

Fading Multiplicative Noise Model Estimation for Low Lightness Images

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ABSTRACT

The study of image processing that deals with low lightness and low contrast has a major importance in many applications in several fields especially surveillance and security systems, under water imaging, night vision in bad vision conditions like: rain, fog, dust, aerial and medical Imaging. Low-light images accompanied by high proportion of noise and defects and this considered big problems for the digital images in this fields. Therefore, this study focused on low lightness effect on captured test images (black and white targets) using two types of cameras (Samsung and Sony). Lighting environment is controlled by a lighting system depending on three fluorescents of light with different sizes and power Estimating mathematical model for the noise accompanied with captured images under low lightness for each camera. Where have been captured several images in different lightness values from (0 Lux to 7 Lux). Then computed the probability distribution for the estimated multiplicative noise separately in each image targets (black target and white target). After that using Table Curve Software to estimate the best matching eq. with noise distributions. This model represents the multiplicative noise that combine between the Gaussian function and exponential function. The results show that best matching is dependent on the correlation factor which exceeds 90 %. Whereas, the best matching of the black region for test images is for Samsung camera sensors for multiplicative noise, while in the white region of the test images, the best matching of the results is for Sony camera.

Keywords: multiplicative Noise, Images Lightness, Lux meter, Least Square Fitting, Table Curve.

1. INTRODUCTION

The amount of light coming to the eye from an object depends on the amount of light striking the surface, and on the proportion of light that is reflected. If a visual system only made a single measurement of luminance, acting as a photometer, then there would be no way to distinguish a white surface in dim light from a black surface in bright light. Yet humans can usually do so, and this skill is known as lightness constancy [1]. Most vision applications such as surveillance, security, etc. require robust detection of image features. Images captured under low-light conditions (e.g. night time, indoor and underexposure), however, suffer from poor lightness and severely distorted color and thus exhibit very little scene information. Therefore, it is imperative to correct for lightness, contrast and color fidelity in order to provide a clearer view of the scene and make vision systems more reliable [2]. Images formed at low-light levels are corrupted by the noise associated with the discrete nature of light. This noise is labeled as Poisson noise, since the emission of photons is governed by a Poisson random process. The noise is clearly signaling dependent; the variance of the Poisson probability density is equal to its mean. Examples of image signals corrupted by Poisson noise occur in scintillation camera imaging, medical imaging, astronomical imaging, and low-light-level television systems. Restoration of images degraded by signal-dependent noise has been extensively studied [3].

Many researchers have been study; the noise in the images under low lighting conditions, and some of these studies are as follows:

Anwar H.M. Al-Saleh (2012) studied the effect of the light distribution on the quality of captured images under different amount of lightness conditions then enhancing these images using an adaptive histogram equalization method and Lee's

method using color YIQ space. The results were analyzed and compute the quality of the enhancement images by using various statistical criteria based on the mean and standard deviation. Where the Adaptive histogram equalization technique gives a good quality for the enhanced images for different lightness conditions [4].

Muna Abdul Hussain Radhi¹ (2014): studied the effect of the light distribution on the quality of captured images under different lightness conditions then enhancing the captured images using Histogram equalization method using YIQ color space. [5].

Devanand Bhonsle.el^t (2015): Introduces a technique of bivariate threshold based dual tree complex Wavelet transform (DTCWT) to remove additive and multiplicative both the noise signals. Since both the noise are different in nature hence it is difficult to remove both types of noise by using single filter. Therefore two different filters are required to denoise medical images which are corrupted by either of the noises simultaneously [6].

2. LIGHT AND CAPTURE IMAGE QUALITY

Light is just one portion of the various electromagnetic waves flying through space, These waves have a frequency and a length, the values which are distinguishing light from other forms of energy on the electromagnetic spectrum. Light is emitted from a body due to radiance, Electric Discharge, Electro luminescence and Photoluminescence [7]. Most of images cannot exist without light. To produce an image, the scene must be illuminated with one or more light sources. In this section can be focus on interaction of light with surface and some artificial light source. Moreover can be determining general factors that the effect on the light equality assessment. Lighting quality depends on several factors, it depends largely on people's expectations and past experiences of electric lighting .The Lighting quality cannot be expressed simply in terms of photometric measures nor can there be a single universally applicable recipe for good quality lighting[8].

3. IMAGE NOISE

The noise in the images is generally defined as unwanted information leading to the distortion of the image and weakens their clarity, Also the capture process or record digital image (Digital Image Acquisition) which is converted visual image into an electrical signal is subject to an ongoing process of division and numbering usually later on lead to the emergence of noise. There are also fluctuations or oscillations occur because of natural phenomena in the vicinity of imaging system or imaging system itself adds random values to the real values for each element of the image [9].

3.1. Mathematical Noise Models:

The existence of noise in the pictures makes them distorted, which makes the analysis of these images difficult process, and therefore the study of noise helps a lot to know the impact and determine the best techniques to remove them possibly at the lowest loss. The noise types can be classified mathematically in to three main groups: [10]

