ \dot{\phi}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t) \quad 6$$

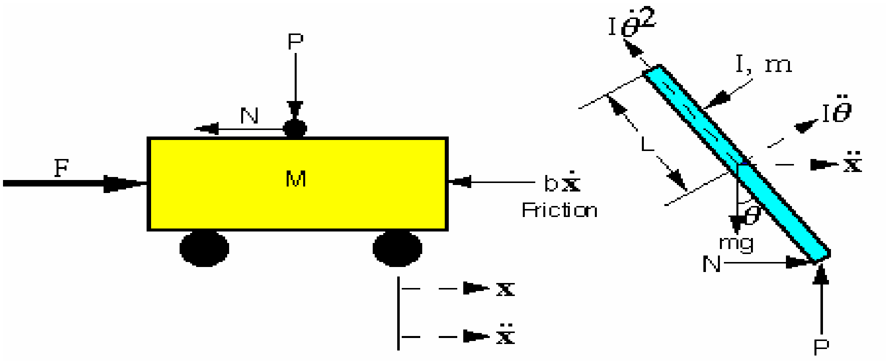

Figure 2. Free body diagrams of the inverted pendulum.[3]

IV. Design of Controllers

In this section, an LQR and a pole placement controllers are presented to control the system under consideration. These controllers work on linearized model of the system.

A. LQR Controller

LQR is a control scheme that provides the best possible performance with respect to some given measure of performance. LQR is a method in modern control theory that uses state-space approach to analyze such a system. Using state-space methods it is relatively simple to work with a multi-output system [6]. Figure 3 shows the full state feedback representation of Inverted pendulum system. The LQR controller is designed using MATLAB. [1]

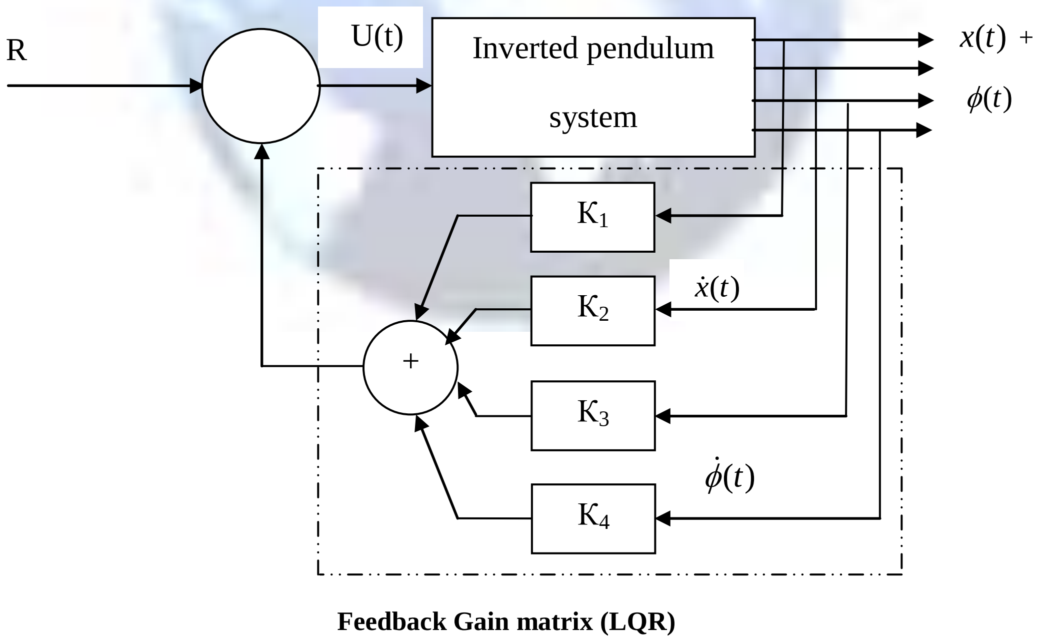

Figure 3. Full-state feedback controller with reference input for the inverted pendulum system.

