An Optimum Signal Perturbation Free Transmit Scheme to Enhance the Channel Estimation of MIMO-OFDM System

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ABSTRACT

There are many algorithms to improve the channel estimation of MIMO-OFDM system based on Blind, Semi-Blind and Training methods. But it can be improved further. Signal Perturbation Free (SPF) transmit is one such scheme. This scheme transmits the data which contains the partial information of the Correlation Matrix of the transmitted signal to the receiver in order to cancel the Signal Perturbation Error. In this paper, we propose an Optimum Signal Perturbation Free transmit scheme which enhances the Channel Estimation (CE) of frequency selective channel of MIMO-OFDM system. In the present work, performance of SPF assisted Semi-Blind channel estimation is enhanced by the selection of a Circularly Shifted Orthogonal Pilot Sequences on different antennas. The Linear Prediction (LP) based SP transmit scheme with optimum pilot sequences known as optimum SPF transmit scheme outperforms the SPF scheme with orthogonal pilot sequences.

Keywords-MIMO-OFDM system, Signal Perturbation Free (SPF), Linear Prediction (LP), Channel Estimation, Optimum Pilot Sequence, Bit Error Rate (BER).

I. INTRODUCTION

The MIMO (Multiple–Input–Multiple–Output) technique is one of key technologies for the next generation wireless communications [1],[6],[10]. This technique can provide either Diversity Gain to combat signal fading or a capacity gain called Spatial Multiplexing. It provides both high data rate and superior system performance without increasing the total transmission power or bandwidth [7]. The performance of MIMO system depends on the availability of knowledge of the channel. Thus, accurate channel estimation of MIMO systems is very important. The frequency selective problem that exists in the wireless system can be solved by Orthogonal Frequency Division Multiplexing (OFDM) technique. Hence, MIMO and OFDM are important techniques for the next generation wireless communication systems.

The channel estimation can be classified into the three categories namely, Training Based, Blind and Semi-Blind methods [1][2][3]. Among them, most of the existing Blind and Semi-Blind MIMO-OFDM channel estimation methods are based on second order statistics of a long vector whose size is equal to or larger than the number of Subcarriers [1],2,[3]. This method requires a large number of OFDM symbols unsuitable for fast varying channels. In addition, matrices are huge in size and their computational complexity is high [1]. In contrast, Linear Prediction based Blind algorithm depends on second order statistics of short vector with a size slightly larger than channel length [2].

The LP based Blind algorithms of MIMO channels are subject to a Signal Perturbation Error (SPE) due to finite data length effect in the calculation of correlation matrix of the received signal. But this results in poor performance [3]. The Semi-Blind algorithm uses Ideal Nulling Constraint on the channel matrix in the absence of noise and gives better performance [4], [5]. In paper [6], the Semi-Blind algorithm proposed with SPF scheme to cancel the Signal Perturbation Error at the receiver for frequency flat MIMO channels. Since the MIMO channels are always frequency selective, LP based Semi-Blind method with SPF scheme was mainly for frequency-selective MIMO systems [7].

In paper [7], the LP based Semi-Blind MIMO OFDM channel estimation with SPF scheme has been reviewed without considering the selection of pilot sequences and their performance. In [8], equi-powered, equi-spaced, phase shift orthogonal called optimal sequences were designed and their performance has been reviewed for flat and frequency selective faded MIMO-OFDM channel. In this work, LP based Semi-Blind MIMO–OFDM system channel estimation with SPF that uses the optimal pilot sequences are simulated. The scheme is then simulated with orthogonal pilot
sequences and their performance is compared. The rest of paper is organized as follows. The system model is described in section II. In section III, we briefly review the SPF assisted LP based Semi-Blind channel estimation [7] and optimal pilot sequence design [8]. In section IV, we present the proposed method. In section V, the results are discussed. Conclusions are drawn in section VI.

II. SYSTEM MODEL

Consider a MIMO-OFDM system with N_T transmit and N_R(\geq N_T) receive antennas. The m^{th} OFDM symbol in i^{th} transmitting antenna can be written as vector of the frequency domain signals, \{X_{iT}(m,0), X_{iT}(m,1), \ldots, X_{iT}(m,K-1)\}^T, where K denotes the number of Sub-Carriers.

