Modified Bat Algorithm (MBA) on non-linear programming

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ABSTRACT

Heuristic optimization techniques have became very general techniques and have diffuse know areas. Main purpose of these techniques is to achieve good performance on problem . One of these techniques is Bat Algorithm (BA). BA is an optimization algorithm based on echolocation usually of bats and promoted by simulate of bats’ behavior. In this study, exploration and exploitation mechanisms of bat algorithm are improved by Using descent directions(Powell) . Performance of proposed and standard version of algorithm is compared on eight basic criterion test problems. Results indicate that proposed version is preferable than standard version in terms of standard deviation and other parameters.

Index Terms: Bat algorithm, continuous optimization,, heuristics, descent directions(Powell), Optimal design .

1- INTRODUCTION

Optimization algorithms can be generally categorized as deterministic and non-deterministic (random) algorithms. Metaheuristics are well-known non-deterministic optimization algorithms that are utilized in engineering optimization problems and are in a progressive state of development. Many researchers are concerned with structural design optimization via various meta-heuristic algorithms .[4]

In decision theory, mathematics and information science, optimization is based on the selection of the best solution within certain given domain, which can minimize or maximize some function. A lot of problems in business and industry belong in the group of intractable continuous or discrete optimization problems. Different approaches have been developed for solving such optimization problems [1]

The bat algorithm (BA) was first presented by Yang (2010). The basic idea behind the BA is that a population of bats (possible solutions) use echolocation to sense distance and fly randomly through a search space updating their positions and velocities. The bats’ flight aims at finding food/prey (best solutions). A loudness decay factor acts in a similar role as the cooling schedule in the traditional simulated annealing optimization method, and a pulse increase factor regulates the pulse frequency. As the loudness usually decreases once a bat has found its prey/solution (in order to not to loss the prey), the rate of pulse emission increases in order to raise the attack accuracy.[9]

Echolocation is an important feature of bat behavior .That means, bats emit a sound pulse and listen to the echo bouncing back from obstacles whilst flying. This phenomenon has been inspired Yang to develop the Bat Algorithm (BA). The algorithm obtained good results where dealing with lower-imensional optimization problems, but may become problematic for higher-dimensional problems because it tends to converge very fast initially. On the other hand, differential evolution is a typical evolutionary algorithm with differential mutation, crossover and selection that was successfully applied to continuous function optimization.[2]

2- BAT ALGORITHM

Bat algorithm has been developed by Xin-She Yang in 2010 . The algorithm exploits the so called echolocation of bats. Bats use sonar echoes to detect and avoid obstacles. It is generally known, that sound pulses are transformed to frequency
which reflects from obstacle. Bats can use time delay from emission to reflection and use it for navigation. They typically emit short loud, sound impulses. The pulse rate is usually defined as 10 to 20 times per second. After hitting and reflecting, bats transform their own pulse to useful information to gauge how far away the prey is. Bats are using wavelengths, that vary from range [0.7,17] mm or inbound frequencies [20,500] kHz. By implementation , pulse frequency and rate has to be defined. Pulse rate can be simply determined from range 0 to 1, where 0 means there is no emission and by 1, bats are emitting maximum.

2.1 - Bat Algorithm Idealized Rules:

1- All bats use echolocation to sense distance, and they also know the difference between food/prey and background barriers.
2- Bats fly randomly with velocity \( v_i \) at position \( x_i \), with a fixed frequency \( f_{min} \), varying wavelength \( \lambda \) and loudness \( A_0 \) to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission \( r \in [0, 1] \), depending on the proximity of their targets;
3- Although the loudness can vary in many ways, we assume that the loudness varies from a large (positive) \( A_0 \) to a minimum value \( A_{min} \).

2.2- Movement of Virtual Bats

In BA, each bat is defined by it’s position \( x_i \), velocity \( v_i \), frequency \( f_i \), loudness \( A_i \), and the emission pulse rate \( r_i \). In a D-dimensional search space. The new solutions \( x_i \) and velocities \( v_i \) at time step \( t \) are given by

\[
\begin{align*}
    f_i &= f_{min} + (f_{max} - f_{min})\beta \quad \ldots \ldots (1) \\
    v_i^{t+1} &= v_i^{t} + (x_i - x^*)f_i \quad \ldots \ldots (2) \\
    x_i^{t+1} &= x_i^{t} + v_i^{t+1} \quad \ldots \ldots (3)
\end{align*}
\]

where \( \beta \in [0, 1] \) is a random vector drawn from a uniform distribution.

