

A survey of entropy based image thresholding techniques

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Abstract: Image thresholding is one of the most widely used segmentation techniques in image processing. The objective of image thresholding is to segment a given image so that the object is more distinguishable from its background. This has been an active area of research in image processing and several methods have been proposed. Some of them are based on entropy of the image and its histogram. In this paper we provide a brief survey of entropy based thresholding techniques which include method based on entropy of histogram, thresholding using Renyi entropy and thresholding using Tsallis entropy.

Keywords: Entropy, Thresholding, Histogram, Renyi entropy, Tsallis entropy

Introduction

Image segmentation is a key step in image analysis. In this process, we divide an image into its components and thus enabling distinction of objects of interest from its background. The process of image segmentation is based on some properties of the image such as color, intensity or texture. Image thresholding is one of the several image segmentation techniques. The problem of thresholding arises due to the fact that the histogram associated with images is bimodal, and in general can be multimodal. This enables us to obtain an optimal threshold value for extracting object from its background. One of the widely adopted approaches for thresholding is based on entropy framework. In this class of methods, we use the entropy of the distribution of the gray scale levels of the image to calculate the threshold value. We now describe some of thresholding methods based on entropy.

Method based on entropy of histogram

Histogram depicts the intensity distribution of the image. It is quite useful in the analysis and processing of an image. On the basis of the entropy of the histogram of the image, Pun [1, 2] proposed two algorithms to calculate the threshold.

A. In the first method, he calculated the optimal threshold value by maximizing the posteriori entropy H , given by-

$$H = H_b + H_w \quad (1)$$

where $H_b = - \sum_{i=0}^t p_i \log_2 p_i$ and $H_w = - \sum_{i=t+1}^n p_i \log_2 p_i$

Here $p_1, p_2, \dots, p_n$ are the probability distribution of the intensity values of the image whereas H_b and H_w are the entropies of the background and object, respectively.

For maximizing H , Pun[1] proposed an evaluating function $f(t)$ which is given as

$$f(t) = \left[\frac{H_t}{H_T} \frac{\ln p_t}{\ln \max_{0 \leq p_0, \dots, p_t}} + \left\{ \frac{1-H_t}{H_t} \right\} \frac{\ln(1-p_t)}{\ln \max_{p_{t+1}, \dots, p_n}} \right] \quad (2)$$

where

$$H_t = - \sum_{i=0}^t p_i \log_2 p_i \quad , \quad (3)$$

$$H_T = - \sum_{i=0}^n p_i \log_2 p_i \quad (4)$$

$$p_t = \sum_{i=0}^t p_i \quad (5)$$

B. In the second method, the threshold value depends on an anisotropic coefficient α ,

$$\alpha = \left[\frac{\sum_{i=0}^m p_i \log_2 p_i}{\sum_{i=0}^{n-1} p_i \log_2 p_i} \right] \quad (6)$$

The threshold value t is chosen such that

$$\sum_{i=0}^t p_i = \begin{cases} 1-\alpha & \text{if } \alpha \leq 1/2 \\ \alpha & \text{if } \alpha \geq 1/2 \end{cases} \quad (7)$$

Pun[2] investigated a few images and found that the second method performs better than the first method.

Image thresholding using Renyi entropy

Sahoo, Wilkins and Yeager[3] proposed a method to find the threshold value using Renyi's entropy. The Renyi entropies of the object (H_o^α) and its background (H_b^α) are given by

$$H_o^\alpha(t) = \frac{1}{1-\alpha} \ln \sum_{i=0}^t \left[\frac{p_i}{p_t} \right]^\alpha \quad (8)$$

$$H_b^\alpha(t) = \frac{1}{1-\alpha} \ln \sum_{i=t+1}^n \left[\frac{p_i}{1-p_t} \right]^\alpha \quad (9)$$

Sahoo et. al.[3] calculated the threshold value by maximizing the sum of above two entropies i.e., ($H_o^\alpha(t) + H_b^\alpha(t)$)

For a particular value of α , the threshold $t^*(\alpha)$ can be obtained from

$$t^*(\alpha) = \underset{\alpha}{\operatorname{argmax}} (H_o^\alpha(t) + H_b^\alpha(t)) \quad (10)$$

An illustration of Renyi entropy based image thresholding is shown in the figure 1 and figure 2. The image is of UofL campus.

