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Study and comparison of fractional delay filters for Sample Rate Conversion

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Abstract: In the receiver architecture of Software Defined Radio (SDR), design of sample rate conversion (SRC) filter can be classified into two tasks, namely, SRC by integral factor and SRC by fractional rate change. SRC by integral factors can be achieved by employing Cascaded Integrator Comb (CIC) filters followed by its compensator. A fractional rate interpolator followed by one or two stages of half band filters is used to achieve the exact sampling rate as required by the wireless standard. In this paper, a comparison between the frequency response of fractional rate interpolator based on Taylor's approximation and Lagrange polynomial with different orders is presented. From the simulation results it is observed that the former method requires a filter of higher order in comparison to the later method to attain the required spectral characteristics, hence employing more number of computational units.

Keywords: CIC filter; Interpolation; symbol rate; Farrow structure; SRC; RF; IF; SDR.

1. Introduction

With the evolution of various wireless standards at a rapid rate, a hardware radio seems to be an incompatible device which can adapt to various wireless standards. State of art communication system demands a software solution, where the radio can adapt to any wireless standards with the help of software rather than hardware i.e. a SDR. However, a complete software solution is impossible due to the limitations posed by ADC [1], [2], [3]. Therefore the architecture is classified into three stages viz, Radio frequency stage, Intermediate frequency stage and Baseband stage [4]. In the RF stage the signal is mixed and brought down to a single Intermediate frequency such that it incorporates all the wireless standards. In the second stage, IF signal which is in the range of few tens of mega Hertz is converted into digital with a sample rate of twice the Intermediate frequency. Hence the signals of different wireless standard gets oversampled and the signal sample rate has to be, digitally down converted and for proper synchronization of transmission and reception of signals the symbol rate has to be matched [4], [5]. In the baseband stage modulation, demodulation, encoding and decoding etc is carried out at the transmission and reception end respectively.

In the present work, different methods for sample rate alterations for a multi-mode multi-standard radio are investigated. Sample rate conversion filter is designed to extract GSM900, CDMA2000, WCDMA and HiperLAN signal from an intermediate frequency signal of 80MHz. The IF signal which is sampled at a Nyquist rate of 160MSPs is to be decimated to a sample rate of 0.270833MSPs,1.2288MSPs,3.84MSPs and 10MSPs with sample rate conversion factors being 590.769, 130.20833, 41.66667 and 16 for GSM900, CDMA2000,WCDMA and HiperLAN respectively. To achieve the process of decimation is carried out in stages namely, CIC decimation stage, CIC compensation, Fractional rate interpolation and half band filtering stage.

In CIC decimation stage, decimation is carried out by an integral factor and realized in stages as proposed in [4], [3], [6]. As the CIC filters have a gain droop in the pass band of the spectral mask a CIC compensation filter is needed and a fractional rate interpolator/decimator is needed to match the symbol rate of the standard. In [4] a joint compensation and interpolation filter is proposed based on frequency domain polynomial approach using Farrow structures [7], [8]. Interpolation by fractional rate can be easily accomplished using Farrow structures just by changing the fractional delay value without actually changing the filter coefficients [9]. However this approach produces a degraded response as the Farrow coefficients are computed in a way such that the error is minimized in the least squares or minimax sense.

Another method employs a discrete CIC compensation filter followed by a fractional rate interpolator based on Farrow structures. In this letter a method based on Newton's backward difference formula is compared with a method based Lagarange's polynomial interpolation. An efficient filtering structure has been proposed by C.Canadan for fractional rate interpolation where the order of interpolation can be changed on the fly. In this structure the computational complexity grows linearly with the order of the filter. In Lagrange's Polynomial Interpolation, the order of the filter cannot be changed flexibly and also the computational complexity grows quadratically with the order of the filter [10], [11]. The performance characteristics of both the structures are to be compared. The objective is to propose a suitable filter with low computational complexity and high performance for sampling rate alteration for SDR transceivers.

This paper is organized as follows. Section 2 describes different methods for CIC compensation and interpolation. Results, performance characteristics and computational complexity of different methods are presented in section 3. Section 4 concludes the paper.

2. Fractional Rate Interpolating Structures

A. Farrow Structure

This structure consists of a polyphase filter bank where the interpolation rate can be changed by varying the fractional delay value. The structure is shown in Fig.1.

