Spectrum Nulling TH Codes with Improved Correlation Performance for Multi-user UWB Systems

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Abstract

The power spectrum of UWB-IR systems can be controlled by proper selection of any of the three entities pulse, modulation scheme and Time Hopping (TH) codes, or their combinations. This paper presents algorithms to design TH codes with good correlation properties for spectrum nulling at a selected frequency, of multiuser UWB signals, to avoid interference to narrow band systems. To improve the correlation between sequences, variation in frequency hopping (FH) codes, LCC and one Coincidence FH codes are applied. Results are presented to verify the analysis.

Keywords: Ultra Wide Band (UWB) system, impulse radio (IR), time hopping (TH) codes, frequency hopping (FH) codes.

I. INTRODUCTION

Ultra Wide Band (UWB) wireless communication technology has recently reached an appreciable degree of development so as to able to support high data rates with low power consumption and low complexity in terms of transmission/reception operation. In the past two decades, UWB has been used for radar, remote sensing, and military communications. On Feb 14, 2002, the Federal Communications Commission (FCC) in the United States issued a ruling that UWB could be used for data communication as well as for radar and safety applications. FCC defined the UWB signal as “A signal is classified as UWB if its fractional bandwidth is larger than 20% or its bandwidth is larger than 500 MHz” [1]. A significant bandwidth of 7.5 GHz (3.1-10.6 GHz) was allocated [1]. This enormous bandwidth is the largest allocation to any commercial terrestrial system, and so has the potential to offer data rates on the order of gigabits per second (Gbps). The UWB allocation was approved with the restriction of very low power levels (below -41.3 dBm/MHz or 0.556 mW/MHz) [1]. This allows UWB technology to overlay already available services such as the global positioning system (GPS) and IEEE 802.11 a/b/g wireless local area networks (WLANs) that lie in the 3.1-10.6 GHz band [2].

UWB systems are mostly based on Impulse Radio (IR) technology [2] [3]. IR based implementation of the receiver structure is very simple, where intermediate frequency (IF) processing is not required. This feature gives UWB systems an advantage in low-cost receiver designs. The low-power spectral density, low-cost features make UWB systems suitable in applications such as realtime, high-data-rate home entertainment systems, sensor networks, and systems that can exploit the geolocation capability of UWB. Impulse radio UWB uses narrow pulses of the order of sub-nanoseconds duration, modulated either in time or in amplitude. In a multipath environment, hundreds or thousands of echos of the narrow pulses can be resolved by the receiver. This results in a large diversity gain which can be exploited to improve the performance and make it suitable for tactical Ad-hoc military communication systems [4].

The modulation schemes generally adopted by these systems are binary pulse position modulation (BPPM) and BPSK. There are two commonly used Multiple Access (MA) techniques for UWB systems, namely Time Hopping (TH) and Direct Sequence (DS) [5]. TH is gaining more popularity over DS among researchers, because 1) It has much better multipath immunity over DS [6]; 2) TH has better spectral efficiency than DS [7]; 3) Recent technology advances allow to generate pulses of sub-nanosecond duration making Time Hopping (TH) an attractive option for UWB systems.

Since UWB signals would range from d.c. to several GHz, UWB systems and conventional narrowband systems cannot help giving interferences to each other [8] [9].Therefore, the coexistence problem between UWB and narrowband systems, and the effects of their mutual interference are being researched very extensively [10] [11] [12].

In the literature only a few analysis for the PSD of UWB signals are available which have resulted in some methods for the reduction of line spectra depending on TH sequences and modulation schemes [2] [7] [16] [17]. Careful design of the TH codes controls the power spectrum for good coexistence with other technologies, and improves synchronization between the transmitter and the receiver [7] [13] [18] [15]. The design of TH codes for narrowband interference mitigation is suggested in [14]. In multiuser environment multiuser interference play a great role on the receiver performance and it needs that the TH codes should have good correlation properties [18] [15]. The researchers found that TH codes are similar in construction to FH Codes and the meaning of correlation properties of TH codes is also similar to that of FH codes [15] [20]. The authors of [15] also found one family of FH code suitable for TH-UWB systems.
In [16] authors proposed some methods to create spectral null in UWB systems to avoid interference. But multiuser UWB system and correlation properties of TH codes have not been addressed. The system model in [16] is valid only for a single user per period and the TH codes are randomly generated. It has been demonstrated in literature, that it is possible to generate codes with better correlation properties than random codes and better correlation properties have direct impact on BER performance of the system [13] [15] [19].