The additive noise is the simplest noise types that possibly deform the digital images, and be written in terms of observed image $I(x, y)$ of the real image $R(x, y)$ additive signal independent noise $N(x, y)$, (In other words caused by external factors that doesn't relate to the original signal) and it is characterized by a zero mean and variability can take specific values adopted by imaging system and the nature of the target or scene. Mathematically Additive signal independent equation given by [11]:

$$I(x, y) = R(x, y) + N(x, y) \quad \dots \dots (1)$$

This type of noise is characterized as: a white noise and Statistical approximations for the distribution of additive noise usually approaching to Gaussian distribution or uniform Distribution. Multiplicative noise can be considered a noise depend on the signal noise (Signal dependent noise), which means that the bright areas of the image are with a high noise, and whenever the intensity of light was less the noise was less, This means that the relationship between the amount of noise and the rate of the light intensity are positively related and this noise characterized as:[12].A color noise and often approaching from Chi -square distribution for coherent noise and Poisson distribution for photonic noise. The resulting observed image $I(x, y)$ is a real image $R(x, y)$ multiplied by the multiplicative noise parameter $F(x, y)$ as shown in the following mathematical formula [13]:

$$I(x, y) = R(x, y)F(x, y) \quad \dots \dots \dots (2)$$

4. MODELING AND LEAST SQUARES FITTING

The Method of Least Squares is a procedure to determine the best fit curve to data; the proof uses simple calculus and linear algebra. The basic problem is to find the best fit function [14]. The oldest (and still most frequent) use of ordinary least squares (OLS) was linear regression, which corresponds to the problem of finding a line (or curve) that best fits a set of data. In the standard formulation, a set of N pairs of observations (x_i, y_i) is used to find a function giving the value of the dependent variable (Y_i) from the values of an independent variable (x) . With one variable and approximated function $f(x_i)$. The prediction is given by the following eq. [14]:

$$Y_i = f(x_i) \dots \dots \dots (3)$$

This equation involves some parameters which be obtained. Where obtained using least square method by minimize the sum of the squares (hence the name least squares) between the measurements and the model $f(x_i, y_i)$ (i.e., the predicted values). This amounts to minimizing the expression [14].

$$s = \sum_{i=1}^n [Y_i - y_i]^2 = \sum_{i=1}^n (f(x_i) - y_i)^2 \dots \dots \dots (4)$$

$f(x_i)$ = approximate function

Can be optimizing eq. (4) to get least square function parameters value to estimate the best function $f(x_i)$.

5. EXPERIMENTAL WORK

In this study, captured test image (Black and white target image) studied in geometry depict in fig. (1). This image is placed one meter apart of two cameras type Samsung (SN-Cam) and Sony (SN-Cam) and light source placed behind cameras. Light source composed of three fluorescent lamps. Intensity light measured using Lux meter device. Then this image captured in different lighting values (0 to 7 Lux) respectively. These images saved in JPEG format.

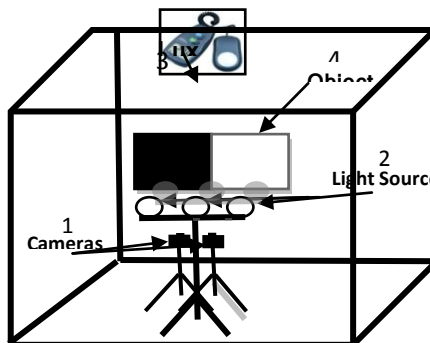


Fig (1): Imaging System for Variant Lighting Condition.

6. The processing Steps and Algorithms

The block diagram shown in Fig (2) the suggested processing steps for determines the mathematical model for Fading of Multiplicative Noise in capture image under low lightness conditions. Where in this study, the work was divided into several steps:

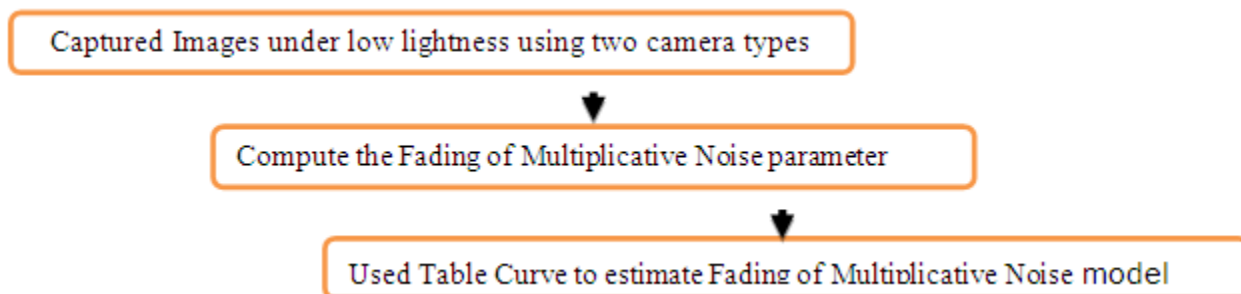


Figure (2): The Block diagram explain the steps of estimate the mathematical model.

The work focused on the analysis of digital images that captured low lightness, were used two camera types (Sony and Samsung) to capture images of one test image of two targets (black and white) as shown in Fig (3), For the purpose of controlling the intensity of lightness have been replaced fluorescent aforementioned lamps. Where light intensity was measured using the Lux meter, then the captured image are divided into targets (black and white) each target used for two camera types (Samsung and Sony) as shown in the Fig (4) and (5) respectively. Then compute some of statistical parameters such as mean (μ), and Fading of Multiplicative Noise for each frame image by using algorithm built in MATLAB program version (7.14) R2012a, according to algorithm (1).

Algorithm (1) Shown the steps of determine multiplicative noise distribution

Input: The images captured under lowlight conditions $I(x, y)$.

Output: Compute Probability Density function for the fading of multiplicative noise $P(F)$.

Start algorithm

1- Load image $I(x, y)$.

2- Compute:

i) Mean of the image.
$$\mu = \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N I(x, y)$$

ii) Fading of Multiplicative Noise:
$$F = \frac{I}{\mu}$$

Compute Probability Density function for the fading of multiplicative noise $P(F)$.

(I.e. compute $P(F)$).

