Performance Comparison of Blind Image Processing Algorithms

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ABSTRACT

Several image processing algorithms have been developed to help recover images distorted by blurring, turbulence, interference, and noise. Such algorithms include Lucy-Richardson (LR); constrained least-squares (CLS), also known as regularized filtering; Wiener filtering (WF), or deconvolution; and the Fractional Fourier Transform (FrFT). If the source of the corruption is known or can be estimated, then all of these techniques perform very high quality image reconstruction. However, when the corruption is not known, we must blindly correct, and only the former three techniques can operate blindly. In this paper, we describe the blind counterparts to the WF, LR and CLS image processing algorithms, and we validate through simulation that CLS performs better than LR, in terms of minimizing the mean-square error (MSE) between the original and reconstructed image, often providing an order of magnitude reduction in MSE over LR; in blind mode WF and LR are nearly identical, so we only consider LR in simulations. CLS is also not as computationally complex as LR, which requires many iterations to produce the image estimate. Hence, for blind image restoration, CLS is the recommended approach.

Keywords: Blind Image Processing, Constrained Least-Squares, Lucy-Richardson, mean-square error (MSE).

1. INTRODUCTION

Image reconstruction, involving recovering an image from impairment, is an important area in the field of image processing. Reconstruction includes image enhancement and/or image restoration. This usually involves processing an image that has been distorted by blurring, turbulence, and/or noise. Removal of noise can often be addressed by spatial or frequency domain filtering [4] and is considered a type of image enhancement. This is often a subjective process, i.e. attempting to make an image more appealing to the eye by removing speckle, salt and pepper noise, or chirp interference, or perhaps by improving contrast to enhance an image. For example, improving the contrast in an x-ray between the white bones and the dark background can be useful to help identify hairline fractures. Image restoration, however, is mostly an objective process in which an image has been degraded in some fashion, such as by blurring or turbulence, and we attempt to recover the original image [4]. The degradation usually takes on some point spread function (PSF), which is just a spatial domain filter, or equivalently an optical transfer function (OTF) in the corresponding frequency domain, and these terms arise from modeling of optical systems [4]. In this paper, we focus on image restoration.

If the PSF is known a priori, the Fractional Fourier Transform (FrFT) can outperform standard image processing algorithms ([5] and [7]). This is because the FrFT enables processing in the optimum dimension of the Wigner-Distribution by allowing for the two dimensions of the image to be transformed using independent rotational parameters [6]. When no a priori knowledge of the image corrupting PSF or OTF function is available, we face a blind image restoration problem. Algorithms that recover images under blind conditions include Wiener Filtering (WF), or deconvolution; Lucy-Richardson (LR); and constrained least-squares (CLS), also called regularized filtering (RF).

The literature on blind image processing algorithms includes [2], where characteristics of the distortion are used to estimate the PSF, followed by performing WF. In [1], these blind algorithms are compared, but the choice of the PSF and the constraints on the LR and CLS algorithms are suboptimum. A hybrid approach of several algorithms in computer tomography (CT) is addressed in [8], but only for noise suppression, and [9] compares the algorithms and develops better WF, but assumes the PSF can be estimated.

In this paper, we describe and compare the WF, LR and CLS algorithms used for blind image restoration. WF performance is not shown, however, because the performance of blind WF is nearly identical to that of LR. The FrFT is also not considered here because it is difficult to estimate both the rotational parameter 'a' and the PSF simultaneously without knowledge of one of them. In other words, since both the image and the PSF are unknown, finding an optimization criteria in another domain, 'a', is difficult and may not even be possible. This is due to the fact that the constraints that are usually applied, such as minimizing the residuals in the image estimate or choosing the image estimate where the noise or PSF takes on a Gaussian or Poisson distribution, can vary with changing domain. Changing
the optimization criteria in each domain is also not a plausible solution as the complexity and computational requirements become formidable.

The paper is organized as follows: Section 2 describes the image distortion and restoration problem. Section 3 presents the three blind image processing algorithms: Wiener Filtering (WF), Lucy-Richardson (LR) and constrained least-squares (CLS). Section 4 shows examples comparing the algorithms, where we show that CLS consistently outperforms LR, without requiring iterations. WF performs similarly to LR, so we do not show both. A summary is given in Section 5.

2. IMAGE RESTORATION PROBLEM

We write a received distorted image \( g(x, y) \) as

\[
g(x, y) = h(x, y) \ast f(x, y),
\]

where \( f(x, y) \) is the original image of size \( M \times N \), \( h(x, y) \) is the degrading PSF, and ‘\( \ast \)’ denotes convolution. The objective is to obtain the best estimate of the original image \( f(x, y) \), denoted \( \hat{f}(x, y) \), without any knowledge of the PSF or of the original image itself. Hence, we wish to blindly estimate \( f(x, y) \). We consider two main types of degradation functions, the first caused by a blurring function, where we define the PSF as a \( 1 \times L \) vector

\[
h_B(x, y) = \left[ \frac{1}{L}, \frac{1}{L}, \ldots, \frac{1}{L} \right],
\]

where \( L \) will be defined later with numerical examples. This PSF has the effect of smearing every pixel out over \( L \) pixels and simulates motion due, for example, to unintended uniform linear motion of a camera as it takes a picture. Note that the pixels may also be smeared out in a different direction by adding a rotational parameter \( \theta \) to \( h_B(x, y) \), which turns it from a vector into a matrix [4].