Motivated by the need to reduce narrowband interference by creating null at the desired frequency and to reduce multiuser interference, this paper provides algorithms to improve correlation performance of spectral nulling TH codes in multiuser UWB systems.

The rest of the paper is organized as follows. In section II, a brief description of the system model is given. Sections III explains about the signal power spectrum, while the spectral nulling technique has been described in section IV. Correlation measure of TH sequences is given in section V. Spectrum nulling TH code with improved correlation performance are discussed in Section VI. Finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL

A Time Hopping Binary Phase Shift Keying modulation (TH-BPSK) scheme has been considered for the system model. The MA scheme under consideration is PPM. We assume that \( N_u \) is the number of user can transmit simultaneously. The system transmits a symbol of duration \( T_s \) sec by transmitting a sequence of \( N_s \) short pulses, called monocycles and denoted by \( w(t) \). The symbol time is divided into \( N_s \) frames of duration \( T_f = T_s / N_h \) sec and a monocycle is placed in each frame. The frame is divided into \( N_h \) chips each lasting \( T_c = T_f / N_h \) sec and a pseudo-random TH code is used to place the monocycle in one of the chips. It is convenient to consider the TH code a sequence of ordered integers \( c_{n,k} \) used to specify the location of the transmitted pulse/user in a specified frame. In other words, \( TH \) code is a sequence of ordered integers \( c_{n,k} \) used to specify the location of the transmitted pulse/user in a specified frame. Fig. 1 shows four users transmitting their signal according to the TH codes assigned to them as per the Table I.

<table>
<thead>
<tr>
<th>Users</th>
<th>TH Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>k1</td>
<td>0 1 2 3</td>
</tr>
<tr>
<td>k2</td>
<td>3 4 5 6</td>
</tr>
<tr>
<td>k3</td>
<td>6 7 8 9</td>
</tr>
<tr>
<td>k4</td>
<td>9 10 11 0</td>
</tr>
</tbody>
</table>

Table I: Numerical example of TH codes for \( (N_s = 4; N_h = 12) \)

Fig. 1. Time Hopping multiple access representation with \( (N_s = 4; N_h = 12) \)

The TH-BPSK multiple access UWB signal with periodic TH Codes, for synchronous transmission, can be written as follows for every code period [7] :

\[
s(t) = \sum_{k=0}^{N_u-1} a_k \sum_{n=0}^{N_s-1} w(t - c_{n,k}T_c - nT_f)
\]

where \( a_k \in \{-1, 1\} \) are information bits modulating the monocycle’s amplitude.

III. SIGNAL POWER SPECTRUM

The power spectrum of the system model (1) can be computed as in [7]. The spectrum shape is given by :

\[
S(f) = |W(f)|^2 G(f)
\]

where \( W(f) \) is Fourier Transform (FT) of the monocycle and the term \( G(f) \) is termed as the code spectrum and is given by [7]

\[
G(f) = \sum_{k=0}^{N_u-1} |C_k(f)|^2
\]
where

\[ C_k(f) = \sum_{n=0}^{N_r-1} e^{-j2\pi f(c_n v_N T_c + n T_f)} \]  

(4)