3- Save the output Data $P(F)$.

4- End algorithm.

Finally, Using table curve fitting program software (TC) “2D version 5.01, 2007” to estimate the probability density function of the fading of multiplicative noise $P(F)$. Then determine a mathematical model for fitting functions to find the relationship between (F) and $P(F)$. After that have be perform verification between real values and theoretical values compute from estimated mathematical model function for the two camera types for different lightness values.

7. THE RESULTS AND DISCUSSIONS

To estimate the best models for (black and White) image target from two camera types (Samsung and Sony) shown Fig (6 a and b) for black image target and Fig (6 c and d) for white image target, It is represents the relationship between distribution probability of fading of multiplicative noise $P(F)$ as a function of multiplicative noise (F) for different lighting, Values the estimate best model for the multiplicative noise to combine between the Gaussian function and exponential function given by:

$$P(F) = e^{a_1 + b_1 F + c_1 F^2} \dots \dots (5)$$

- $(a_1, b_1 \text{ and } c_1)$: parameters of the fitting equation for the $P(F)$.

From the Fig (6) the (y) axis represents the distribution probability of fading of multiplicative noise $p(F)$, and (x) axis represent fading of multiplicative noise values (F) . Fitting process using image at lightness $(L=0, 1, 2, 3, 5, 6, 7 \text{ Lux})$ and ignore $(L=4 \text{ Lux})$ from calculate the model by the tables (1 and 2) for (Black and White) image targets respectively.

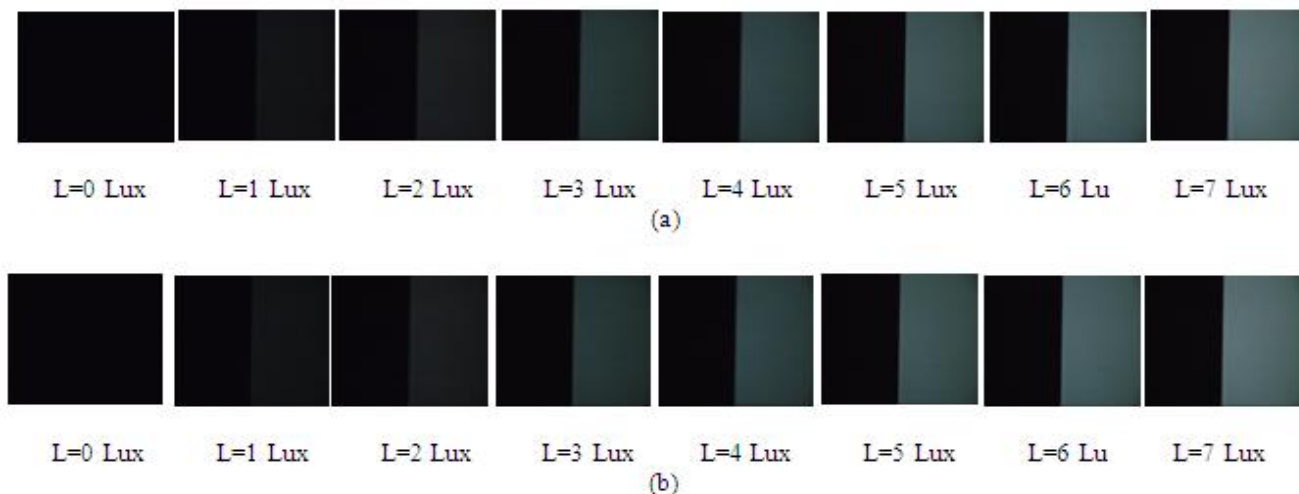


Fig (3) Shows the captured images in different lightness values of two type's camera (a) Samsung (b) Sony camera.



Fig (4) Shows the black target from the captured images in different lightness values for the two camera types (a) Samsung (b) Sony.

