

New Revised Ones Assignment Method for Solving Traveling Salesman Problem

Mohammed Ahmed Shihab Alkailany

College of Computer Sciences and Mathematics, Dept: Operation Research, University of Mosul, Iraq

ABSTRACT

Traveling Salesman Problem (TSP) is special case from assignment problem (AP) except there is a conditional restriction, $C_{ij} = \infty$, if $i=j$. Traveling Salesman Problem has to visit n cities. In this paper we present new revised ones assignment method to solve TSP, We obtain an optimal solution for Traveling Salesman Problem by New Revised Ones Assignment Method, the results of the tests show that New Revised ones assignment is superior to some other methods such as ones assignment method, Revised Ones Assignment method.

Keyword: Traveling Salesman Problem (TSP), ones assignment method, Linear integer programming.

1. INTRODUCTION

Assignment problems deal with the question how to assign n objects to m other objects in an injective fashion in the best possible way. An assignment problem is completely specified by its two components the assignments, which represent the underlying combinatorial structure, and the objective function to be optimized, which models "the best possible way". The assignment problem refers to another special class of linear programming problem where the objective is to assign a number of resources to an equal number of activities on a one to one basis so as to minimize total costs of performing the tasks at hand or maximize total profit of allocation. In other words, the problems is, how should the assignment be made so as to optimize the given objective [7]

The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization studied in operations research (O R). a map of cities is given to the salesman and he has to visit all the cities only once to complete a tour such that the length of the tour is the shortest among all possible tours for this map.[3]

We have so far already discussed the examples and algorithm for solving a traveling salesman problem using a new revised ones assignment method. Now in this Paper we discuss how the new revised ones assignment method for can be applied for Travelling Salesman Problem. For this we have considered an example related with travelling salesman problem and explain in detail how to find optimal solution using new revised ones assignment method of assignment problem.

2. Mathematical formulation of travel salesman problem

The mathematical formulation of the problem can be Optimize

$$\begin{aligned}
 [\min \text{ or } \max] z &= \sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij} \\
 \sum_{j=1}^n x_{ij} &= 1 \quad i = 1, 2, \dots, m \\
 \sum_{i=1}^m x_{ij} &= 1 \quad j = 1, 2, \dots, n \\
 x_{ij} &= 1 \text{ or } 0 \quad \forall i, j
 \end{aligned}$$

Solution must be a cycle. As in the Problem, d_{ij} , with $i, j = 1..n$, is the distance (or cost, etc.) to go from city i to city j , and x_{ij} is to be integer positive or zero, so the condition of $x_{ij}=0$ or 1, is automatically satisfied [2]

3. New Revised Ones assignment Method

We presents a new algorithm to Solve the Traveling Salesman Problem (TSP).
Now , Consider the distance matrix $[d_{ij}]$ where d_{ij} is the distance from i to city j .

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & . & . & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ . \\ n \end{matrix} & \begin{bmatrix} d_{11} & d_{12} & d_{13} & . & . & d_{1n} \\ d_{21} & d_{22} & d_{23} & . & . & d_{2n} \\ d_{31} & d_{32} & d_{33} & . & . & d_{3n} \\ . & . & . & . & . & . \\ d_{n1} & d_{n2} & d_{n3} & . & . & d_{nn} \end{bmatrix} \end{matrix}$$

If $d_{ij} = d_{ji}$ then problem is called symmetric traveling salesman problem (stsp)

The new algorithm is as follows:

Step 1:

Calculated $K_i = \text{sum } r_i / \min \text{sum } r_i$ for all $i, i=1,2,3,\dots,n$

Where:

$\text{Sum } r_i = \text{sum of elemets of each row}$, if $k_i > 2$ then go to step 2 otherwise applied theorem then step 2.

Step 2:

Find the min (max) of each row in the distance matrix (say a_i) also find min (max) of each column in the distance matrix (say b_j).

Step 3:

Calculated $d_{ij} / a_i * b_j$ for all i, j

This means the divide each element d_{ij} by corresponding product of i row and j column.