It can be seen from (3) and (4) that the shape of the spectrum can be changed with the help of TH codes. The code spectrum \( C_k(f) \) can be made zero at particular frequency of interest to avoid Narrow Band (NB) interference [16]. In order to validate code spectrum null creation, we need to consider some assumptions and facts. The measurement need to be done only in the band [-1/(2Tc) : 1/(2Tc)] since \( G(f) \) of (3) is periodic with period 1/Tc. To consider the effect of code spectrum, we are using UWB signal constructed with a monocycle [16] \( w(t) = \sin(\pi t/T_c) / (\pi t/T_c) \). Since the FT of that monocycle is 1 in the band [-1/(2Tc) : 1/(2Tc)], and it is zero elsewhere. (2) as \( S_{Th,BPSK}(f) = G(f) \) [16]. For getting signal spectrum, the sequence of samples of \( s(t) \) is obtained by replacing \( w(t) = \sin(\pi t/T_c) / (\pi t/T_c) \) in (1) and sampling at \( t = i T_c \). The equation (1) can be written as:

\[ s_i = u(i T_c) = \sum_{k=0}^{N_r-1} \delta_k \sum_{n=0}^{N_r-1} \delta(i - c_n - n N_h) \]  

(5)

where \( \delta(k) \) is the discrete impulse function, defined by \( \delta(0) = 1 \) and \( \delta(k) = 0 \) for \( k \neq 0 \). The signal spectrum can be estimated by grouping the samples in blocks of \( N \), taking the Discrete Fourier Transform (DFT) of the blocks and squaring it [16].

IV. SPECTRAL NULLING TECHNIQUE

This section explains about the method of creating Null at a desired frequency in the multi-user UWB spectrum. The method is given by [16] and we are using it as a reference method. We have changed the system model to multi-user transmission per period in place of single user per period considered in [16].

Consider (3), in order to zero the code spectrum at a given frequency \( f_0 \) we have to zero \( |C_k(f_0)| \) for every value of \( k \). Let us write the generic \( C_k(f) \) at frequency \( f_0 \), by replacing \( T_f = N_h T_c \) and by dropping the radix \( k \) for the sake of simplifying the notation. Because in this method each code vector independently tries to create a null at desired frequency in the spectrum. We obtain

\[ C(f_0) = \sum_{n=0}^{N_r-1} e^{-j2\pi f_0 c_n v_N T_c} \]  

(6)

From the last equation we see that at any given frequency \( C(f) \) is the sum of \( N_r \) unitary vectors [16]

\[ v_n = e^{-j2\pi f_0 (c_n v_N T_c + n N_h T_c)} \]  

(7)

The phase of these vectors is controlled by the TH code. For the particular case \( f_0 = 1/(K T_c) \), where \( K \) is an integer, we have

\[ v_n = e^{-j2\pi f_0 (c_n + n N_h)} \]  

(8)

Since \( (c_n + n N_h) \) is integer the last equation shows that the vectors take values on the points of a \( K \)-ary Phase Shift Keying (PSK) constellation. And as long as \( K \leq N_h \) we can drive every vector in every point of the constellation by properly choosing the corresponding \( c_n \).

Before proposing the methods for creating spectral null, code correlation properties are given in the next section, which are useful in designing good TH codes.

V. CORRELATION MEASURE OF TH SEQUENCES

A performance measure that is commonly used for the evaluation of the code characteristics is the code correlation function (known as Hamming correlation function in FH literature) [13] [18] [15],

\[ C_{m,n}(\tau) = \sum_{j=0}^{N_r-1} h[j N_h + c_{m,[j+a] N_r}, j N_h + c_{n,j} + b] \]

\[ + \sum_{j=0}^{N_r-1} h[(j + 1) N_h + c_{m,[j+a+1] N_r}, j N_h + c_{n,j} + b], \]  

(9)

where the function \( h[c_A, c_B] \) for any two integers \( c_A, c_B \) is given by

\[ h[c_A, c_B] = \begin{cases} 
0, & \text{if } [c_A \neq c_B] \mod(N_r, N_h) \\
1, & \text{if } [c_A = c_B] \mod(N_r, N_h)
\end{cases} \]  

(10)

The time delay is defined as \( \tau = a N_h + b \), where \( a \) is the integer number of time frames within the delay duration, and \( b \) is the remaining number of chip durations. The code correlation function in (9) shows the number of hits between two different
TH sequences with some relative delay with respect to each other, or between the delayed versions of the same TH sequence. For a synchronous system, taking $\tau = 0$ (i.e., $a = 0$ and $b = 0$), (9) can be seen as Normalized Occurrence of Coincidence (NOC) as [19] [15]

$$C(m, n) = \frac{1}{N_s} \sum_{j=0}^{N_s-1} h[c_{m,j}, c_{n,j}]$$

(VI. Spectrum Nulling TH Code Design with Improved Correlation Performance)

In this section we are proposing some methods to create spectral null with lower NOC value than proposed by [16].