The Inverse Discrete Fourier Transform (IDFT) processing gives the time-domain OFDM signal denoted as,

\[ x_i(n) = [x_{iT}(m,0), x_{iT}(m,1), \ldots, x_{iT}(m,K-1)]^T. \]

By omitting the time index m, we can write,

\[ x(n) = [x_1(n), \ldots, x_{N_T}(n)]^T \]

and

\[ x(n) = [x^T(n) \ldots x^T(n-L+1)]^T \quad (n = 0,1,\ldots, K-1) \]

Let,

\[ R_m = [H(0)H(1) \ldots H(L-1)] \]

Where, H(l) is \( l \)th tap of the L-tap MIMO FIR filter.

The Channel Matrix for \( l \)th tap and the output are given as,

\[ H(l) = \begin{bmatrix}
    h_{1,1}(l) & h_{1,2}(l) & \ldots & h_{1,N_T}(l) \\
    \vdots & \vdots & & \vdots \\
    h_{N_R,1}(l) & h_{N_R,2}(l) & \ldots & N_{N_R,N_T}(l)
\end{bmatrix} \]

\[ y(n) = H_n x(n) + v(n) \]

where, \( v(n) = [v_1(n) \ldots v_{N_R}(n)]^T \) is Zero Mean , Unit Variance Gaussian Noise. In present work, the Frequency Selective Channel of MIMO-OFDM system, qualifies to be the system model.

III. LP BASED SEMI BLIND CHANNEL ESTIMATION

Let \( y(n) = [y_1(n) \ldots y_{N_R}(n)]^T \) be the output signal.

The Prediction is then given by,

\[ y_p(n-1) = [y^T(n-1) \ldots y^T(n-P)]^T \]

\[ \Hat{R}_{n-1} = E[y_p(n-1) y_p^H(n-1)] \]

\[ \Hat{R}_n = E[y(n) y_p^H(n-1)] \]

where, \( P \) is Prediction Filter Length.

The Prediction Error will be,

\[ \delta_{p,p}^2 = R_y(0) - P_p \Hat{R}_n \]

where, \( R_y(0) = E[y(n) y^H(n)] \)

and

\[ P_p = \Hat{R}_n \Hat{R}_{n-1}^{-1} \]

is an \( N_R \times N_R \) matrix representing \( n \)th tap of the Prediction Filter.

It is known that if the transmitted signals are Uncorrelated i.e., \( P \cdot N_R \geq (L+P-1)N_T \), then,

The Prediction Error is \( \delta_{p,p}^2 = H(0) H^H(0) \) in[2]

Basically, \( H(0) \) is estimated from the Prediction Error. From the SVD of prediction error, \( W_0 \) Whitening Matrix can be estimated. \( Q_0 \) is a Unitary Rotation Matrix. The Rotation Matrix is computed from Training Pilots during the first twenty OFDM symbols. The \( H(0) \) matrix can be obtained as, \( H(0) = W_0 Q_0^H \)
Defining the following equations plays an important role in our proposed work.

\[ Y_Q(k) = W_0^H P_{RF}^H(k) Y_P(k) X_P^H(k) \quad \text{--- (1)} \]
\[ Y_Q = \sum_{k=1}^{K_T} Y_Q(k) \quad \text{--- (2)} \]

The SVD of \( Y_Q \) gives \( Y_Q = U_Q \Sigma_Q V_Q^H \) \( \text{--- (3)} \)

From this, the Rotation Matrix will be,

\[ Q_0 = V_Q U_Q^H \quad \text{--- (4)} \]

The Channel Matrix \( H(0) \) is \( W_0 Q_0^H \) \( \text{--- (5)} \)

From the value of \( H(0) \), the values of \( H(1), \ldots, H(L-1) \) can be predicted.

\[ H(l) = P_R(l) H(0) \quad \text{--- (6)} \]

where \( P_R(l) \) is obtained as,

\[ P_R(1) = P_p(1) \]
\[ P_R(2) = P_p(2) + P_p(1) P_p(1) \]