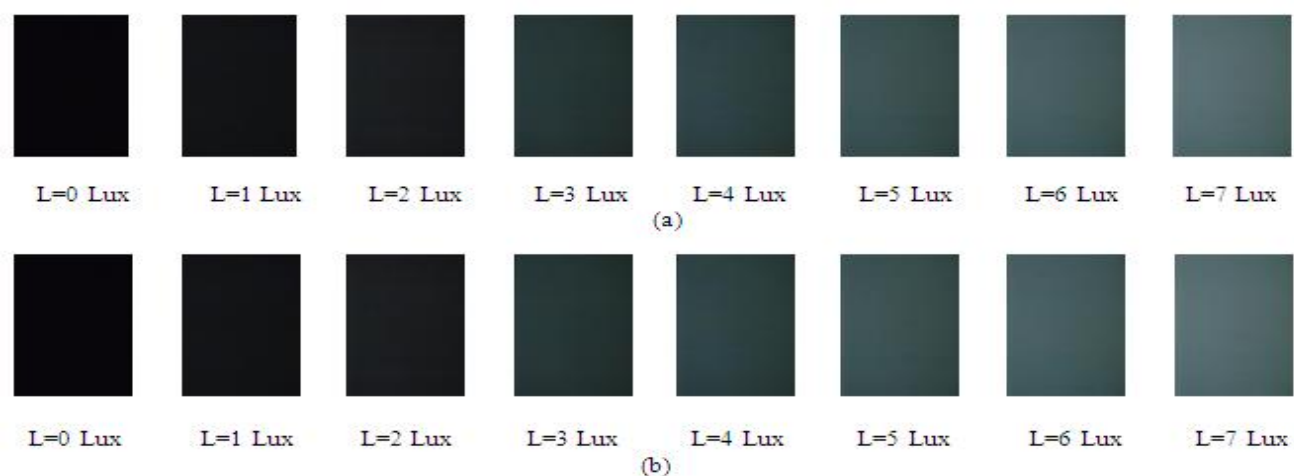
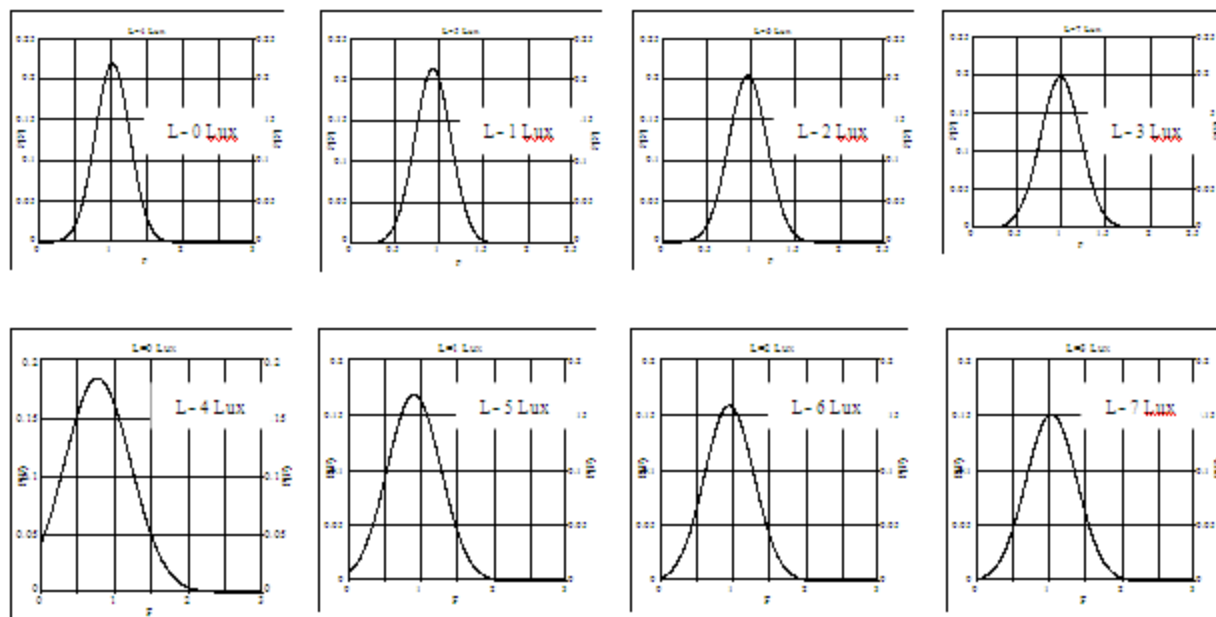
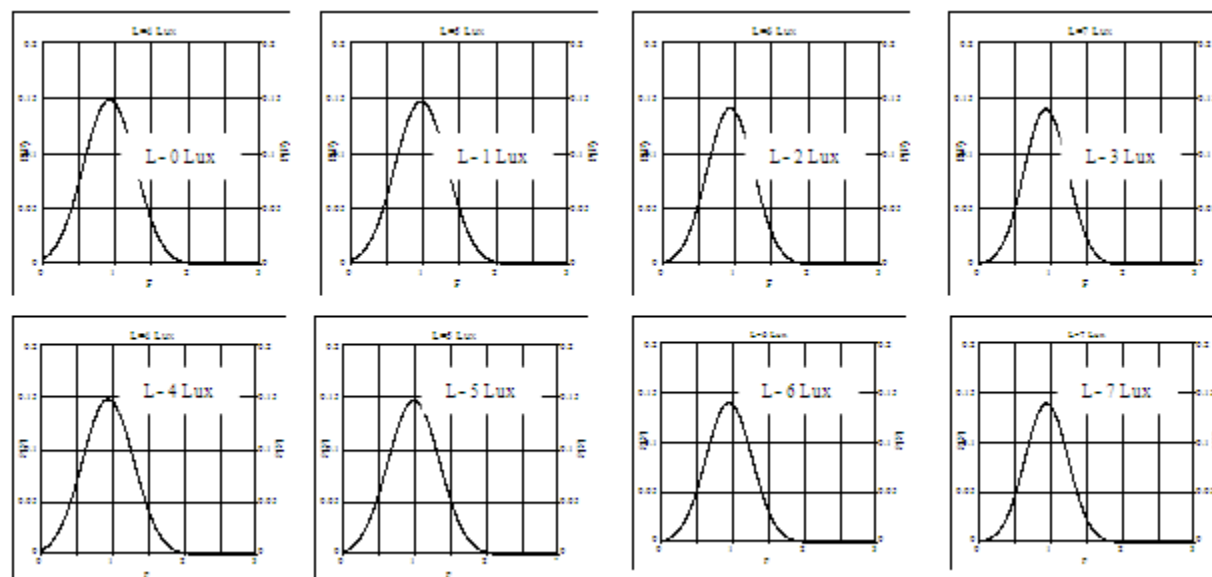


