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Performance of Polar Code over Wireless Networks

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ABSTRACT

Due to the different kinds of channel impairments, FEC schemes were developed. Polar coding is one of FEC schemes that has a very remarkable behavior and low decoding complexity as compared to other FEC schemes. Polar coding refers to polarizing a set of identical channels, from capacity perspective, to a new set of the same number of channels each one having a different capacity level. Efficient channels are the ones used for data transmission. This paper covers the code encoder\ decoder models, simulation tests were used to measure BER performance, code complexity and BW efficiency for polar codes, the latter results has been compared to the corresponding BER, complexity and BW efficiency of TCM, LDPC and Turbo codes. The paper also discussed the changing behavior of Polar codes that depends on the data transmission rate, as smaller the rate as better gets the performance. Finally system behavior has been tested over AWGN, fading and multipath fading channels.

Keywords: Forward Error Correction, Polar, Polarization, Encoder, Decoder, BER, Code Complexity, BW Efficiency.

1. INTRODUCTION

Data transmission over wireless channels is characterized by fading effects in addition to additive noise leading to considerable errors. Thus methods for dealing with such errors are required to improve system performance. Forward error correction (FEC) schemes are suitable to correct possible number of errors without requiring retransmitting the data [1]. In 1950 hamming codes were developed [2]. Algebraic codes were invented during 1950-1960 such as reed Solomon and reed Muller codes [3]. Convolutional codes are introduced in 1970 [4] followed by the coded modulation invention in 1980 [5]. In 1990 turbo and LDPC codes were introduced [6,7] and finally polar codes were developed in 2009 by Erdal Arikan [8]. Trellis coded modulation (TCM), Turbo Codes (TC), and Low Density Parity Check (LDPC) are among the useful codes introduced during the last 3 decades [9]. Later on different efforts have been devoted to introduce codes that are suitable for wireless networks applications and achieving at the same time performance close to Shannon limit. In 2009 Erdal Arikan introduced the most promising FEC technique with an encouraging performance by presenting the polarization concept that achieves channel capacity. The code is called polar codes and has an efficient performance [8].

The author in [8] achieved the symmetric capacity of a BDMC by introducing channel polarization concept in the code construction. In [10] the author decoded polar codes using BP decoding algorithm. The author in [11] showed that polar codes perform better than LDPC codes over AWGN and fading channels. The research aim here is to measure the performance of polar codes together with conventional codes such as TCM, Turbo, and LDPC codes when operating over different wireless channel environment. The measured performance covers bit error rate, code complexity, and bandwidth efficiency.

2. SYSTEM MODEL

Fig.1 shows the system model used. The source generates a bit stream to be fed to the encoder shown.

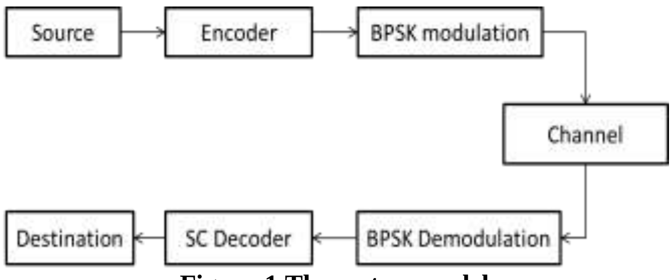

Figure-1 The system model

The encoder performs some linear transformations and modulo-2 addition. Channel polarization here should take place which includes channel combining and splitting:

- channel combining [12]

$$W_N(r^N|x^N) = W_{\frac{N}{2}}(r^{\frac{N}{2}}|x^N_{odd}\oplus x^N_{even})W_{\frac{N}{2}}(r^N_{(N/2)+1}|x^N_{even}) \quad (1)$$

W_N: N channels before polarization. r^N: received binary sequence. x^N: message sequence. Odd and even refers to odd and even indices.

- Channel splitting [13]

$$W_N^i: s \rightarrow r \times s^{i-1} \quad 1 \le i \le N \quad (2)$$

$$W_N^i(r, x_1^{i-1}|x_i) \triangleq \sum_{x^N_{i+1}\in s^{N-i}} \frac{1}{2^{N-1}} W_N(r, x) \quad (3)$$

W_N^i: ith channel after polarization. s: the coded sequence.