Step 4

Draw the minimum number of lines to cover all the ones of the matrix. If the number of drowned lines less than n , then the complete solution is not possible, while if the number of lines is exactly equal to n , then the complete solution is obtained.

Step 5

If a complete solution is not possible in step 3, then select the smallest (largest) element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new ones to this row or column.

Steps (4-5) are actually same as discussed [5][6]

3-1 Theorem :

If a number is added to or subtracted from all of the entries of any one row or column of a cost matrix, then on optimal assignment for the resulting cost matrix is also an optimal assignment for the original cost matrix. [8]
By used the theorem we will making k_i least from 2, through added or subtract a number to any row.

4. Numerical examples

We take some examples to help us understand the topic:

Example 1: Consider the following traveling salesman problem. Design a tour to five cities to the salesman such that minimize the total distance. Distance between cities is shown in the following matrix.[1]

	1	2	3	4	5
1	—	10	3	6	9
2	5	—	5	4	2
3	4	9	—	7	8
4	7	1	3	—	4
5	3	2	6	5	—

Step 1 gives us

	1	2	3	4	5	sum	Ki
1	-	10	3	6	9	28	1.86
2	5	-	5	4	2	16	1.06
3	4	9	-	7	8	28	1.86
4	7	1	3	-	4	15	1
5	3	2	6	5	-	16	1.06

Since all the values of k_i less than 2 .So start the solution without use theorem, go step 2. Then proceed all steps

	1	2	3	4	5	min
1	-	10	3	6	9	3
2	5	-	5	4	2	2
3	4	9	-	7	8	4
4	7	1	3	-	4	1
5	3	2	6	5	-	2
Min	3	1	3	4	2	

→

	1	2	3	4	5
1	-	10/3	1/3	1/2	3/2
2	5/6	-	5/6	1/2	1/2
3	1/3	9/4	-	7/16	1
4	7/3	1	1	-	2
5	1/2	1	1	5/8	-

→

	1	2	3	4	5
1	-	10	1	3/2	9/2
2	5/3	-	5/3	1	1
3	1	27/4	-	21/16	3
4	7/3	1	1	-	2
5	4/5	8/5	8/5	1	-

Optimal Solution is $3+2+4+1+5=15$,this means from city 1 to city 3 distance equal 3, from city 2 to city 5 distance equal 2, from city 3 to city 1 distance equal 4, from city 4to city 2 distance equal 2, from city 5 to city 4 distance equal 5.

Discussion :

In ones assignment method get an optimal distance is 16 while new revised ones assignment method , we get an optimal distance is 15 that means the suggest method (new revised) is the better, also the solution consist in two cycle which are (1,3),(3,1) ; (4,2),(2,5),(5,4).

Example 2: Consider the following traveling salesman problem. Design a tour to five cities to the salesman such that minimize the total distance. Distance between cities is shown in the following matrix. [1]

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} - & 11 & 10 & 12 & 4 \\ 2 & - & 6 & 3 & 5 \\ 3 & 3 & 12 & - & 14 & 6 \\ 4 & 6 & 14 & 4 & - & 7 \\ 5 & 7 & 9 & 8 & 12 & - \end{bmatrix} \end{matrix}$$

	1	2	3	4	5	sum	Ki
1	-	11	10	12	4	37	2.3
2	2	-	6	3	5	16	1
3	3	12	-	14	6	35	2.1
4	6	14	4	-	7	31	1.9
5	7	9	8	12	-	36	2.2

ki contains values from 2, So use theorem 1 , subtract 2 from row 1 and Add 1 in row 2 , so that ki becomes less than 2,

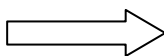
	1	2	3	4	5	sum	Ki
1	-	9	8	10	2	29	1.4
2	3	-	7	4	6	20	1
3	3	12	-	14	6	35	1.7
4	6	14	4	-	7	31	1.5
5	7	9	8	12	-	36	1.8

step 2. Then proceed all steps

	1	2	3	4	5	min
1	-	9	8	10	2	2
2	3	-	7	4	6	3
3	3	12	-	14	6	3
4	6	14	4	-	7	4
5	7	9	8	12	-	7
Min	3	9	4	4	2	