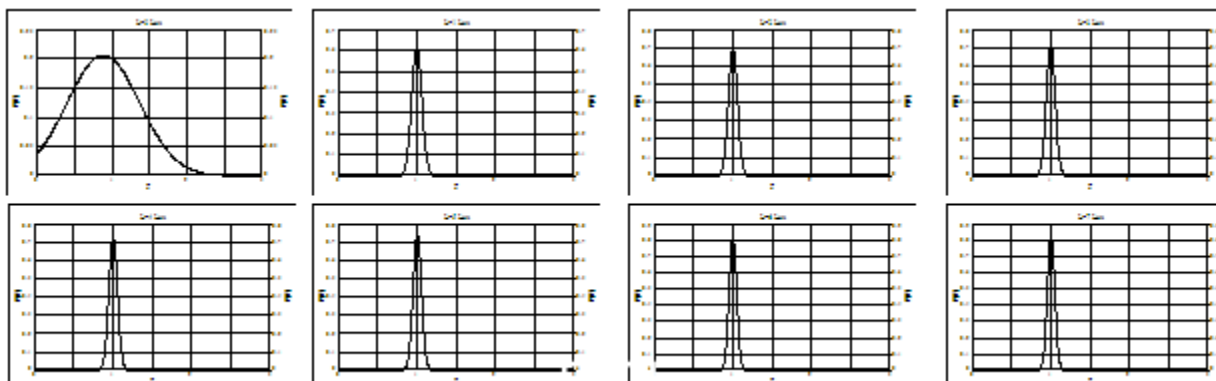
Fig (5) Shows the white target from the captured images in different lightness values for the two camera types (a) Samsung (b) Sony.



(a)



(b)



(c)

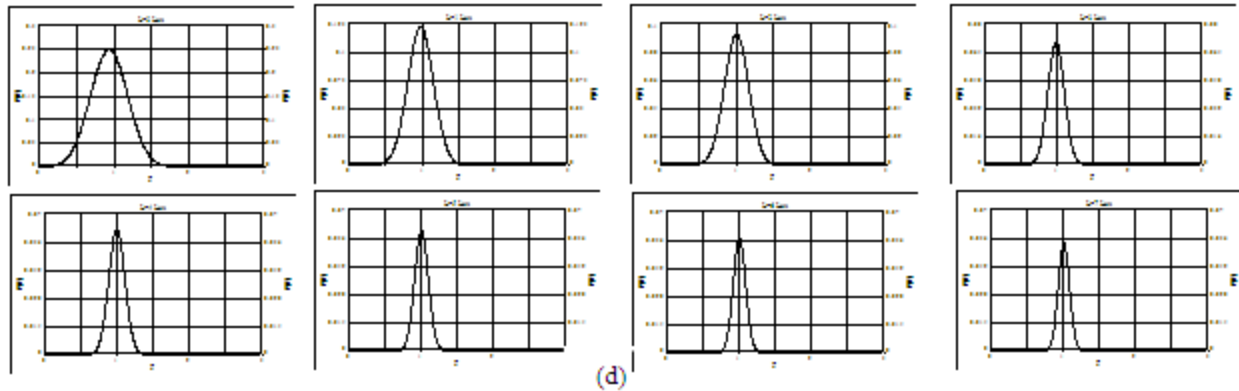


Fig (6) Shows the Relationship between probability and fading of multiplicative noise (F) for black target of ((a) Samsung (b) Sony) while for black target for ((c) Samsung (d) Sony) at different lightness values

Table 1: The (a_i , b_i and c_i) parameters of the fitting eq. for the P (F) for black target for two camera types (Samsung and Sony) at different lightness values.								
L(Lux)	a-parameter		b-parameter		c-parameter		r ² -parameter (correlation)	
	Samsung	Sony	Samsung	Sony	Samsung	Sony	Samsun g	Sony
0	-8.403	-3.199	14.834	3.902	-7.878	-2.524	0.943	0.831
1	-8.969	-4.875	15.402	6.818	-7.878	-3.761	0.960	0.954
2	-8.992	-5.819	16.445	8.376	-9.016	-4.414	0.969	0.968
3	-10.352	-6.367	17.973	8.643	-9.129	-4.177	0.975	0.983
4	-10.512	-5.327	17.745	7.385	-8.755	-3.990	0.974	0.978
5	-11.933	-5.68	22.416	7.710	-12.095	-3.950	0.970	0.981
6	-11.579	-6.33	20.816	9.324	-10.844	-4.973	0.964	0.980
7	-11.288	-6.919	19.520	10.632	-9.855	-5.708	0.955	0.987

Table 2: The (a_i , b_i and c_i) parameters of the fitting eq. for the P (F) for white target for two camera types (Samsung and Sony) at different lightness values								
L(Lux)	a-parameter		b-parameter		c-parameter		r ² -parameter (correlation)	
	Samsun g	Sony	Samsung	Sony	Samsung	Sony	Samsung	Sony
0	-8.55	-3.279	15.30	3.83	-8.17	-2.164	-2.164	0.967
1	-17.74	-122.40	31.86	244.50	-16.23	-122.57	-122.5	0.993
2	-22.16	-159.00	39.22	317.52	-19.43	-158.87	-158.8	0.998
3	-39.92	-165.81	75.58	332.00	-38.33	-166.51	-166.5	0.999
4	-42.56	-173.90	79.89	346.62	-40.10	-173.02	-173.0	0.998
5	-59.34	-183.66	112.45	365.52	-55.93	-182.1	-182.1	0.996
6	-70.44	-226.48	134.57	453.34	-66.99	-227.05	-227.0	0.991
7	-86.25	-233.07	164.92	465.39	-81.61	-232.50	-232.5	0.991