This procedure polarizes a given set of N identical channels from capacity perspective to a new set of N channels having different capacities. Efficient channels will be used for transmission leaving others frozen after the modulation operation. The code rate will define how many channels should be used.

The signal will be conveyed through the channels used (AWGN, single tap flat fading and multipath fading channels). At the receiver the received sequence will go through BPSK demodulation operation followed by decoding algorithm.

The noisy stream r is received and decoded by SC decoding algorithm. The following steps are performed by this decoding algorithm [8]

- as a result from channel polarization procedure, inefficient channels are known and their values are set to 0.
- at the decoder zeros of all frozen channels (of the value 0) will be flowed backward going through Mod-2 addition operations and linear transformations, see Fig-2 where N is chose 8 for simplicity, in simulation results different values for N was chosen (256, 512 and 1024).
- at the other side when r is receiver, LR for each noisy bit of r is calculated by using pdf function
- LR of all x values are calculated by:

$$LR(x_i) = \frac{LR(r_i)LR(r_{i+1})+1}{LR(r_i)+LR(r_{i+1})} \quad (4)$$

$$x_i = \begin{cases} 0 \text{ if } LR(x_i) \ge 1 \\ 1 \text{ if } LR(x_i) < 1 \end{cases} \quad (5)$$

$$LR(x_{i+1}) = \begin{cases} LR(r_i)LR(r_{i+1}) & \text{if } x_i = 0 \\ \frac{LR(r_{i+1})}{LR(r_i)} & \text{if } x_i = 1 \end{cases} \quad (6)$$

- Next node LR calculation of v values take place, the difference is that v_i values are predetermined by 0 due to the backward flowing of zeros, so $LR(v_{i+1})$is definitely calculated by multiplication expression of Eq.28.
- This process is repeated until the decoded vector y is recovered.

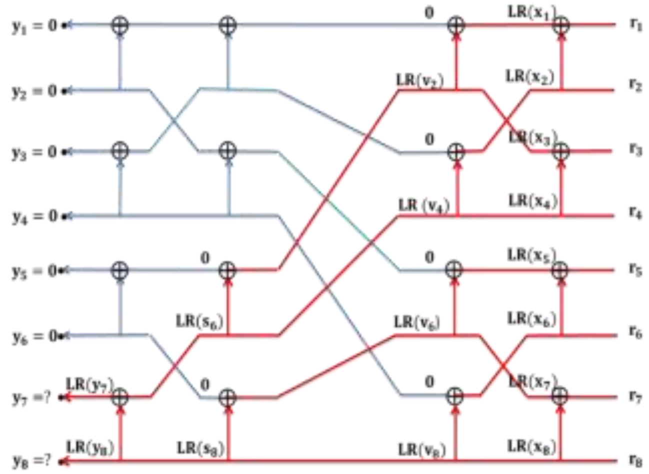

Figure-2: SC decoding of polar codes when N=8

3. SIMULATION TEST RESULTS

3.1 Performance over AWGN Channel

Simulation results of (8PSK-TCM-4 state, 8PSK-TCM-8 state, 1-iteration Turbo, 3-iteration Turbo, LDPC(12,4), LDPC(12,8) and Polar of rate 1/4 when N=1024) over AWGN channel is shown in Fig-3