**Signal Perturbation Cancellation Scheme:**

Let \( R_x'(l) = \frac{1}{K} \sum_{m=1}^{K-1} x(n)x^H(n-l) \) \( \text{--- (7)} \)

is the Estimate of the correlation matrices of the transmitted signal, \( x(n) \). When the transmitted frequency domain signals is a Gaussian Process with zero mean and unit variance, the transmitted time domain MIMO OFDM signals are uncorrelated [5], i.e.,

\[ R_x(l) = E\{x(n)x^H(n-1)\} = \delta(l) I_{N_T} \quad \text{--- (8)} \]

The equation (7) can be written as,

\[ R_x'(l) = R_x(l) + \Delta R_x(l) \]

where \( \Delta R_x(l) \) is perturbation term of \( R_x(l) \) and is given by,

\[ \Delta R_x(l) = \frac{1}{K} \sum_{n=0}^{K-1} x(n)x^H(n-l) - \delta(l) I_{N_T} \]

This Perturbation Error due to the MIMO channel has to be cancelled so that channel estimation performance can be improved.

Let the information bearing data \( T(k) \) be obtained from a \( K_T \) size DFT of \( \Delta R_x(l) \) with gain of \( \frac{1}{K_T} \)

\[ T(k) = \frac{1}{K_T} \left[ \sum_{l=0}^{L-1} \Delta R_x(l) e^{j2\pi kl/K_T} + \sum_{l=L_1}^{K_T-1} \Delta R_x(l - K_T), e^{j2\pi kl/K_T} \right] \quad (k = 0, 1 \ldots K_T - 1) \]

\[ T(k) \]

\[ T_L(k) \] and \( T_R^H(k) \) are transmitted to the receiver to cancel the Signal Perturbation error at the receiver.

A. **Pilot Sequence**

Semi-Blind estimation algorithm have been reviewed without considering the selection of pilot sequence and their performance in [4],[5]. In [6], SPF assisted whitening rotation based semi-blind method has been reviewed with the complex orthogonal pilot sequence. The pilot sequence themselves can be optimized and designed to obtain the minimal MSE of LS MIMO-OFDM channel estimation methods. In [7], the SPF assisted LP based semi-blind MIMO-
OFDM channel estimation has not reviewed the design of pilot sequence and their performance. In [8], optimal pilot sequences are designed with respect to minimal MSE of LS channel estimate and the optimal placement of pilot tones.

The MSE of LS channel estimate is given by:

\[
    \text{MSE} = \frac{1}{L_N} \mathbb{E} \left\{ \| \tilde{h} - h \|^2 \right\}
\]

For zero mean white noise,

\[
    \text{MSE of estimation} = \sigma_n^2 \text{Tr} \left\{ \left( \tilde{A}^H \tilde{A} \right)^{-1} \right\}
\]

For a fixed power \( P \), \( \tilde{A}^H \tilde{A} = PI_{L_N} \). Then,

\[
    \text{MSE}_{\text{min}} = \frac{\sigma_n^2}{P}
\]

### B. Optimal Pilot Sequence Design

Pilot sequence are designed and placed in such a way that they can be optimized to obtain their minimum MSE of LS channel estimate.

When \( L=1 \), it results in flat fading where the pilot sequences on different antennas must be orthogonal. When \( L>1 \), there will be frequency selective fading where the pilot sequences on different antennas must be not only orthogonal, but phase shift is also orthogonal for phase shift in the range, \( \Phi \epsilon \{-L+1, \ldots, L-1\} \). It is a well known fact that the Phase Shift Orthogonality in Frequency Domain is equivalent to Circular Shift Orthogonality in Time Domain [9], [10].

In other words, Pilot Sequences of one antenna must not only be orthogonal to pilot sequences of other antennas, but to Circularly Shifted Replicas of these sequences as well.