In this problem, R represents the commanded step input to the cart. The four states, (x , $\dot{x}$, θ , $\dot{\theta}$) represent the position and velocity of the cart and the angle and angular velocity of the pendulum. The output y contains both the position of the cart and the angle of the pendulum.. A controller will be designed so that when a step input is given to the system, the pendulum should be displaced, but eventually return to zero (i.e. the vertical) and the cart should move to its new commanded position. [6]

The state and output matrix equations describing the inverted pendulum can be written as the following equation.[5]

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \qquad 7$$

And that all of the four states are available for the controller. The feedback gain is a matrix K of the optimal control vector

$$K = [K_1 \quad K_2 \quad K_3 \quad K_4] \qquad 8$$
$$u(t) = -K\,x(t)$$

So as to minimize the performance index

$$J = \int_0^\infty (x^T Q\,x + u^T R u)\,dt \qquad 9$$

Where Q is state-cost matrix and R is performance index matrix..For designing LQR controller, the value of the feedback gain matrix, K, must be determined. Figure 4 shown how to determine the values of K.[5]

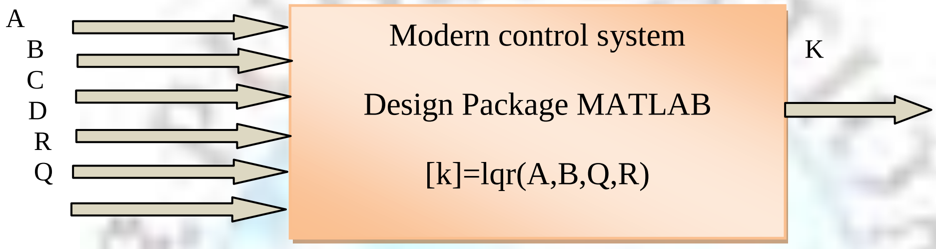

Figure 4. Determine the values of matrix K.

B. Pole Placement Controller Design

In pole placement we aim to place the poles of the closed loop transfer function in reasonable positions. A full state feedback controller based on the pole assignment method can improve the system characteristics such that the closed loop system performance will satisfy the requirement criteria.Figure.6shows the block diagram for the Pole-placement controller. [4]

Algorithm

For a given system:

$$\dot{x} = Ax + Bu \text{ -----} \quad (10)$$

Where,

$x\mathfrak{R}^N$ is the state vector

$u\mathfrak{R}^P$ is the input vector

$A\mathfrak{R}^{N*N}$ is the basis matrix

$B\mathfrak{R}^{N*P}$ is the input matrix

The control law for the Pole-placement controller is given as

u(t) = -K x(t) ----- (11)

where, K is the state feedback gain matrix.

Then the closed loop state equation can be obtained as

$\dot{x} = (A - BK)x$ -----(12)

This state equation describes the system formed by combining the plant and the controller. It is a homogeneous state equation, which has no input. The solution of this state is given by:

$x(t) = e^{(A-BK)t}\ x(0)$ ----- (13)

The state feedback controller u(t)=-K x(t) drives the state to zero for arbitrary initial conditions, provided that the closed loop poles ---the Eigen values $(A - BK)$ of --all have negative real parts. By setting pole locations, we can make the closed loop system not only stable but also satisfy a given set of transient specifications.

A state feedback gain K that yields the closed loop poles {p1,p2,p3 -----pn_x}is obtained by solving the equation
det (sI-A+BK) = (s-p1) (s-p2)---------(s- pn_x) ----- (14)

For designing pole placement controller, the value of the feedback gain matrix, K, must be determined. The following block is shown how to determine the values of K.

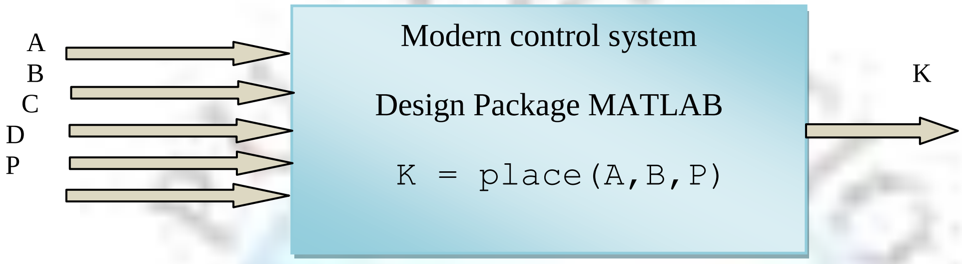

Figure 5. Determine the values of matrix K.