Then using Table curve to drawn the relationship between (a_i , b_i and c_i) parameters as a function of (L) see Fig (7) for black image target for Samsung and Sony cameras respectively, the fig mentioned above notice that three parameters for Sony camera are high matching with real data according to the information matching (r^2), the r^2 represent parameter (correlation) while for Samsung camera are (less information matching) as in Table (4). Fig (8) represent the relationship

between $(a_i, b_i \text{ and } c_i)$ parameters as a function of (L) for white image target for Samsung and Sony cameras respectively. Also notice that the $(a_i, b_i \text{ and } c_i)$ parameters for Samsung camera are high matching with real data according to the information matching (r^2) while for Sony camera is (less information matching) where the parameters $(a_i \text{ and } c_i)$ are decreasing with increase (L), while the parameter b_i is increase with increase (L). The estimated function for $(a_i, b_i \text{ and } c_i)$ parameters for two cameras (Samsung and Sony) for black image target given by:

$$a_i = \frac{1}{(a_{i1} + b_{i1}L + c_{i1}L^2 + d_{i1}L^3 + e_{i1}L^4 + f_{i1}L^5)} \dots (6)$$

$$b_i = \frac{1}{(a_{i2} + b_{i2}L + c_{i2}L^2 + d_{i2}L^3 + e_{i2}L^4 + f_{i2}L^5)} \dots \dots \dots (7)$$

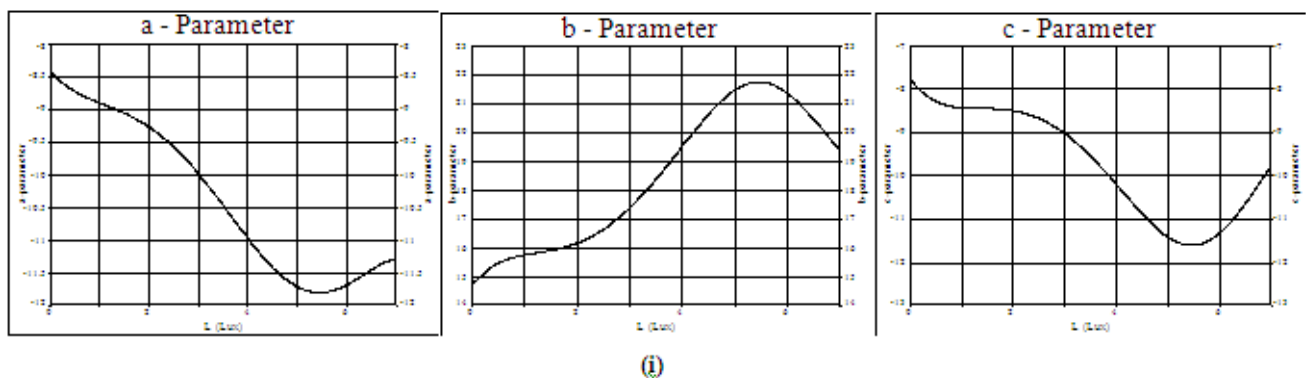
$$c_i = \frac{1}{(a_{i3} + b_{i3}L + c_{i3}L^2 + d_{i3}L^3 + e_{i3}L^4 + f_{i3}L^5)} \dots \dots \dots (8)$$

Table 3: The parameters values for the eq. (11, 12 and 13) for two type's camera (Samsung and Sony) for black image target.								
Black Region	cameras	a_{i1}	b_{i1}	c_{i1}	d_{i1}	e_{i1}	f_{i1}	r^2
	Samsung	-0.118	0.011	-0.008	0.003	-0.0006	3.705	0.96
	Sony	-0.307	0.115	-0.010	-0.010	0.002	-0.0001	0.97
		a_{i2}	b_{i2}	c_{i2}	d_{i2}	e_{i2}	f_{i2}	r^2
	Samsung	0.067	-0.010	0.008	-0.003	0.0005	-2.779	0.89
	Sony	0.249	-0.135	0.036	0.001	-0.001	0.0001	0.99
		a_{i3}	b_{i3}	c_{i3}	d_{i3}	e_{i3}	f_{i3}	r^2
	Samsung	-0.128	0.025	-0.022	0.008	-0.001	6.113	0.77
	Sony	-0.394	0.171	-0.042	-0.005	0.002	-0.0001	0.99

For **black** image target from eq. (10), eq. (11) and Table (4) for two types of Cameras (Samsung and Sony) can be getting the following:

$$P(F) = e^{(1/(a_{i1} + b_{i1}L + c_{i1}L^2 + d_{i1}L^3 + e_{i1}L^4 + f_{i1}L^5)) + (1/(a_{i2} + b_{i2}L + c_{i2}L^2 + d_{i2}L^3 + e_{i2}L^4 + f_{i2}L^5))F + (1/(a_{i3} + b_{i3}L + c_{i3}L^2 + d_{i3}L^3 + e_{i3}L^4 + f_{i3}L^5))F^2} \dots \dots (9)$$

Eq. (9) represents a mathematical model between noise distributions $P(F)$ as a function of lightness (L). Here if anyone knew any two variables of the three variables ($P(N)$, N , and L) can be easily Conclude to the third variable. From eq. (9) can be compute the value of distribution probability for the fading of multiplicative noise $P(F)$ at the lightness value ($L = 4$ Lux), To compare with the actual values that have been get them using the algorithm (1) for the same lighting value and compared the theoretical and practical data for two cameras shown Fig. (9).