2.3 - Loudness and Pulse Emission

In order to provide an effective mechanism to control the exploration and exploitation and switch to exploitation stage when necessary, we have to vary the loudness \( A_i \) and the rate \( r_i \) of pulse emission during the iterations. Since the loudness usually decreases once a bat has found its prey, while the rate of pulse emission increases, the loudness can be chosen as any value of convenience, between \( A_{min} \) and \( A_{max} \), assuming \( A_{min} = 0 \) means that a bat has just found the prey and temporarily stop emitting any sound. With these assumptions, we have

\[
\begin{align*}
    A_i^{t+1} &= \alpha A_i^t \quad r_i^{t+1} = r_i^t[1 - \exp(-\gamma t)] \quad \ldots \ldots \ldots (4)
\end{align*}
\]

where \( \alpha \) and \( \gamma \) are constants. In essence, here \( \alpha \) is similar to the cooling factor of a cooling schedule in simulated annealing. For any \( 0 < \alpha < 1 \) and \( \gamma > 0 \), we have

\[
\begin{align*}
    A_i^t \rightarrow 0, \quad r_i^t \rightarrow r_i^0, \quad \text{as} \quad t \rightarrow \infty \quad \ldots \ldots (5)
\end{align*}
\]

2.4- Original Bat Algorithm

1: Objective function \( f(x) \), \( x = (x_1, \ldots, x_d)^T \)
2: Initialize the bat population \( x_i \) and \( v_i \) for \( i = 1 \ldots n \\)
3: Define pulse frequency \( f_i \) at \( x_i \\
4: Initialize pulse rates \( r_i \) and the loudness \( A_i \\
5: while (t < T_{max}) // number of iterations
6: Generate new solutions by adjusting frequency and update velocities and locations/solutions
7: if(rand(0, 1) > r_i)
8: Select a solution among the best solutions
9: Generate a local solution around the best solution
10: end if
11: Generate a new solution by flying randomly
12: if(rand(0, 1) < \( A_i \) and \( f(x_i) < f(x^*) \))
13: Accept the new solutions
14: Increase \( r_i \) and reduce \( A_i \)
15: end if
16: Rank the bats and find the current best
17: end while
18: Postprocess results and visualization

3- STATEMENT OF UNCONSTRAINED OPTIMIZATION

Conjugate Directions in Minimization Methods

In the minimization algorithm

\[
x_{i+1} = x_i + \alpha_i d_i, \quad i = 0, 1, ... .... (6)
\]

If directions \( d_i \) are chosen to be conjugate, when (6) is applied to quadratic function

\[
f(x) = a + b^T x + \frac{1}{2} x^T G x \quad ...........(7)
\]

Where \( G \) is an \( n \times n \) symmetric matrix and \( b \in \mathbb{R}^n \) both of which have constant elements from the necessary conditions for a minimum \( \bar{x} \).

\[
g(\bar{x}) = G\bar{x} + b = 0 \quad ...........(8)
\]

If \( G \) is positive definite, then

\[
\bar{x} = -G^{-1} b \quad ...........(9)
\]

Is a strong local minimum.

The only difference seems to be the lengths of the conjugate direction step. However, we will show that the two algorithms are identical.

Consider the function

\[
H(x) = (\bar{x} - x)^T G (\bar{x} - x) = \bar{x}^T G\bar{x} - 2x^T G\bar{x} + x^T Gx \quad ...........(10)
\]

\[
= -\bar{x}^T b + 2b^T x + x^T Gx \quad ...........(11)
\]

\[
= 2a + 2b^T x + x^T Gx (say) = 2f(x) \quad ...........(12)
\]

From (7). Now ,

\[
H(x_i + \alpha_i d_i) = H(x_i) + \alpha_i^2 d_i^T Gd_i + 2\alpha_i d_i^T g_i \quad ...........(13)
\]

In order to find the \( \alpha_i \) for which \( H(x_i + \alpha_i d_i) \) is a minimum, we put

\[
dH/d\alpha_i = 0 \quad .......(14)
\]

That gives

\[
\alpha_i = -\frac{g_i^T d_i}{d_i^T G d_i} \quad .......(15)
\]

All the optimization algorithms must be iterative. [8]
An initial guess $x^0$ for a local minimum $x^*$ need be given, and a sequence of iterates $x_{k+1}$,

$$k = 0, 1, 2, \ldots \ldots$$

will be generated. It is expected that the sequence $\{x_k\}$ converges to $x^*$ as $k \to \infty$. A general approach is to specify a direction $d_k$, called the search direction, along with direction from $x_k$ we determine the new iterate, i.e. we determine $x_{k+1}$ in terms of

$$x_{k+1} = x_k + \alpha_k d_k$$

\hspace{10cm} \text{……….}(16)

where $\alpha_k$ called the step size, represents how far to move along the direction $d_k$ from $x_k$. Two questions need to be answered to furnish the iteration step: how to choose the search direction $d_k$ and how to choose the step size $\alpha_k$. [3]

4. MODIFY BAT ALGORITHM

A new bat algorithm, called Modify Bat Algorithm (MBA) is proposed in this paper. That is, the original bat algorithm was modified by using the unconstrained optimization. The pseudo-code of the MBA is illustrated in Algorithm