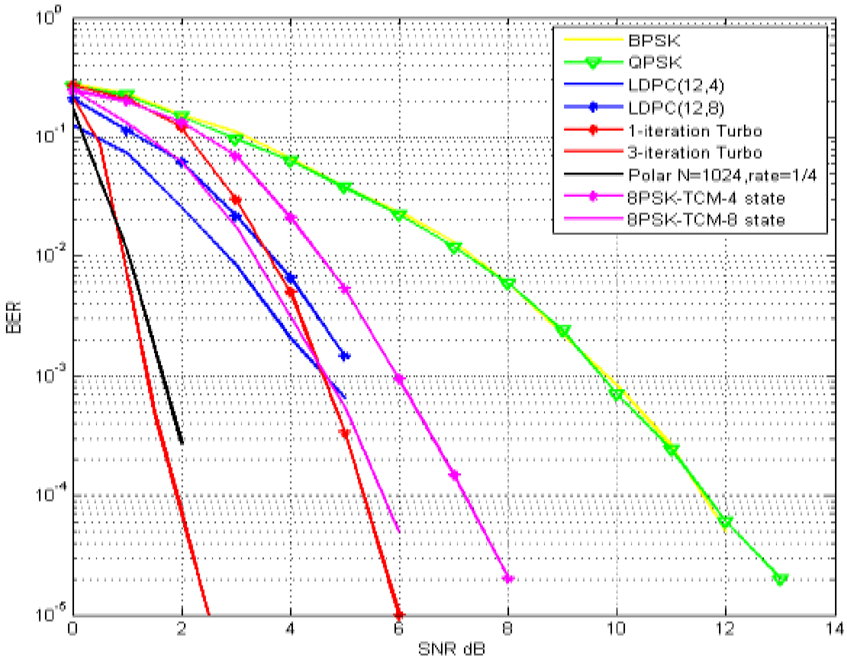

Figure-3: different coding schemes performance over AWGN channel

The simulated polar code is constructed by using 1024 channels, coding rate is 1/4. Polar code performance is better than all the other codes except 3-iteration Turbo codes when BER < 0.01. Fig-4 shows different behaviors of Polar codes according to the used rate in data transmission.

Fig-4 and Table-1 summarize that polar code good performance could be achieved by using a large set of channels N and a small rate for data transmission. The case when N=1024 and rate=1/4 is the one that showed the best performance.

Fig-5 shows that system is improved when using a large value of N and a small transmission rate. For example system improvement of 1.3 dB could be noticed by comparing the systems of (k=300 and rate=1/4 and 3/4).

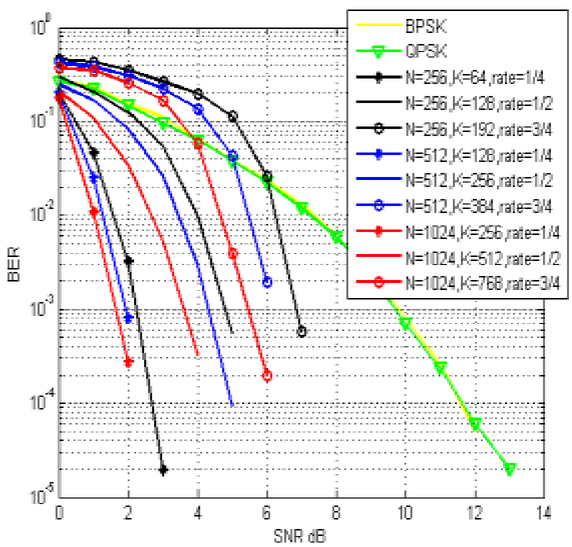

Figure-4: Polar code performance using different rates

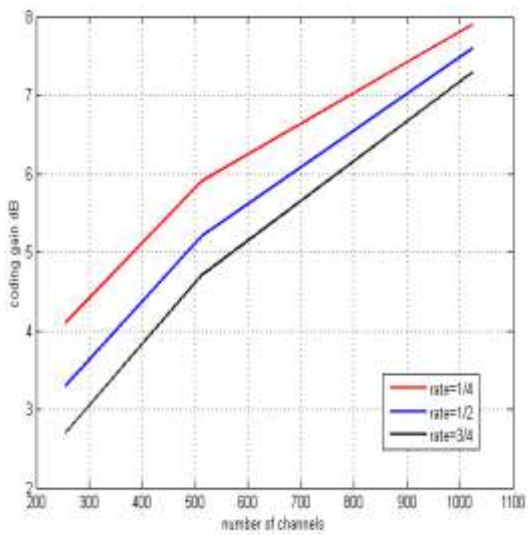

Figure-5: Performance improvement of Polar codes over AWGN channel

Table-1: SNR gains according to the changing values of N and coding rate over AWGN channel