The selection of closed loop system Eigenvalues needs an understanding of the system characteristics and the limitation of the actuator. Different pole locations determine different system performance. This is the critical part of the controller design. By comparing the system performance in simulation, we can select the suitable pole locations.[4]

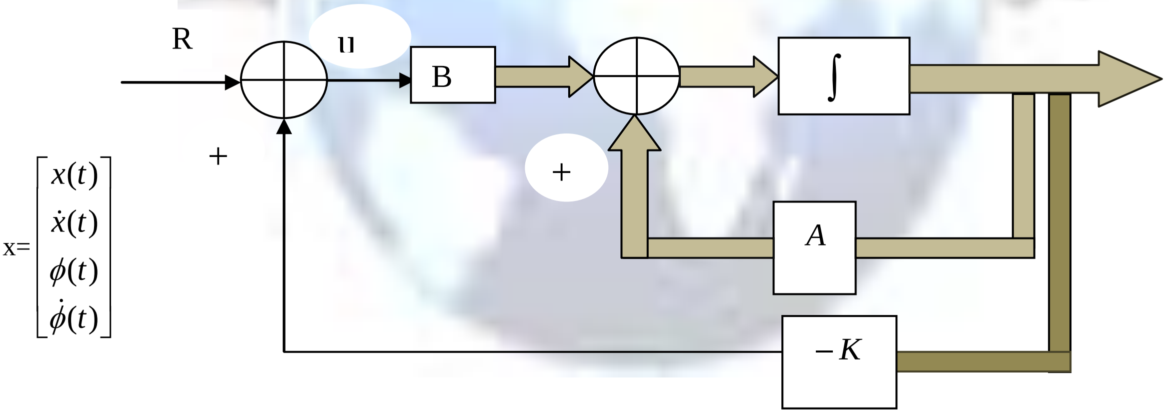

Figure 6: Block diagram for Pole-placement controller.

V. Simulation and Results

The system under consideration and the proposed controllers are modeled and simulated in the MATLAB/Simulink environment. The step response performance of the two controllers is compared. Figure.7shows the Impulse response of the system under consideration in absence of a controller and is found to be unstable. The open loop poles is shown in Table 1.1

Open loop poles	0	-5.6041	-0.1428	5.5651

Table 1.1 Open loop poles

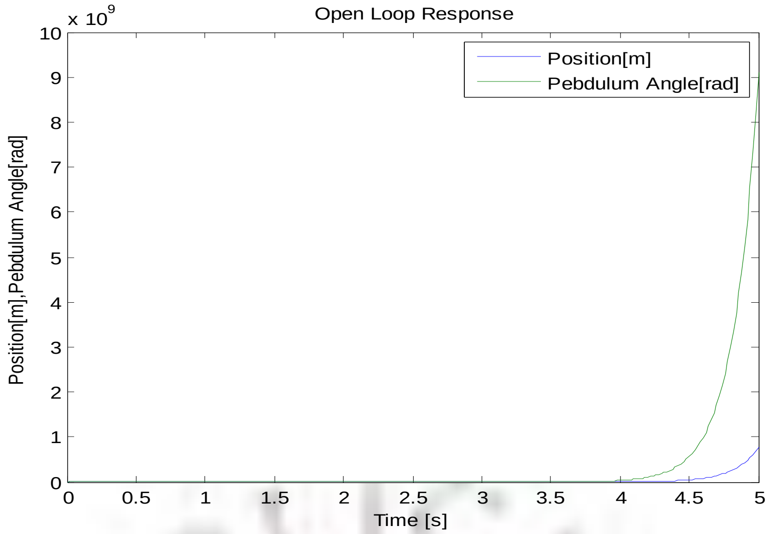

Figure 7 Open loop response for pendulum angle and cart's position using state-space

Figure.8 shows the step response of pendulum angle and cart position by using LQR controller.