A. Method 1

This method differs from method 1 of [16], called reference method, in the sense that it does not choose $c_{2n}$ randomly each time, whereas it fix the even elements of a sequence to reduce the repetition of elements in the sequence. Assumption for this methods are $N_s$ is even and consider a frequency $f_0 = 1/(KT_c)$ where $K$ is even and $2 \leq K \leq N_h$. The $N_s$ vectors that sum up to form $T(f_0)$ belong to a K-ary PSK constellation [16]. Since $K$ is even every point of the constellation has a corresponding point in phase opposition. We can zero $T(f_0)$ by grouping the vectors in couples and by posing the second vector of every couple in phase opposition with the first one. Specifically consider, without loss of generality, the couples formed by the vectors $v_{2n}$ and $v_{2n+1}$ for n=0,1,2,... From (8) the two vectors will have a phase difference of $\pi$ provided that [16]

$$[c_{2n} + 2nN_h - c_{2n+1} - 2nN_h - N_h]K = K/2$$

(12)
where \( [x]_y \) denotes \( x \) modulo \( y \). Manipulating the last equation yields
\[
[c_{2n+1}]_K = [c_{2n} - N_h - K/2]_K
\] (13)

As long as (13) is satisfied the two vectors will add up to 0. Thus we can summarize modified method 1 for producing the TH code as follows, where \( S = N_h/K \) denotes the integer division of \( N_h \) by \( K \):

1) for \( n = 0, 1, 2, \ldots, N_h/K - 1 \),
2) Get a vector \( \vec{c} \) by random permutation of values \([0, N_h - 1] \).
3) Fix \( c_{2n} \) from \( \vec{c} \).
4) Choose an integer \( s \) randomly in \([0, S-1] \).
5) Compute \( c_{2n+1} = [c_{2n} - N_h - K/2] + sK \).
6) For Next code sequence now Shift \( \vec{c} \) right by one value circularly and assign it as \( \vec{c} \).
7) Repeat step (3) to (6) \( N_h \) times.

So In this way we will get \( N_h \) code sequences for transmitting maximum of \( N_h \) users simultaneously and still able to get spectral null at frequency \( f = 1/(KT_c) \) in multiuser environment. However we will get nulls at other frequencies also at \( f_m = 2m+1/KT_c \), where \( m \) is integer [16]. TH codes generated by this algorithm have Lower NOC value than Method 1 of [16], because even elements of a sequence are non repeating. This can be verified by the following analysis.

**Analysis**

In this analysis we calculate and compare the probability that at least one element of a TH Code coincide in the code sequence generated by **reference method** and proposed Method 1. Since in both the methods odd elements are always independent on even elements, so we can consider two pairs of elements. Let \( x_1, x_2 \) are any two even elements and \( y_1, y_2 \) are any two odd elements. Assume that \( A = (x_1 \neq x_2) \) and \( B = (y_1 \neq y_2) \). Then
\[
P_{co} = 1 - P_{nc}
\]
(14)
Where, \( P_{co} \) is the probability of at least one coincidence and \( P_{nc} \) is the probability of no coincidence between elements of a code sequence.
\[
P(AB) = P(A)P(B/A)
\]
(15)
Where \( P(AB) \) is the pairwise no coincidence probability. (15) can be extended as:
\[
P(AB) = P(A)P(B/A, |x_1 - x_2|_k = 0)P(|x_1 - x_2|_k = 0) + P(B/A, |x_1 - x_2|_k \neq 0)P(|x_1 - x_2|_k \neq 0)]
\]
(16)

For **reference method**: \( P(A) = \frac{N_s - 1}{N_s} \), \( P(B/A, |x_1 - x_2|_k = 0) = \frac{S-1}{S} \), \( P(|x_1 - x_2|_k = 0) = \frac{S-1}{N_h-1} \), \( P(B/A, |x_1 - x_2|_k \neq 0) = 1 \) and \( P(|x_1 - x_2|_k \neq 0) = 1 - \frac{S-1}{N_h-1} \).