Number of total channels N	Number of good channels K	Rate K/N	SNR gain over uncoded BPSK at 10^{-3}BER
256	64	1/4	4.1 dB
256	128	1/2	3.3 dB
256	192	3/4	2.7 dB
512	128	1/4	5.9 dB
512	256	1/2	5.2 dB
512	384	3/4	4.7 dB
1024	256	1/4	7.9 dB
1024	512	1/2	7.6 dB
1024	768	3/4	7.3 dB

3.2 Performance over Single Tap Flat Fading Channel

Simulation results of the previously mentioned systems are shown in Fig-6. Polar code parameters are N=512, and rate=1/4.Polar code have the best performance as compared to the other codes except 3-iteration Turbo code when BER<0.05. Fig-7 illustrates different polar code performances by using different parameters. Table-2 shows the changing coding gain of polar codes that is obtained by changing N and transmission rate. It could be concluded that to achieve good results small rates as well as large values of N should be used.

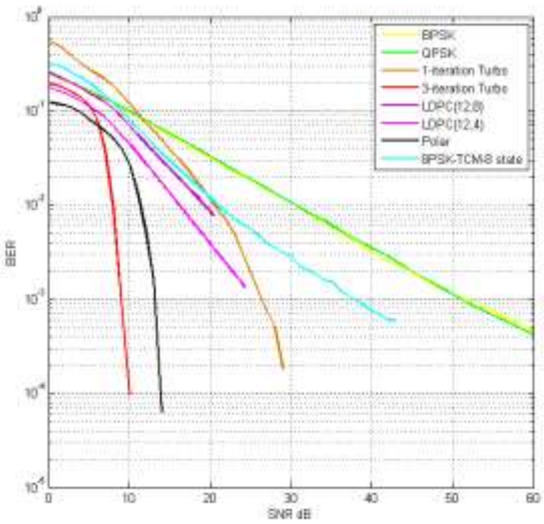

Figure-6 BER performance of different coding schemes of single path flat fading channel

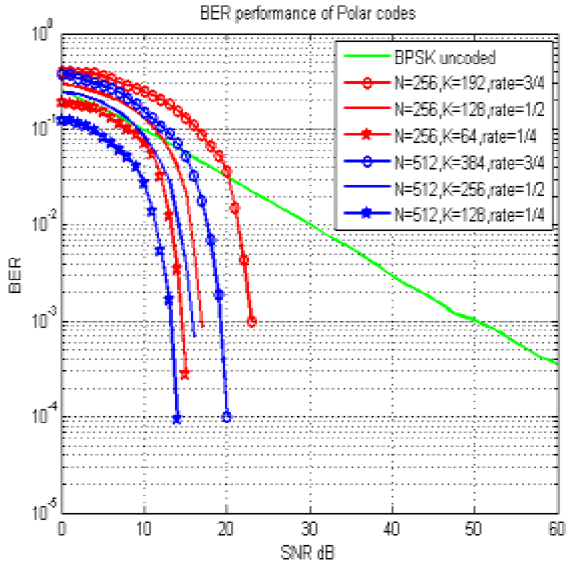

Figure-7 Performance of Polar code with different code parameters and uncoded BPSK over single tap flat fading channel

Table-2 SNR gains according to the changing values of N and coding rate over flat Fading channel

Number of total channels (N)	Number of good channels (K)	Rate K/N	SNR gain over uncoded BPSK at BER of 10^{-3}
256	64	1/4	35.5 dB
256	128	1/2	33 dB
256	192	3/4	27 dB
512	128	1/4	37 dB
512	256	1/2	34.2 dB
512	384	3/4	30.7 dB

Fig-8 shows that the system over flat fading channel could be improved as well depending on system parameters.