### IV. THE PROPOSED OPTIMUM SPF METHOD

In the proposed system, the received signal correlation without signal perturbation error is calculated using the equation, \( \tilde{R}_y(l) = \tilde{R}_y(l) - R_T(l) \). Here, \( \tilde{R}_y(l) \) is the received signal correlation estimate and \( R_T(l) \) can be obtained from the transmitted user specific data \( T_L(k) \) and \( T_R(k) \).

After cancelling the Perturbation Error at the receiver, the proposed method consists of:

(i) Computation of the Prediction Error from the received signal correlation which is free from the perturbation error.
(ii) \( W_0 \) Whitening Matrix can be computed from the Prediction Error.
(iii) \( \Phi_0 \) a Unitary Rotation matrix is computed using the equations, (1) to (4).
(iv) \( H(0) \) can be computed using equation (5)
(v) \( H(1) \ldots H(L-1) \) can be computed using the Recursive Equation (6).

The Optimal Pilot Sequences can be designed using the following equations,

\[
    \sqrt{\frac{P}{P}} e^{-j2\pi n_p} \frac{P}{P} \\
    \forall p \epsilon \{0, \ldots, P - 1\} \\
    \forall r \epsilon \{1, \ldots, N_t\}
\]

where \( \{n_p\}_{p=1}^{N_p} \) is the set.

Thus, equi-powered, equi-spaced, phase shift orthogonal sequences called Optimal Sequences can be designed. In this paper, LP based Semi-Blind MIMO – OFDM system channel estimation with SPF that uses the optimal pilot sequences are simulated and the same scheme with orthogonal pilot sequences are simulated and their performance are compared.

### V. RESULTS AND DISCUSSION

The optimum SPF transmit scheme employed for the MIMO-OFDM system is simulated and performance is compared with SPF scheme with orthogonal pilot sequences. In most simulation runs, the optimum SPF transmit scheme outperform the BER, MSE performance of SPF scheme with orthogonal pilot sequences.
In the experiments, a two input, two outputs MIMO – OFDM system is considered and wireless channels are simulated randomly i.e. the number of paths \(1 \leq L \leq 3\). As in [7], the prediction filter length \(P\) is 4. The Signal-to-Noise Ratio (SNR) of the channel is changed between 0 and 20 dB.

![Graph](image)

**Fig. 1.** BER computation for different CE schemes

In Figure 1, the BER is calculated for three different cases (i) No Channel Estimation (ii) Channel Estimation with Orthogonal Pilots (iii) SPF assisted Channel Estimation with Orthogonal Pilots.

![Graph](image)

**Fig. 2.** BER Comparison for different approaches of SPR

In Figure 2, the BER is calculated by employing two different approaches (i) SPF assisted channel estimation with orthogonal pilots (ii) SPF assisted channel estimation with optimal pilots.

![Graph](image)

**Fig. 3.** Performance of BER with SNR

In Figure 3, the BER is calculated with pilot length 20 and <20+SPF.
Figure 3 shows the variation of BER with the SNR for two different approaches (i) SPF assisted channel estimation with twenty optimal pilot symbol (ii) SPF assisted channel estimation with ten optimal pilot symbols.

![Mean Square Error vs SNR](image)

**Fig. 4. Performance of MSE with SNR**

Figure 4 shows the variation of Mean Square Error (MSE) with SNR for the proposed optimum SPF scheme. From the above discussion, we conclude the SPF assisted LP based Semi-Blind method outperforms the Walsh-Hadamard code by 0.045dB. Moreover the Optimum SPF scheme with 10 pilot symbols outperforms the SPF scheme with 20 pilot symbols by 0.005dB and with fewer pilots overhead.

**CONCLUSION**

The Signal Perturbation Free transmit scheme enhances the LP based Semi-Blind channel estimation of MIMO-OFDM system by sending the SPF data to cancel the Perturbation Error at the receiver. The performance of SPF is further improved by using the Optimum Training Signal as Pilot Signal. The performance improvement of Optimum SPF scheme is about 0.04dB. The BER, MSE performance of optimum SPF scheme outperform the SPF assisted LP based estimation scheme. As future work, the optimum SPF scheme can be extended for multi-cell environment.

**REFERENCES**