For the second degradation, we consider space-varying atmospheric turbulence. In this case, the PSF is given by

\[
h_T(x, y) = e^{-j\pi\alpha_1^2(x, y)(x-x_0)^2+(y-y_0)^2},
\]

for all \( 1 \leq x \leq X, 1 \leq y \leq Y \), where

\[
\alpha_1(x, y) = \alpha_0 + \beta(x, y),
\]

\( \alpha_0 = 0.01 \), \( \beta(x, y) \) are AWGN samples with unity variance, \( x_0 = X/2 \), and \( y_0 = Y/2 \). The parameters \( X, 1 \leq X \leq M \), and \( Y, 1 \leq Y \leq N \), determine the severity of the turbulence and will be defined using simulation examples in Section 4. Note that the turbulence function produces a more random distortion of the pixels than the smearing operation produced by the blurring function.

3. BLIND IMAGE RECONSTRUCTION ALGORITHMS

In this section, we describe three algorithms that are used to blindly reconstruct images after they have been degraded by some unknown PSF. These algorithms have not been described and compared in the literature; they can be implemented easily with MatLab (© Math Works, Inc.).

A. Wiener Filtering (WF) Algorithm

The WF algorithm is a linear approach that is based on the concept of minimizing the MSE between the image and its estimate. That is, minimize

\[
\epsilon_0 = E\{(f - \hat{f})^2\},
\]

where \( E\{\cdot\} \) denotes expected value. The solution is straightforward and can be written in the frequency domain as [4]

\[
\hat{F}(u, v) = \frac{|H(u, v)|^2}{H(u, v)\{|H(u, v)|^2 + S_n(u, v)/S_f(u, v)\}} G(u, v),
\]
where $S_f(u,v)$ and $S_n(u,v)$ are the image and noise power spectrums, respectively. The ratio $S_n(u,v)/S_f(u,v)$ is known as the noise-to-signal ratio (NSR). Note that even when the NSR is unknown, it can be assumed, estimated, or determined experimentally. However, since $H(u,v)$ is unknown, the blind WF solution entails assuming a statistical distribution for the unknown quantity and choosing $\hat{F}(u,v)$ which has maximum likelihood. This typically requires iteration, and we see that performance of blind WF is very similar to that of the blind LR, so we only show simulation results for the latter algorithm, described next. Also note that the spatial domain estimate can always be obtained from the frequency domain estimate using

$$\hat{f}(x,y) = ifftn\{\hat{F}(u,v)\}. \quad (7)$$

### B. Lucy-Richardson (LR) Algorithm

The LR algorithm is a blind, iterative, and non-linear approach. Non-linear techniques can be problematic as their results may be non-predictable, so convergence may not be possible. However, non-linear algorithms such as LR have been shown to outperform linear techniques such as Wiener Filtering (WF) [4]. Many iterations may also be required, hence there is computational complexity, but given the advances in digital computing, this is also often not a significant problem. The blind LR algorithm works by attempting to solve a maximum likelihood problem, and results in the following iterative solution, for $i = 1, 2, \ldots, I$ iterations, [4]

$$\hat{f}_{i+1}(x,y) = \hat{f}_i(x,y)[h(-x,-y) \ast \frac{g(x,y)}{h(x,y) \ast \hat{f}_i(x,y)}]. \quad (8)$$

Note that since $h(x,y)$ is assumed unknown, we must choose it blindly. This is typically done by setting $h(x,y)$ equal to an $M \times N$ matrix with all pixels set to unity, except for 4 or 5 pixels along the borders. The blurring PSF distorts the pixels along the border differently, so we do not want those values to affect the estimate, potentially causing errors. The OTF is then $\hat{H} = ftfn(h)$.

An issue that arises with the LR algorithm is that if $I$ is too small, e.g. $I < 10$, the blur cannot be completely corrected, but if $I$ is very large, e.g. $I > 50$, there may be a large computational factor. We deal with this issue by decimating the images, so we can even let $I = 100$ here and the algorithm still runs in just a couple of seconds.