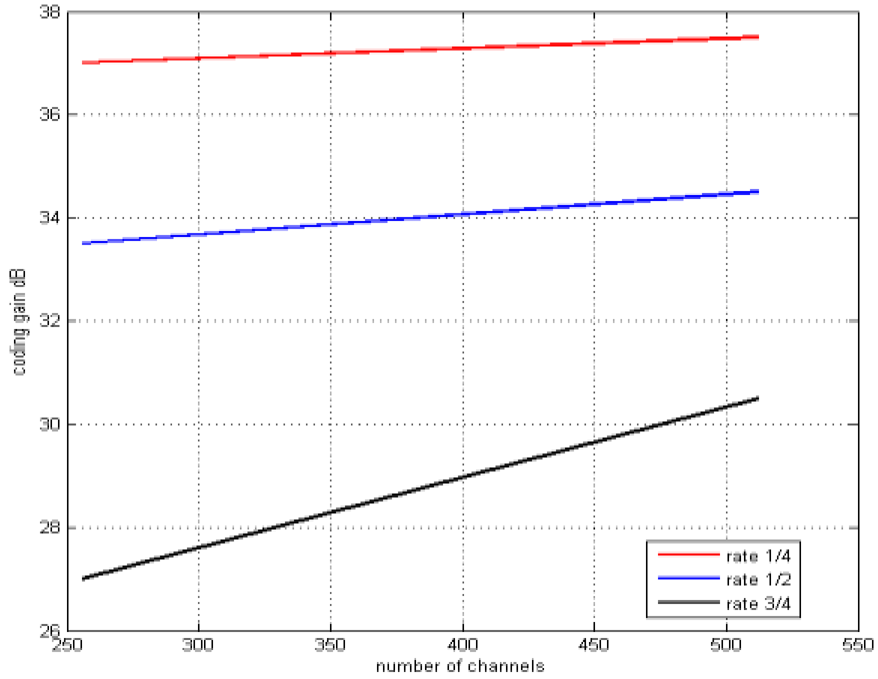

Figure-8 Performance improvement over single tap flat fading channel

3.3 Performance over SUI-3 Channel

Fig-9 illustrates the performances of the earlier mentioned systems, polar code simulated with N=256, rate=1/4. 3-iterations Turbo code almost performs better than all codes. Polar codes perform better than all other codes except the 3-iteration turbo code.

Fig-10 shows different performances of polar codes over SUI-3 channel obtained by changing system parameters N and coding rate. Using lower rates and larger values of N end up with a good performance.

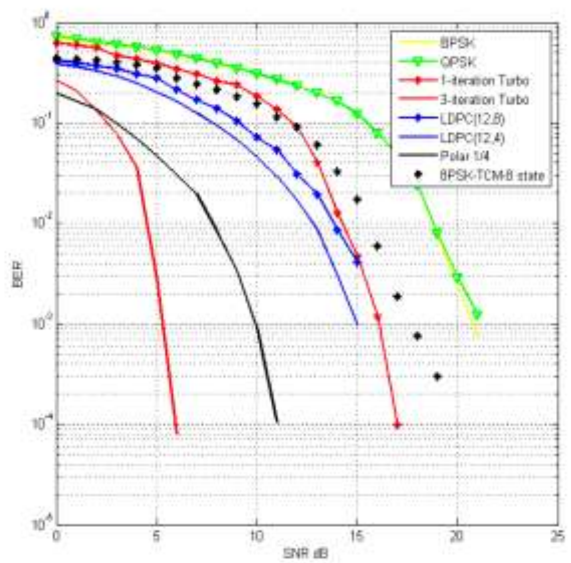

Figure-9 BER performance of different coding code schemes over SUI-3 channel

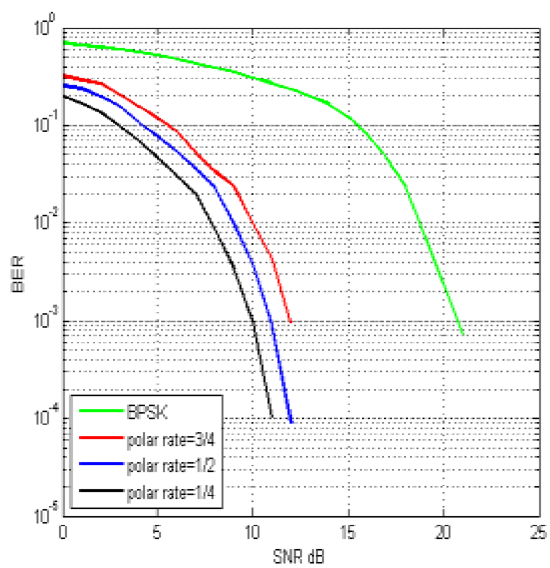

Figure-10 Performance of Polar code with different parameters over SUI-3 channel.