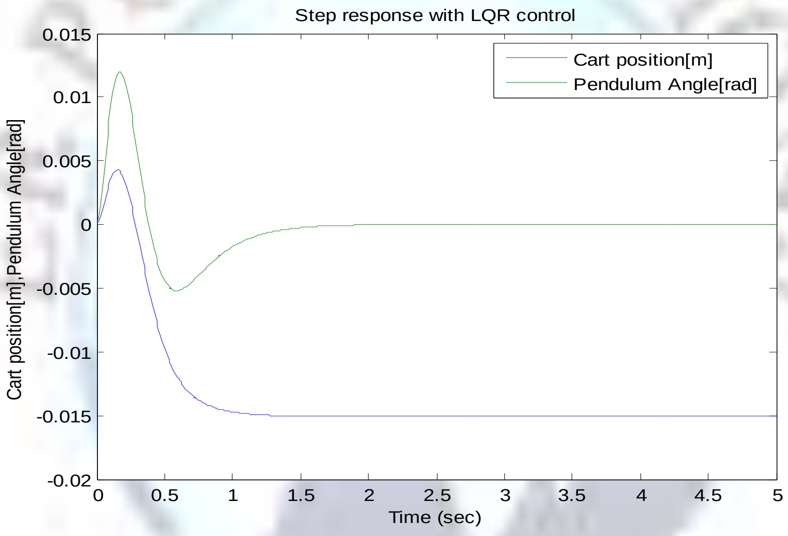

Figure 8. Step response of pendulum angle and cart position with LQR controller.

The LQR controller parameters are $Q = \begin{bmatrix} 4400 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = [1]$

The elements of gain matrix K obtained by LQR method are.

K = [-66.3325 -35.6882 100.4177 19.9371]

The closed loop poles are.

[-8.1092 + 7.5930i-8.1092 - 7.5930i-4.8495 + 0.6516i-4.8495 - 0.6516i]

Figure.9 shows the step response of pendulum angle and cart position by using pole placement controller.

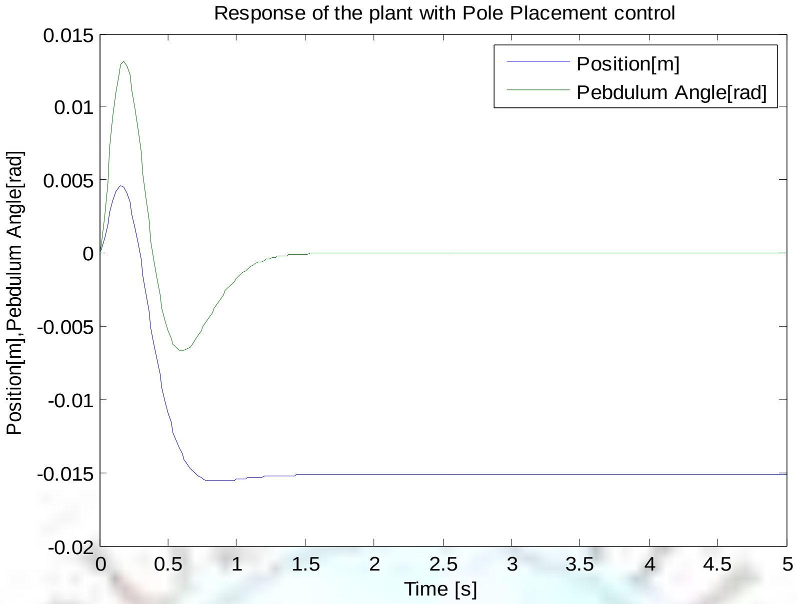

Figure 9. Step response of pendulum angle and cart position with pole placement controller.

The elements of gain matrix K obtained by pole placement controller are.