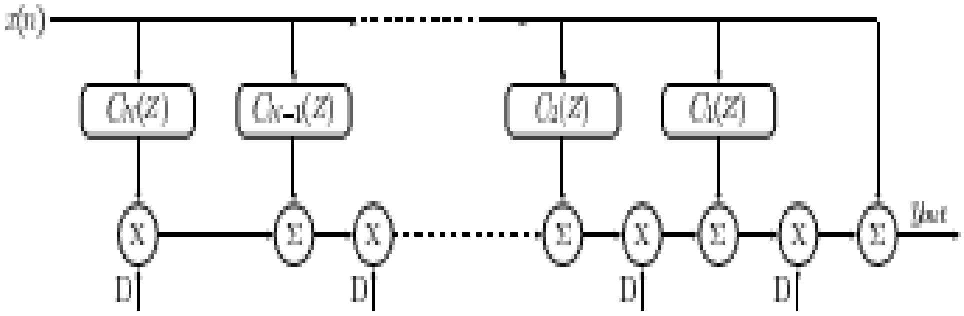

Fig. 1. Farrow Filter structure for Fractional rate Interpolation

Farrow Coefficients are computed using Lagrange's polynomial interpolation as stated in [12]. The coefficients of an Nth order Lagrange interpolation filter can be computed by calculating the inverse of the Vandermode matrix and then they are mapped appropriately in the Farrow structure. As an example, Farrow coefficients for Lagrange's Cubic polynomial interpolation are depicted as follows.

$$\hat{x}_a\left(\bar{t}\right) = \hat{x}_a\left(t_0 + DT_{in}\right) = \sum_{n=-N1}^{N2} P_k\left(D\right) x\left[n+k\right] \qquad (1)$$

where,

$$P_k\left(D\right) \prod_{l=-N1, l\neq k}^{N_2} \left(\frac{D-l}{k-l}\right) \qquad (2)$$

A Lagrange cubic polynomial with N_1=3, N_2=0, D as a fractional delay parameter is given as:

$$\begin{aligned} x\left(n+D\right) = D^3 & \left(-\frac{1}{6}x_{-3} + \frac{1}{2}x_{-2} - \frac{1}{2}x_{-1} + \frac{1}{6}x_0\right) \\ & + D^2\left(-\frac{1}{2}x_{-3} + 2x_{-2} - \frac{5}{2}x_{-1} + \frac{1}{6}x_0\right) \\ & + D\left(-\frac{1}{3}x_{-3} + \frac{3}{2}x_{-2} - 3x_{-1} + \frac{11}{6}x_0\right) \\ & + x_0 \end{aligned} \qquad (3)$$

Eq. 3 can be realized as a Farrow filter with $-\frac{1}{6}, \frac{1}{2}$ …………etc., as the coefficients of the polyphase filters in the Farrow structure and D being a fractional delay parameter which lies in the range of $0 \le D \le 1$.

Joint compensation and interpolation method has been proposed based on frequency domain approach [4], [7], [8]. In this method no separate compensation filter is required and the Farrow coefficients are computed such that they approximate the desired response as required by the compensation filter and interpolate the sample rate by the required factor. However, the coefficients are computed such that the least squares error or maximum error is minimized. The technique for computation of Farrow coefficients is briefly described as shown below.