Table-3 SNR gains according to the changing values of N and coding rate over SUI-3 channel

Coding Scheme	SNR required to achieve BER of 10^{-3}	SNR gain over uncoded BPSK at BER of 10^{-3}
Polar rate=3/4, N=256, K=192	8.5 dB	12 dB
Polar rate=1/2, N=256, K=128	10 dB	11 dB
Polar rate=1/4, N=256, K=64	11 dB	10 dB

Code rate and system performance relationship over SUI-3 channel could be shown in Fig-10, the same case as AWGN and flat fading channels; performance is improved by using lower coding rates.

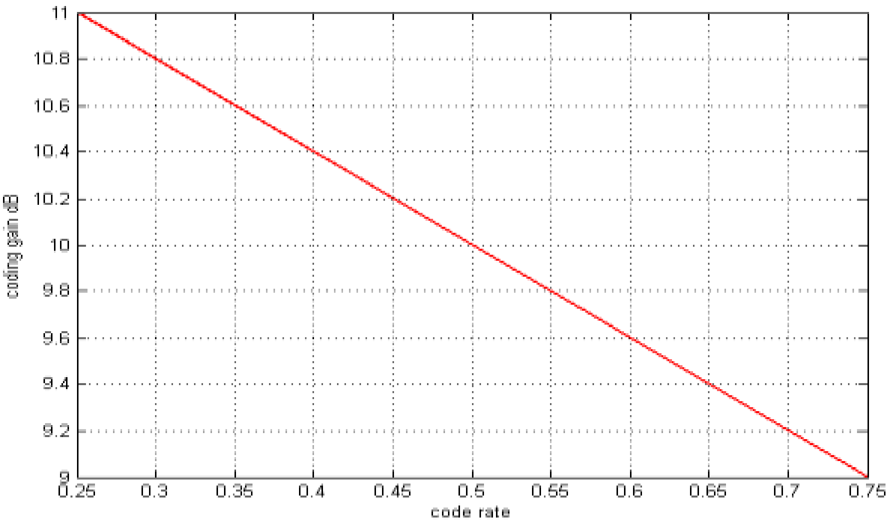

Figure-10 Performance improvement against Polar coding rates over SUI-3 channel

4. COMPLEXITY COMPARISON AMONG DIFFERENT SYSTEMS

Table-4 gives an approximate coding and decoding complexity comparison among all coding techniques in the work.

Table-4 Encoding and decoding complexity for different FEC schemes.

FEC Scheme	TCM	1-iteration Turbo	3-iteration Turbo	LDPC	Polar
Encoder used	Convolutional encoder	Convolutional encoder	Convolutional encoder	Matrix multiplications	Mod-2 addition and linear transformations
Encoder complexity	O(n)	O(n)	O(n)	O(n)	O (N $\log_2$N)
Decoder used	Viterbi	BCJR	BCJR	Simplified SPA	SC
Decoder complexity	$O_v(2^{\vartheta}, 2^k)$	$2O_v$	$3(2O_v)$	N_c (n)+ N_ε (n)	O (N $\log_2$N)

Where: O_v Stands for computational complexity. ϑ Presents number of stored paths. k Is message length all in Viterbi decoding algorithm. N_ε Denotes the number of summation. N_c Represents numper of comparison in SPA decoding algorithm.

As could be seen, Turbo decoder is very complex, LDPC decoder complexity depends on the size of the generator matrix, as gets bigger as becomes more complex nevertheless it improve decoding performance. Finally Polar decoder is the most modest decoding algorithm even though its complexity increases as N gets larger. Table-5 gives an approximation of operation numbers needed by the decoding algorithms.

Table-5 Operations required by SPA, Viterbi, BCJR and SC decoding.