### C. Constrained Least-Squares (CLS) Algorithm

The CLS solution attempts to find the estimate of the original image by applying the technique of least-squares along with a constraint. The optimal restoration criteria is based on maintaining smoothness of the image, which is measured by its second derivative. Hence, we minimize the second derivative of the image function, known as the Laplacian [4]

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \nabla^2 f(x,y)^2, \quad (9)$$

under the least-square constraint [4]

$$\|g - H\hat{f}\|^2 = 0. \quad (10)$$

Here, $g$, $H$, and $f$ correspond to $g(x,y)$, $h(x,y)$, and $f(x,y)$ in matrix and vector form. This yields the solution, more easily expressed in the frequency domain as

$$\hat{F}(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + \lambda |P(u,v)|^2} \hat{G}(u,v), \quad (11)$$

where $H(u,v)$, $G(u,v)$, and $P(u,v)$ are the Fourier Transforms of $h(x,y)$, $g(x,y)$, and $p(x,y)$, respectively, and $p(x,y)$ is defined by

$$p(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \quad (12)$$

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The Lagrange multiplier, $\lambda$, is chosen to minimize the residuals between the estimated and given image; i.e. we choose $\lambda$ to meet

$$
\arg \min_{\lambda} \lambda^2 \sum \frac{(||P||^2 \cdot ||G||^2)}{||H||^4 + 2\lambda||H^2||^2 ||P^2|| + \lambda^2 ||P||^4 + \epsilon},
$$

(13)

and $\epsilon$, defined earlier, prevents $\lambda$ from becoming unstable. It is this constraint that makes the CLS algorithm robust, but $\lambda$ is still found iteratively [4] using Eqs. (9) - (13). From simulations, we see that convergence is quick, even with a badly distorted image. Once again, since $h(x,y)$ is unknown, we set it to the mostly unity PSF described above. The function $p(x,y)$ implements the derivative in Eq. (9). We can see this by writing a well-known second derivative approximation as

$$
\nabla^2 f(x, y) = f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1) - 4f(x, y),
$$

(14)

which is equivalent to writing

$$
\nabla^2 f(x, y) = p(x, y) * f(x, y).
$$

(15)

4. EXAMPLES

We consider an image taken by the author in Lower Antelope Canyon, near Page, AZ, taken standing on the floor of the slot canyon and looking upward towards the sky. It is a 1,600 by 1,200 pixel JPEG image that has been mapped from color to a gray scale image and decimated by a factor of 5 to keep the image small. After decimation, $M = 320$, and $N = 240$. Figs. 1, 2, and 3 show the original image and the image distorted by blur motion with $L = 39, 81, \text{ and } 153$, respectively. The MSEs between the true image and recovered images are $\text{MMSE}_{LR} = 0.0219$ and $\text{MMSE}_{CLS} = 0.0029$, $\text{MMSE}_{LR} = 0.0343$ and $\text{MMSE}_{CLS} = 0.0028$, and $\text{MMSE}_{LR} = 0.0511$ and $\text{MMSE}_{CLS} = 0.0027$, respectively. Note that the CLS consistently outperforms LR, and the CLS also maintains robustness even as the blur becomes very severe.

Fig. 1: Blur Motion; $L = 39$

Fig. 4 shows the figure distorted by a turbulence model with $X = 27, Y = 33$. We set the turbulence parameter to $\alpha_0 = 0.01$. The MSEs between the true image and recovered images are $\text{MMSE}_{LR} = 0.0311$ and $\text{MMSE}_{CLS} = 0.0049$. The CLS algorithm outperforms LR even when the distortion occurs across only $10 - 15\%$ of the pixels in the image.
Fig. 2: Blur Motion; $L = 81$

Fig. 3: Blur Motion; $L = 153$

Fig. 4: Turbulence; $X = 27, Y = 33$
Fig. 5 shows turbulence with X = 56 and Y = 71. Now, we get MMSE_{LR} = 0.0451 and MMSE_{CLS} = 0.0107. The CLS algorithm again does a good job of removing the turbulence but the LR results in an image that is still somewhat blurry and distorted.

Finally, Fig. 6 shows turbulence with X = 126 and Y = 111. Note from Fig. 6b that we start seeing significant image distortion and the CLS performance starts to degrade, but is still good, whereas LR performs poorly. We get MMSE_{LR} = 0.0555 and MMSE_{CLS} = 0.0452. The LR performance is also degraded and not as good as that of CLS. In summary, LR does better with blur motion than turbulence, but CLS remains robust to either; CLS also consistently outperforms LR.

**SUMMARY**

In this paper, we compare Wiener Filtering (WF), Lucy Richardson (LR), and constrained least-squares (CLS) algorithms for blind image restoration, i.e. when the point spread function (PSF) that degrades an image is unknown. We describe the algorithms and show by numerical examples that CLS outperforms LR in the presence of blurring or turbulence; WF performs very similarly to LR, and hence is not simulated. CLS is also computationally more efficient and maintains robustness over LR as the PSF distortions become more severe. We show that the mean-square error (MSE) between a true image and the recovered image is up to an order of magnitude better for CLS than LR. Therefore, for blind image restoration, the CLS algorithm, using the proper constraint, or Lagrange multiplier, is the preferred method. Future work includes developing blind image processing algorithms that can correct other types of image distortion, such as chirp interference and non-linear image distortions (e.g. image rotation or clipping), or can operate blindly in noise.
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