By putting above values in (16) we get:
\[
P(AB) = \frac{N_hS - 2S + 1}{N_hS}
\]
(17)
Since total combinations of pairs to be compared are \( \left( \frac{N_s}{2} \right) \), So
\[
P_{nc} = P(AB)^{\frac{N_s}{2}}
\]
(18)
Therefore, with the help of (14) and (18), the probability of at least one coincidence \( P_{com} \) for reference method can be written as:
\[
P_{com} = 1 - \left( \frac{N_hS - 2S + 1}{N_hS} \right)^{\frac{N_s}{2}}
\]
(19)

For **Proposed Method 1**: \( P(A) = 1 \) and following the above steps We get:
\[
P_{com} = 1 - \left( \frac{N_hS - 2S + 1}{(N_h - 1)S} \right)^{\frac{N_s}{2}}
\]
(20)
Where \( P_{com} \) is the probability of at least one coincidence for proposed Method 1. It is clearly indicated by (19) and (20) that \( P_{com} < P_{com} \). As probability of coincidence in a code sequence is lesser in Method 1, so it implies that NOC between these code sequences will also be lesser than sequences created in reference method. For \( N_s = 16, N_h = 16, K = 4 \) and \( S = 4 \), \( P_{com} = 0.9610 \) and \( P_{com} = 0.7622 \), which verifies the analysis.
Simulation Results

For the validation and investigation of our method we present simulation results. We have also assumed $T_c = 1 \text{nsec}$ to compare our results with [16]. In figure 1, we have verified the creation of spectral null for 16 simultaneous users at 250 $MHz$ for $K=4$. In the upper plot for $N_s = 16$ and $N_h = 64$. And in the bottom one $N_s = 16$ and $N_h = 16$. It can be seen in figure 1 that results are as per our expectations.

In our case also the $N_h$ has direct relation with spectral null width. If we compare the -10 dB width of the lower plot of figure 2 ($N_h = 16$) with upper plot ($N_h = 64$), it indicates that null width vary inversely with the value of $N_h$. Correlation properties of sequences can be verified with the help of figures 3 to 5. In figure 3 and 4, all NOC plot of sequences has been plotted for reference method and proposed Method 1 respectively, which indicates the magnitude of NOC between each sequence. Since 16 plots has been plotted per figure, in order to facilitate the viewing the plots is normalized as $\text{NOC magnitude}(n) = \frac{2}{n}$, where n is sequence index, on vertical axis. The horizontal axis indicates the sequence index. In figure 5 we compared the worst case NOC (NOC plot of the sequences having largest NOC magnitude) results among different methods. Considering the legend of figure 5, we can see the worst case NOC magnitude of proposed method 1 is very less than reference method. From figure 3 and 4, overall improvement in NOC magnitude among sequences can be verified. Results also indicate that in proposed method 1 as we keep on increasing value of $K$ the NOC magnitude among sequences decreases and as $K = N_h$, sequences becomes orthogonal to each other.

For further improvement in the correlation value we also propose two modifications in method 1. In method 2 and 3, we will keep all assumptions same and try to improve the sequences in term of NOC magnitude.

$$c_j^{(k)} = [k + j - 1] mod(N_h),$$

(21)

B. Method 2

This method uses the a variant of linear congruence code (LCC) given by [5]
where \(0 < j \leq Ns - 1\) and \(1 < k \leq Nh\). These circularly shifted codes assure that codes for different users are orthogonal. Therefore, as long as \(Nu \leq Nh\), addition of new users to the system does not affect the performance, yielding an invariant bit error rate (BER) \[13\]. Method 2 can be summarized as below:

1) Get \(Nh\) TH code sequences using Method 1.
2) Now choose a code sequence which has least coincidence among its elements.
3) Fix this code sequence as sequence number one as \(c^{(1)}\).
4) Now, for \(k=1,2,...Ns\) and \(j=0,1,2,...Ns - 1\), generate other \(Ns - 1\) code sequences using formula \(c^{(k+1)}_j = c^{(k)}_j\).