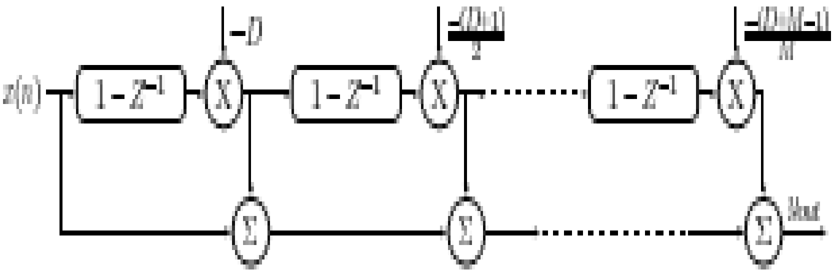

Fig. 2. Fractional Interpolating structure proposed by Canadian

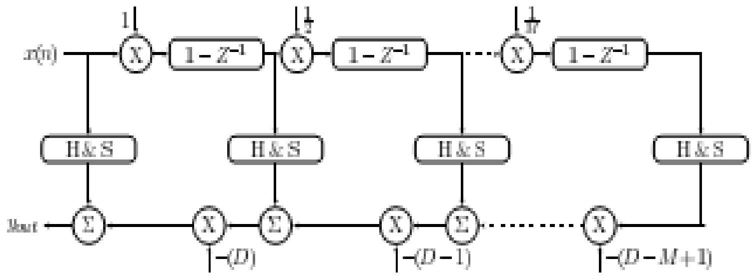

Fig. 3. Newtons's Interpolation structure for SRC. The Hold & Sample (H & S) block performs Sampling at output sampling instants

$$H_{inv}(\omega)=\begin{cases}\dfrac{(\omega)}{\sin(\omega)}, & 0\le\omega\le\omega_p\\ \dfrac{\sin(\omega_p)}{\omega_p}\left[1-\dfrac{(\omega-\omega_p)}{(\omega_s-\omega_p)}\right], & \omega_p\le\omega\le\omega_s\\ 0 & \text{elsewhere}\end{cases} \tag{4}$$

Where, ω_p is the pass band frequency, ω_s is the stop band frequency. The interpolated output sample with input sampling rate T_{in} in can be computed by the convolution equation 5.

$$y_a(t_1)=\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} x(n_1-k)h(k,D) \tag{5}$$

where, $y_a(t_1)$ represents the sample at the interpolated sample rate, n l is integer part of $\frac{t_1}{T_{in}}$, N is the length of the FIR filter, D is fractional part of $\frac{t_1}{T_{in}}$ in usually less than one, $x(n)$ being the sample value at input sampling rate and $h(k,D)$ being the impulse response of the filter. The impulse response $h(k,D)$ of the hybrid analog digital model is

$$h(k,D)=h_a(t)=h_a\left[(k+D)T_{in}\right] \tag{6}$$

The sampled impulse response, approximated as a polynomial in fractional delay parameter D and in terms of unknown polyphase filter coefficients $c_m(n)$ is

$$h_a\left[(k+D)T_{in}\right]=\sum_{m=0}^{M} c_m(k)(2D-1)^m \tag{7}$$

For $k=\frac{-N}{2},\frac{-N}{2}+1,\ldots\ldots\ldots\ldots\frac{N}{2}-1,\quad D\in[0,1)$, M represents the number of polyphase branches in the filter and N being the length of each sub-filter in the polyphase branch.

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As stated by Vesma [7] and if filter coefficients are symmetrically designed, Eq. 7 can be expressed in terms of basis functions $g(n,m,t)$ as

$$h_a(t)=\sum_{n=0}^{\frac{N}{2}-1}\sum_{m=0}^{M}c_m(n)g(n,m,t) \tag{8}$$

The basis function can be expressed as

$$g(n,m,t)=\begin{cases}(-1)^m\left(\dfrac{2\left(t+(n+1)T_{in}\right)}{T_{in}}-1\right)^m, & -t_1\le t\le -t_2\\ \dfrac{\sin(\omega_p)}{\omega_p}\left[1-\dfrac{(\omega-\omega_p)}{(\omega_s-\omega_p)}\right], & t_2\le t\le t_1\\ 0 & \text{elsewhere}\end{cases} \tag{9}$$

where $t_1=(n+1)T_{in},\ t_2=nT_{in}$.

The Fourier transform of the equation 8 can be given as

$$H_a(f)=H_a(j2\pi f)\sum_{n=0}^{\frac{N}{2}-1}\sum_{m=0}^{M}c_m(n)G(n,m,f) \tag{10}$$

where, $G(n,m,f)$ is the Fourier transform of the function $g(n,m,t)$ expressed as a function of sine and cosine terms in n,m,f and T_{in} in [7]. Thus the task of finding unknown polyphase coefficients is simplified as they remain unchanged in the frequency response of the filter as well as the time domain impulse response. Once the desired frequency response is known, the unknown coefficients are computed by solving the set of linear equations resulting from equation 10.