	1	2	3	4	5
1	-	1/2	1	5/4	1/2
2	1/3	-	7/12	1/3	1
3	1/3	4/9	-	7/6	1
4	1/2	7/18	1/4	-	7/8
5	1/3	1/7	2/7	3/7	-



	1	2	3	4	5
1	-	1	2	5/2	1
2	1	-	7/4	1	3
3	1	4/3	-	7/2	3
4	2	14/9	1	-	7/2
5	7/3	1	2	3	-

Optimal Solution is $4+3+3+4+9=23$

Discussion :

We note that optimal distance of this new revised ones assignment method is 23, also the optimal path is (1-5),(5-2),(2-4),(4-3),(3-1) , which is same as obtained by ones assignment methods , therefore the two methods are success in get optimal solution

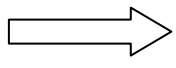
Example 3- Consider the following traveling salesman problem an Eight city TSP for which the cost between the city pairs [5].

	1	2	3	4	5	6	7	8
1	—	2	11	10	8	7	6	5
2	6	—	1	8	8	4	6	7
3	5	12	—	11	8	12	3	11
4	11	9	10	—	1	9	8	10
5	11	11	9	4	—	2	10	9
6	12	8	5	2	11	—	11	9
7	10	11	12	10	9	12	—	3
8	10	10	10	10	6	3	1	—

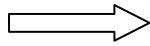
Sol:

	1	2	3	4	5	6	7	8	sum	Ki
1	-	2	11	10	8	7	6	5	49	1.22
2	6	-	1	8	8	4	6	7	40	1
3	5	12	-	11	8	12	3	11	62	1.55
4	11	9	10	-	1	9	8	10	58	1.45
5	11	11	9	4	-	2	10	9	56	1.4
6	12	8	5	2	11	-	11	9	58	1.45
7	10	11	12	10	9	12	-	3	67	1.67
8	10	10	10	10	6	3	1	-	50	1.25

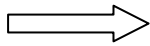
Since all the values of ki less than 2 .So start the solution without use theorem.



	1	2	3	4	5	6	7	8	min
1	-	2	11	10	8	7	6	5	2
2	6	-	1	8	8	4	6	7	1
3	5	12	-	11	8	12	3	11	3
4	11	9	10	-	1	9	8	10	1
5	11	11	9	4	-	2	10	9	2
6	12	8	5	2	11	-	11	9	2
7	10	11	12	10	9	12	-	3	3
8	10	10	10	10	6	3	1	-	1
min	5	2	1	2	1	2	1	3	



	1	2	3	4	5	6	7	8
1	-	1/2	11/2	5/2	4	7/4	3	5/6
2	6/5	-	1	4	8	2	6	7/3
3	1/3	2	-	11/6	8/3	2	1	11/3
4	11/5	9/2	10	-	1	9/2	8	10/3
5	11/10	11/4	9/2	1	-	1/2	5	3/2
6	6/5	2	5/2	1/2	11/2	-	11/2	3/2
7	2/3	11/6	4	5/3	3	2	-	1/3
8	2	5	10	5	6	3/2	1	



	1	2	3	4	5	6	7	8
1	-	1	11	5	8	7/2	6	5/3
2	6/5	-	1	4	8	2	6	7/3
3	1	6	-	11/2	8	6	3	11
4	11/5	9/2	10	-	1	9/2	8	10/3
5	11/5	11/2	9	2	-	1	10	3
6	12/5	4	5	1	11	-	11	3
7	2	11/2	12	5	9	6	-	1
8	2	5	10	5	6	3/2	1	-

Discussion:

We note that optimal distance of this new revised ones assignment method is 17, the solution consist in three cycle which are (1,2), (2,3), (3,1); (4,5), (5,6), (6,4); (7,8), (8,7) and which is same as obtained by algorithm revised ones assignment methods , therefore the two methods are success in get optimal solution.

5. Conclusion:

In this paper , we have discussed and studied an New Revised Ones Assignment Method for solving TSP, where new method proved to be better in some case or a similar in finding optimal solution when compare with ones assignment ,or revised ones assignment method.

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