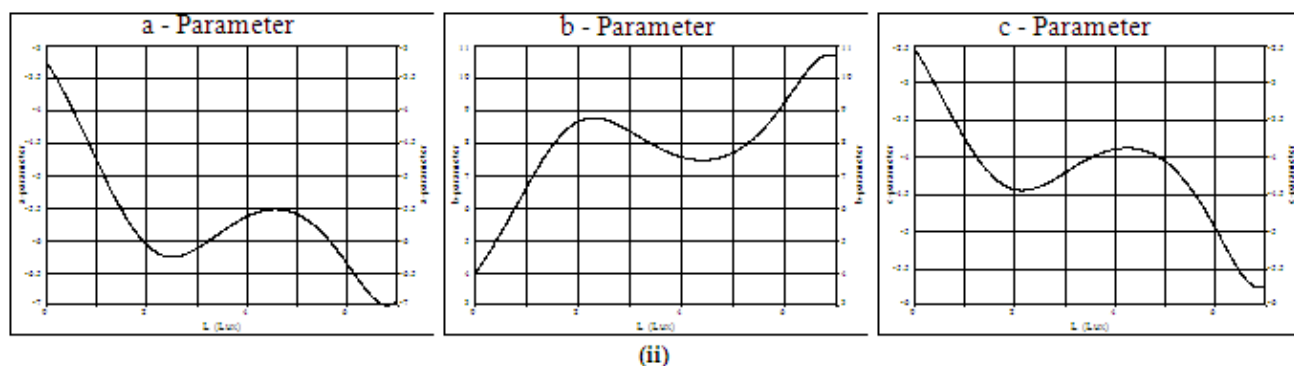


Fig (7) Shows The Relationship between (a , b and c) parameters as function of lightness (L) in Black image target for two camera types (i) Samsung (ii) Sony camera

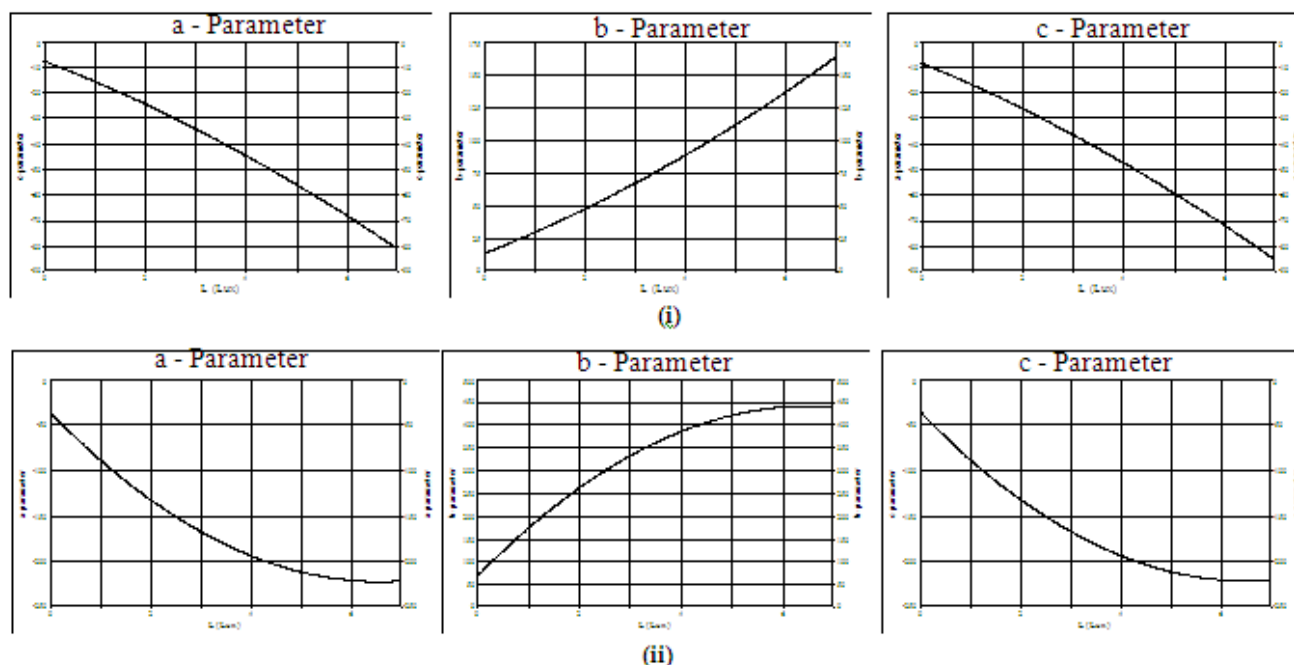


Fig (8) Shows The Relationship between (a , b and c) parameters as function of lightness (L) in White image target for two camera types (i) Samsung (ii) Sony camera

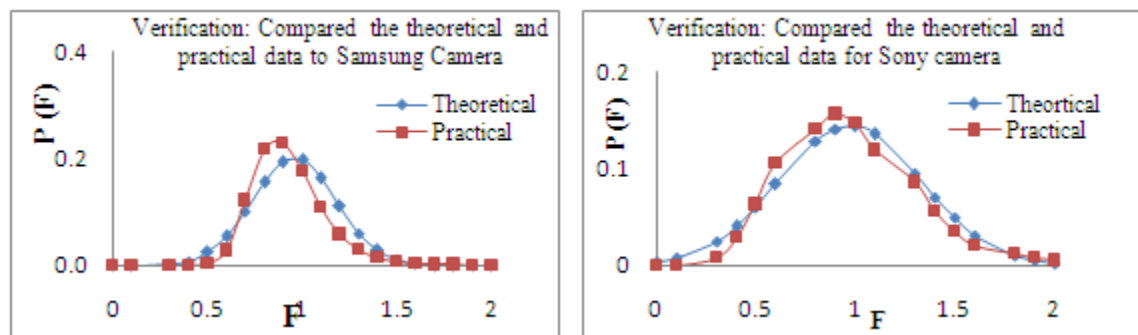


Fig (9) shows Comparing the theoretical and practical data curve for the two Camera types for Black image Target when (L=4 Lux)