K=[-65.9860 -32.0726 93.6270 18.3022]

The closed loop poles are [-5.53 +2i -5.53– 2i -7+ 6i -7- 6 i]

Figure.10.shows the step response of cart position using LQR and pole placement controllers

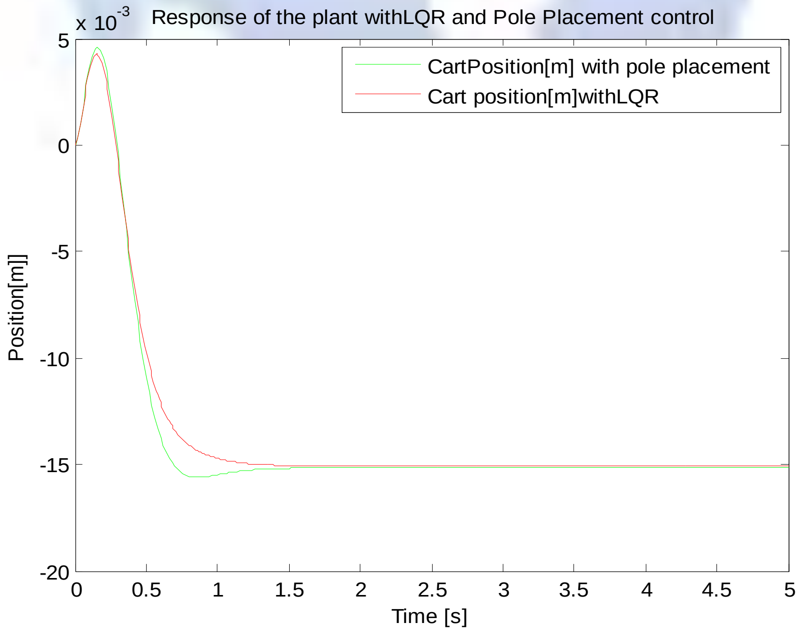

Figure 10. Comparison of step responses for cart position

Figure.11 shows the step response of pendulum angle using LQR and pole placement controllers

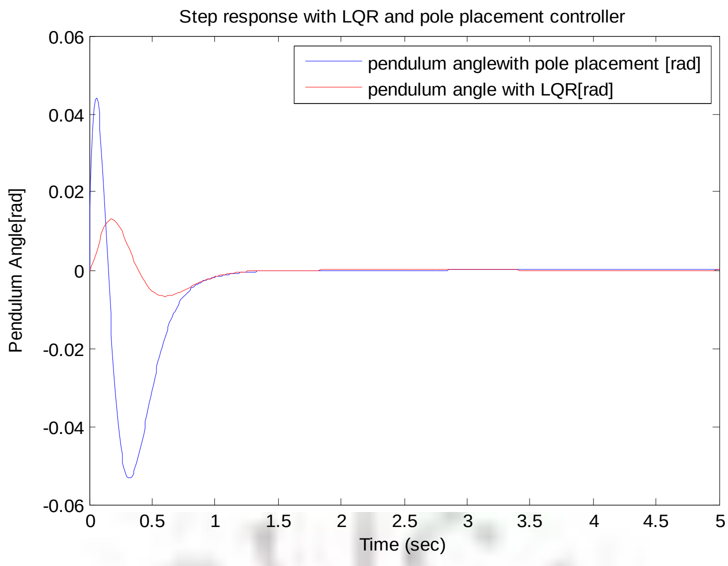

Figure 11. Comparison of step responses for pendulum angle

The time response specifications for the system under consideration equipped with the proposed controllers are given in Tables 2 and 3.

Table 2. Summary Of The Performance Characteristics ForCart Position.

Time response specifications	LQR	Pole placement
Settling Time $\mathbf{T_s}$	0.989 s	0.936 s
Rise Time $\mathbf{T_r}$	0.405 s	0.276 s
Peak amplitude	-0.0151	-0.151
Steady state error $\mathbf{e_{ss}}$	0	0

Table 3. Summary Of The Performance Characteristics For pendulum angle.

Time response specifications	LQR	Pole placement
Settling Time $\mathbf{T_s}$	1.5 s	1.29 s
Peak amplitude	0.012	0.043
Steady state error $\mathbf{e_{ss}}$	0	0

The results show that both the controllers have been successfully designed but the pole placement controller exhibits better response and performance.
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Conclusion

In this paper, an LQR and a pole placement controllers are successfully designed for the inverted pendulum system. Based on the results, it is concluded that both the control methods are capable of controlling the inverted pendulum's angle and the cart's position of the linearized system. However, the simulation results show that pole placement controller has a better performance as compared to the LQR controller in controlling the inverted pendulum system.

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