Method 2 propose set of codes which will have a constant NOC value among all the codes. Similar analysis, as in method 1, can be done for this method also to show that for \(Ns \leq Nh/2\), probability of coincidence among a code sequence element is very less.

**Simulation Results**

Simulation results indicates that for large value of \(Nh\) we can find out some code sequences, as in step 2 of method 2, with zero coincidence among its elements. Then method 2 will provide us set of \(Ns\) orthogonal code sequences with desired properties. Low NOC magnitude can be verified with the help of figure 5, considering its legend. Spectral nulling properties are same as codes generated by method 1.

![Worst Case NOC Plot](image)

**Fig. 6.** Worst case NOC plot comparison of 16 code sequences \((Ns = 16, Nh = 16)\), generated by proposed method 1,2,3, and reference method, the topmost plot is for method 3, second from the top is for method 2, third from the top is for method 1 and lowest one is for reference method

**C. Method 3**

This method uses a well known One coincidence FH Sequence construction to generate even elements of the desired TH Sequence and apply method 1 on that. In one coincidence FH sequences the maximum non trivial Hamming CCF value is 1. \[20\]. These One coincidence FH sequences are non repeating and orthogonal to each other, in case of synchronous transmission.

For generating a One coincidence FH Sequence- Construction I, Given a finite field \(GF(p)\), first select a primitive element \(\alpha\) of \(GF(p)\) from Appendix A.1 of \[20\]. The first construction of a One coincidence FH Sequence set \(\{a^{(r)}_n\}\) can be described by

\[
a^{(r)}_n = [\alpha^n + r]_p, \quad 0 \leq n < p - 2, \quad 0 \leq r < p - 1
\]  

This construction provides a set of \(M = p\) one coincidence sequences, each of period \(N = p - 1\). So we can take \(Ns = p - 1\), which is an even number satisfying the criterion of method1. But \(Nh\) in this case can not be more than \(p\), we can take \(Nh = p - 1\). Rest all parameters can be kept same as method1. Method3 can be summarized as below:

1) Generate \(Nh\) one coincidence sequences \(c^{(r)}_{fh}\) by appropriately selecting \(p\) and \(\alpha\) using (22).
2) for each FH Sequence,
3) for \(n = 0, 1, 2,...Ns/2 - 1\).
4) Fix \(c_{2n} = c^{(r)}_{fh}\).
5) Choose an integer \(s\) randomly in [0,S-1].
6) Compute \(c_{2n+1} = [c_{2n} - Nh - K/2] + sK\).
7) Repeat step (3) to (6) \(N_h\) times.

Sequences generated by this method are better than given in method 1 in the sense of low NOC value. As method 1 guarantees non coincidence among even elements of a sequence, but method 3 guarantees non coincidence between even elements of a sequence as well as among all sequences also. So would provide low NOC magnitude, theoretically this can be verified by procedure given in analysis of method 1.

**Simulation Results**

Simulation results are shown in figure 5 to compare the worst case NOC. using the legend of figure 5, It can be seen that method 5 provide better NOC plot than all other methods. Spectral nulling properties are same as codes generated by method 1.

**VII. Conclusions**

In this paper, three methods to improve the correlation performance of spectrum nulling TH codes multiuser UWB systems have been proposed. For this purpose modifications in some frequency hopping (FH) codes, LCC and one coincidence FH codes have been proposed. The code constructions given will be useful for the co-existence problem of UWB systems with other narrow-band systems.

Improvement in correlation performance have been demonstrated analytically as well as through simulations. Similar idea can be used to improve correlation properties of other constructions also.

**References**