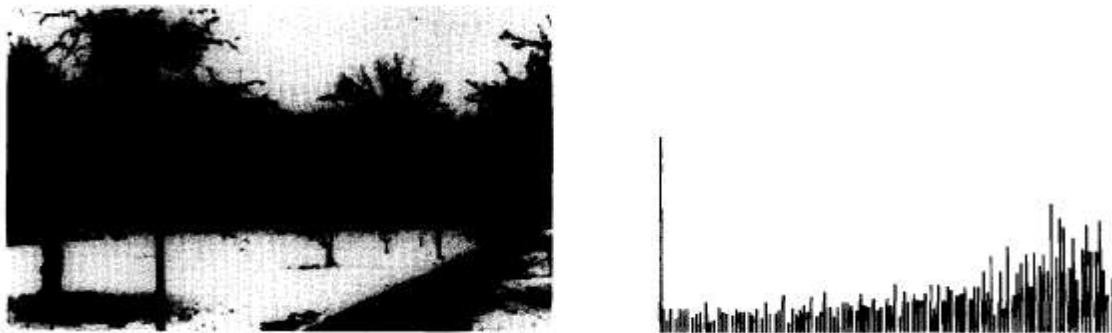


Fig 1: Original image of UofL campus and its histogram [Sahoo et al.(3)]



Fig 2: Thresholded at $t^*=155$ and at $t^*=177$ [Sahoo et al (3)]

Image thresholding using Tsallis entropy

Using Tsallis entropy Albuquerque, Esquef, Gesualdi and Albuquerque [4] proposed a method to obtain an optimal threshold value for different values of the non-extensive parameter q . The entropies of object and background are:

$$S_o^q = \frac{1 - \sum_{i=1}^t (\frac{p_i}{p_o})^q}{q-1} \quad (11)$$

$$S_b^q = \frac{1 - \sum_{i=t+1}^n (\frac{p_i}{p_b})^q}{q-1} \quad (12)$$

Unlike Shannon and Renyi entropies, the Tsallis entropy is pseudo additive in nature. Hence, the sum of the above two entropies comes out to be

$$S^q = S_o^q + S_b^q + (1-q)S_o^q \cdot S_b^q \quad (13)$$

Now the optimal threshold value

$$t_{opt}^* = \text{argmax}_q (S^q)$$

$$= \text{argmax}_q (S_o^q + S_b^q + (1-q)S_o^q \cdot S_b^q) \quad (14)$$

Albuquerque et. al. [4] demonstrates the influence of parameter q in the thresholding.

Figure 3 shows the Tsallis entropy based image thresholding with varying q parameter. However, these authors point out that they could not find evidence which relate q and image categories.

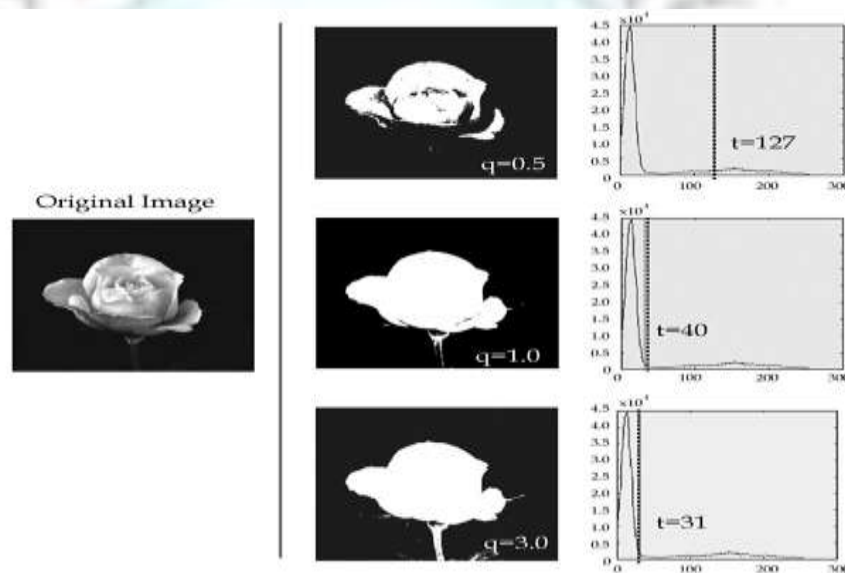


Fig 3: Tsallis entropy based image thresholding [Albuquerque et al.(4)]

Conclusion

We presented a brief survey of image thresholding techniques using entropy. One of them is based on entropy of the histogram. On the basis of histogram entropy, there are two algorithms proposed for optimal threshold value calculation, the first method is based on maximization of the posteriori entropy while the other is based on anisotropic coefficient calculation. Renyi entropy based mechanism has been also studied. The study of thresholding scheme based on Tsallis entropy in which the parameter q plays an important role has also been studied in this paper.

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