While, the estimated function for (a, b and c) parameters for two cameras (Samsung and Sony) for white image target as:

$$a_i = a_{i1} + b_{i1}L + c_{i1}L^2 \dots \dots \dots (10)$$

$$b_i = a_{i2} + b_{i2}L + c_{i2}L^2 \dots \dots \dots (11)$$

$$c_i = a_{i3} + b_{i3}L + c_{i3}L^2 \dots \dots \dots (12)$$

Table 4: The parameters values for the eq. (8 and 9) for two Camera Types (Samsung and Sony) for White image target.					
White Region	cameras	a_{i1}	b_{i1}	c_{i1}	r^2
	Samsung	-8.327	-8.080	-0.421	0.993
	Sony	-36.192	-56.458	4.305	0.882
	cameras	a_{i2}	b_{i2}	c_{i2}	r^2
	Samsung	14.360	15.125	0.890	0.992
	Sony	70.527	113.610	-8.689	0.879
	cameras	a_{i3}	b_{i3}	c_{i3}	r^2
	Samsung	-7.560	-7.550	-0.424	0.990
	Sony	-35.625	-56.675	4.336	0.879

For white image target from eq. (10), eq. (11), eq. (12) and Table (4) for two types of Cameras (Samsung and Sony) can be getting:

$$P(F) = e^{(a_{i1}+b_{i1}L+c_{i1}L^2)+(a_{i2}+b_{i2}L+c_{i2}L^2)F+(a_{i3}+b_{i3}L+c_{i3}L^2)F^2} \dots \dots \dots (13)$$

Eq.(13) represents a mathematical model between noise distribution $P(F)$ as a function of lightness (L) and here if anyone knew any two variables of the three variables ($P(F)$, F , L) can be easily infer to the third variable. From eq. (13) can be compute the distribution probability of the fading of multiplicative noise $P(F)$ at the lightness value ($L = 4$ Lux), to compare with the actual values that have been get them using the algorithm (1) for the same lighting value and compared the theoretical and practical data for two cameras see in Fig. (10b).while the Fig. (10a) Represent Comparing the theoretical and practical normalize data curve with respect to maximum pick for the Samsung Camera to White image target when ($L=4$ Lux) .

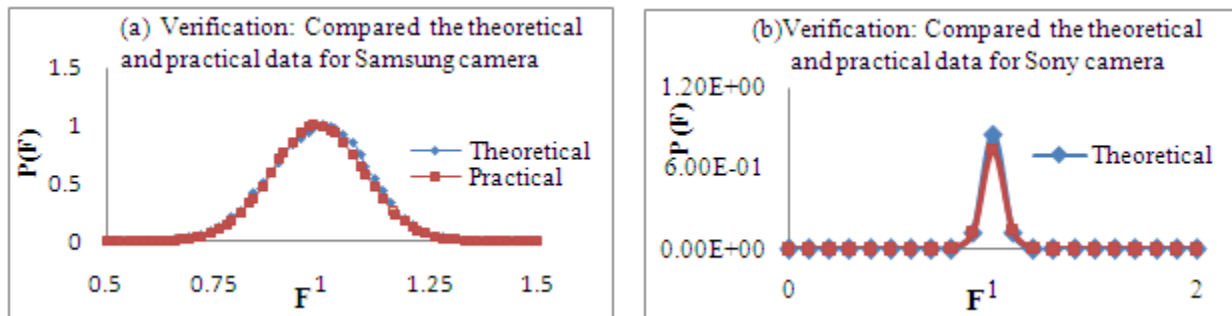


Figure (10) Shows Comparing the theoretical and practical normalize data curve with respect to maximum pick for the Samsung Camera to White image target when ($L=4$ Lux)

CONCLUSION

It has been get efficient best model for the multiplicative noise to combine between the Gaussian function and expositional function for the two type camera (Samsung and Sony), and for both image target s Black and White.

1. Estimate the best mathematical model for the two camera types (Samsung and Sony) and for **Black** image target for fading of multiplicative noise distribution as a function of lightness this model given by eq:

$$P(F) = e^{(1/(a_{i1}+b_{i1}L+c_{i1}L^2+d_{i1}L^3+e_{i1}L^4+f_{i1}L^5))+(1/(a_{i2}+b_{i2}L+c_{i2}L^2+d_{i2}L^3+e_{i2}L^4+f_{i2}L^5))F + (1/(a_{i3}+b_{i3}L+c_{i3}L^2+d_{i3}L^3+e_{i3}L^4+f_{i3}L^5))F^2}$$

2. Estimate the best mathematical model for the two camera types (Samsung and Sony) and for **White** image target fading of multiplicative noise distribution as a function of lightness this model given by eq :

$$P(F) = e^{(a_{i1}+b_{i1}L+c_{i1}L^2)+(a_{i2}+b_{i2}L+c_{i2}L^2)F+(a_{i3}+b_{i3}L+c_{i3}L^2)F^2}$$

3. The $(a_i, b_i, \text{ and } c_i)$ parameters for Sony camera are high matching with real data according to the information matching (r^2) while for Samsung camera are (less information matching) for Black image target.
4. the $(a_i, b_i \text{ and } c_i)$ parameters for Samsung camera are high matching with real data according to the information matching (r^2) while for Sony camera is (less information matching) for White image target.
5. The $(a_i \text{ and } c_i)$ parameters are decreasing with increase (L) , while the parameter (b_i) is increase with increase (L) , for the two camera types (Samsung and Sony) and for two image target (Black and White).
6. The obtained results from the estimated models are excellent and very close to the practical value.

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