Operations performed	Decoding algorithm			
	Viterbi	BCJR	SPA	SC
Mod 2 addition	____	____	____	2N
Linear transformations	____	____	____	N
LLR calculation	____	____	2n+2(µk) Or 2n+2(N_{one})	$\sum_{K=N\frac{\dot{k}}{\dot{n}}\in 2^i}^{N} K$
No. of comparisons	$(2^{2k}-2^k)$	$2(2^{2k}-2^k)$	N	$(\sum_{K=N\frac{\dot{k}}{\dot{n}}\in 2^i}^{N} K)/2$
No. of summations	$(2^i*2^k)/2$	2^i*2^k	2(µk) or 2(N_{one})	____

In Table-5, 2n+2(µk) is used to calculate number of regular LDPC codes LLR values, 2n+2(N_{one}) is used in the work for the adopted irregular LDPC codes, N_{one} is the number of ones in the parity check matrix. The number 2^i is the number of stored paths of the trellis in Viterbi decoding algorithm. Finally, $\frac{\dot{k}}{\dot{n}}$ represents the ratio of the number of high capacity channels used K to the input channels.

5. BANDWIDTH EFFICIENCY COMPARISON

Table-6 BW efficiency comparison is shown below:

Table-6: bandwidth efficiency for simulated coding techniques

	Uncoded		Coding techniques				
	BPSK	QPSK	8PSK-TCM	Turbo	LDPC(12,8)	LDPC(12,4)	Polar
BW ef. bps/Hz	1	2	2	1/3	2/3	1/3	K/N
Bitrate kbps	100	200	200	100/3	200/3	100/3	$\bar{B}$

8PSK-TCM is the scheme with the best bandwidth efficiency. Polar code best BW efficiency could be obtained when using large code rates, $\bar{B}$ denotes the bitrate of Polar code which may be 25, 50 or 75 kbps, 75 kbps is the best among them this leads to the use of 3/4 coding rate.

6. CONCLUSION

Polar codes give the best tradeoff between complexity and capacity as compared with other coding schemes although it gives poor bandwidth efficiency when choosing lower rates for good performance. Thus a compromise between system performance and bandwidth efficiency should take place. TCM coding scheme could be chosen in case of designing systems require high BW efficiency.

REFERENCES

[1]. J. Moreira and P. Farrell, "Essentials of Error-Control Coding", John Wiley & Sons Ltd, The Atrium, Southern Gate, England, 2006.
[2]. R. Hamming, "Error Detecting and Error Correcting Codes", the bell sys. Tech. journal, Vol. XXIX, No. 2, 1950.
[3]. Reed, I. S. and Solomon, G., "Polynomial Codes Over Certain Finite Fields," SIAM Journal of Applied Math., vol. 8, 1960.
[4]. A. Viterbi, "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm", IEEE Trans. Inf. Theory, Vol. IT-13, No. 2, pp. 260-269, April, 1967.
[5]. G. Ungerboeck, "Trellis Coded-Modulation With Redundant Signal Sets Part 1: Introduction", IEEE Comm. Magazine, Vol. 25, No. 2, February, 1987.
[6]. P. Robertson and T. Worz, "Bandwidth-Efficient Turbo Trellis-Coded Modulation Using Punctured Component Codes", IEEE Journal, on Selected Areas in Comm., Vol. 16, No. 2, February 1998.

[7]. R. Gallager, "Low-Density Parity-Check Codes", IRE Trans. Inform. Theory, Vol. IT-8, pp. 21–28, Jan. 1968.
[8]. E. Arıkan, "Channel Polarization: A Method For Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels", IEEE Trans. Inf. Theory, No.55 (7), pp 3051–3073, 2009.
[9]. G. Ungerboeck, "Trellis Coded-Modulation with Redundant Signal Sets Part 2: State of the Art", IEEE Comm. Magazine, Vol. 25, No. 2 February 1987.
[10]. R. Mori, "Properties and Construction of Polar Codes," Master Thesis, Kyoto University, 2010.
[11]. P. Shi, S. Zhao, B. Wang and L. Zhou, "Performance of Polar Codes on Wireless Communications Channel", arXiv: 1204.6106v1 [cs.IT], 27 April, 2012.
[12]. R. Pedarsani, "Polar Codes: Construction and, Performance Analysis", Master Thesis, Information Theory Laboratory (LTHI) School of Computer and Communication Sciences (IC) Swiss Federal Institute of Technology (EPFL),Swiss, June 2011.
[13]. S. Korada, "Polar Codes for Channel and Source Coding", Master Thesis, Ecole Polytechnique Federale de Lausanne, Suisse, 2009.