B. Newton's Backward Difference Interpolation

A structure based on Newton's backward difference formulae and Taylor's series approximation is depicted as follows. The sample value at instant 't' is given as

$$\tilde{x}_a(t)=\sum_{k=0}^{\infty}\Delta^n x[k]\frac{(-D)^{[n]}}{(n!)} \tag{11}$$

where $\Delta x[n]=x[n]-x[n-1],\ D=k-t,$
$(D)^{[n]}=D(D+1)....D(D+n-1)$.

A structure based on Eq.11 has been proposed by Canadan as shown in Fig.2. This filter structure does not account for input and output sampling rates for the signal processing blocks used in its implementation. A more elegant structure has been proposed by Lehtenin and et.al, as shown in Fig.3. In this structure the blocks on the top of hold and sample circuit operates at the input sampling rate and at the bottom operates at the output sampling rate [11]. Filter structures in Figs. 2 and 3 has the advantage that the filter order can be altered flexibly on the fly as no filter coefficients are involved in its realization. However, the performance characteristics of these structures have to be compared with other fractional delay interpolators.

3. Results

A joint compensation and interpolation filter as proposed by Sheikh et al, CIC compensation and fractional rate interpolation based on Lagrange polynomial and CIC compensation and fractional rate interpolation based on Newton's backward difference formula is simulated in MATLAB. The performance characteristics of the three techniques are brought out and their computational complexities are compared.

From Table 1 it can be inferred that the structure based on the Newton's backward difference formula with third order interpolation has least computational complexity when compared with others. This is due to the fact that the computational complexity in this structure is proportional to O(N) where as it is proportional to $O(N)^2$ in the other structures.

For GSM spectral mask the gain droop at the passband edge of 80KHz is 1.64dB which has to be compensated for zero. The performance measures of the three techniques are tabulated in Table 2. It can be inferred that the joint compensation and interpolation technique does not provide required attenuation at the stop band edge. Error at the passband edge is 0.18dB, 0.08dB and 0.9dB with joint compensation and interpolation, Lagrange cubic interpolation and Newton's third order interpolation respectively. Of the three techniques discrete CIC compensation with Lagrange cubic interpolator attains the required spectral characteristics. However, higher order interpolators achieve good attenuation characteristics but they deviate from the passband characteristics introducing high errors. Also the higher order interpolators based on Lagrange and Newton's method show undesirable gain in the high frequency regions.

Table 1. Computational Complexity of Compensation and Interpolation techniques. First element and second element are Number of Multipliers and Adders respectively

Method	Order	CIC Comp.	Interp. Filter	Total Complexity
Joint Method	M=3 N=16	(36,31)		(36,31)
Lagrange	3	(22,21)	(12,12)	(34,33)
Lagrange	4	(22,21)	(20,16)	(42,37)
Lagrange	5	(22,21)	(30,25)	(52,47)
Lagrange	6	(22,21)	(42,36)	(64,57)
Newton	3	(22,21)	(8,7)	(30,28)
Newton	4	(22,21)	(11,10)	(33,31)
Newton	5	(22,21)	(14,13)	(35,34)
Newton	6	(22,21)	(17,15)	(38,36)

Table 2. Performance metrics of different compensation and interpolation techniques for GSM spectral Mask

Method	Passband Edge (80KHz) (in dB)	Stopband Edge (100KHz) (in dB)
Required	1.64	-18
Joint Method	1.46	0.65
Lagrange Order=3	1.56	-19.7
Lagrange Order=4	1.49	-19.8
Lagrange Order=5	1.41	-20
Lagrange Order=6	1.39	-20.1
Newton Order=3	2.31	-16.21
Newton Order=4	-1	-20
Newton Order=5	-1	-35
Newton Order=6	1.83	-21.25

4. Conclusion

Different techniques for CIC compensation and interpolation for Sample rate conversion are investigated. Joint compensation and interpolation method has slightly lower computational complexity when compared with the discrete compensation and interpolation technique. However, this method could not attain the required attenuation characteristics. Hence discrete CIC compensation and interpolation based on Farrow structures are proposed. Though the order of the interpolator can be changed on the fly, this interpolator could not attain the required spectral characteristics when compared with Lagrange's interpolator. Also Lagrange's interpolator of higher orders produces undesirable response in the high frequency range. Thus a Lagrange's cubic interpolator with discrete compensation filter can be employed for Sample rate conversion in SDRs.

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