4.1- Modify Bat Algorithm

1: Objective function $f(x), \quad x = (x_1, \ldots, x_d)^T$
2: Initialize the bat population $x_i$ and $v_i$ for $i = 1 \ldots n$
3: Define pulse frequency $f_i$ at $x_i$
4: Initialize pulse rates $r_i$ and the loudness $A_i$
5: while $(t < T_{max})$ // number of iterations
6: Generate new solutions by adjusting frequency and updating velocities and locations/solutions
7: if(random(0,1) > $r_i$)
8: Modify the solution using "UO [eq 15 - 16] "
9: end if
10: Generate a new solution by flying randomly
11: if(random(0,1) < $A_i$ and $f(x_i) < f(x)$)
12: Accept the new solutions
13: Increase $r_i$ and reduce $A_i$
14: end if
15: Rank the bats and find the current best
16: end while
17: Postprocess results and visualization

4.2- Experiment results

In order to verify efficiency of proposed modifications (MBA), both of the algorithms are tested on benchmark test functions with dimensions as seen in Table I. The values of “Best, worst, mean, median, standard deviation” are shown in Table I.

<table>
<thead>
<tr>
<th>Fu \hspace{0.1cm}c \hspace{0.1cm}ti \hspace{0.1cm}on</th>
<th>General Bat</th>
<th>\hspace{0.1cm}Modified Bat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function Dimension $f_{min}$</td>
<td>Best</td>
<td>Worst</td>
</tr>
<tr>
<td>$F_1$</td>
<td>10</td>
<td>1.8046</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.1907</td>
</tr>
</tbody>
</table>

Table I.
Algorithms are tested with 20 independent runs, the number of individual (bat) in population is fixed to 20. Maximum cycle number is set 2000, for $d=10$. Limit parameter is set to 200, boundaries of inertia factors $W_{max}$ and $W_{min}$ are fixed to 0.9 and 0.2 respectively. $f_{\min}$ is 0 and $f_{\max}$ is 0.1 while $\alpha$ and $\gamma$ are 0.9 for both algorithm. As seen in table I, Although the performance of BA and MBA decreases as dimension of functions increases, it is seen that MBA has better performance than BA on all benchmark test functions with all dimensions except function $f_{4}$ with $d=10$ was std.dev larger.

**CONCLUSION**

We have surveyed some recently introduced nature inspired algorithms like: Bat algorithm, the main of the work is expansion standard bat and test performance expansion bat by used benchmark test functions (look Appendix), which appear results best from bat standard. also we are improved by Using descent directions(Powell).

**REFERENCES**

[3]. Hassam, B.A.(2010)"Modified Variable Metric Algorithms With Exact Line Searches Based on the Quadratic Models " PhD. Thesis Mathematics Computational (not isser), University of Mousl ,Iraq. 9
[7]. Wyk D.v. (1977)" Generalization of Conjugate Direction Methods In the Optimization of Functions, Jurnal of Optimization Theory and Applications Vol.21 No.4 April 1977, New York. 10
[8]. Xrause, Jo., Cordeiro,Je., Parpinell,R.S. and Lopes, H.s.(2013) "A Survey of Swarm Intelligence and Bio-Inspired Computation Elsevier Inc. Brazil. 2
APPENDIX:

THE TEST FUNCTIONS

1. \[ f = \sum_{i=1}^{3} \left( (\exp(x_{4i-3} - x_{4i-2}))^4 + 100 \ (x_{4i-2} - x_{4i-1})^6 + (\tan(x_{4i-1} - x_{4i})^4 + (x_{4i-3})^8 + (x_{4i} - 1)^2 \right) \]

2. \[ f = \sum_{i=1}^{5} (100(x_{2i} - x_{2i-1})^2 + (1 - x_{2i-1})^2) \]

3. \[ f = (1 - x_1)^2 + (1 - x_{10})^2 + \sum_{i=2}^{9} (x_i^2 + x_{i+1})^2 \]

4. \[ f = \sum_{i=1}^{3} \left( (\exp(x_{4i-3} - x_{4i-2}))^4 + 100 \ (x_{4i-2} - x_{4i-1})^6 + (\tan(x_{4i-1} - x_{4i})^4 + (x_{4i-3})^8 \right) \]

5. \[ f = \sum_{i=1}^{5} (x_{2i-1}^2 - (x_{2i})^2 + (1 - x_{2i-1})^2) \]

6. \[ f = \sum_{i=1}^{3} (x_{4i-3} - 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-2} - x_{4i})^4 \]

7. \[ f = \sum_{i=1}^{5} (100(x_{2i} - x_{2i-1})^2 + (1 - x_{2i-1})^2) \]

8. \[ f = \sum_{i=1}^{10} (x_i^4 - i)^2 \]