# Consumer's Price Elasticity of Demand Modeling on Electricity Markets using Ant Colony Optimization

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Abstract: Automated Metering Infrastructure (AMI) is a energy technology that would allow consumers to exhibit price elasticity of demand under smart-grid environments. The market power of the generation and transmission companies can be mitigated when consumers respond to price signals. Such responses by consumers can also result in reductions in price spikes, consumer energy bills, and emissions of greenhouse gases and other pollutants. Here, proposes a novel binary ant colony optimization (NBACO) method. The proposed NBACO is based on the concept and principles of ant colony optimization (ACO), and developed to solve the binary and combinatorial optimization problems. It simulates restructured electricity markets, to explore the impact of consumers' price elasticity of demand on the performance of the electricity market. An 11-node test network with eight generation companies and five aggregated consumers is simulated for a period of one month. At presents of new updating rule for local and global search, the proposed NBACO is applied to test power systems of up to 100-unit along with 24-hour load demands.

Index Terms: Ant colony optimization modeling, automated metering infrastructure, price elasticity of demand, smart grid.

# I. INTRODUCTION

IN Deregulated electricity markets, market power and imbalances in the supply demand associated with the marginal cost of the last unit dispatched have resulted in large of the fluctuations in wholesale electricity prices [1]. In many of the existing electricity markets, only generation companies (GenCos) can respond to the price signals through supplyside offers to the independent system and/or market operator (ISO). The majority of consumers in deregulated markets have contracts with load aggregators or load serving entities who, in turn, submit demand bids to the market operator. If the contract is a pass through contract, there is no incentive for the load aggregator to provide a mechanism for consumers to respond to prices. On the other hand, if it is a fixed price contract, consumers do not see the market prices and will not respond to price signals. Moreover, because most consumers do not have access to hourly or daily electricity price information, their responses to price changes may lag behind[8].

Ant Colony Optimization (ACO) is one of metaheuristic and evolutionary approaches to find the optimal solutions of the combinatorial or binary search problems. ACO was first developed by Dorigo et al. In ACO, artificial ants search for good solutions in a cooperative way. Artificial ants move randomly along paths and deposit chemical substance trails, called pheromone, on the ground when they move, then collect and store information in pheromone trails. This pheromone trails motivates them to follow the path and can choose the shortest path in their movement. The ACO method has been researched in various aspects and successfully applied to the various optimization problems.

There has been considerable research on consumer response to electricity prices. In addition, efforts have been under taken recently to model and simulate the price elasticity in electricity markets. Such studies have shown that reductions in electricity consumption in response to prices, particularly by residential customers, are relatively inelastic in the short term; even high price increases produce fairly small changes in electricity usage. Large consumers, on the other hand, are relatively price sensitive [11].

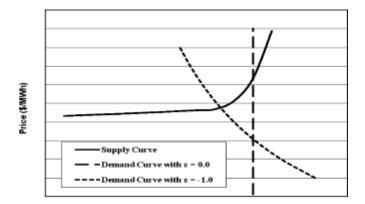
Recently, AMI and smart grid have become widely accepted as promising technologies to provide increased awareness of electricity usage and cost to consumers. As a result, those technologies could enable consumers to overcome the technical and market barriers to participating in electricity markets through improved price elasticity.

In this paper, we have set up a model for exploring consumers' price elasticity of demand by Ant Colony Optimization that simulates the deregulated markets [14].

The remainder of this paper presents demand side response modeling with price elasticity in Section II. The Section III describes the experimental investigation and provides results and discussion. Section IV offers a results and discussion on electricity price markets. Section V presents our conclusions.

# II. DEMAND-SIDE RESPONSE MODELING WITH PRICE ELASTICITY

# A. Price Elasticity



Quantity (MWh)

Fig. 1: Typical demand and supply curves

TABLE I: Estimates of Electricity Price Elasticity

	Price Els	asticity
	Short-Run	Long-Run
Residential	-0.06 to -0.49	-0.45 to -1.89
Commercial	-0.17 to -0.25	-1.00 to -1.60
Industrial	-0.04 to -0.22	-0.51 to -1.82

In economics literature, price elasticity is defined as the percentage change in demand or load result in percentage change in price, for mathematically it can be expressed as:

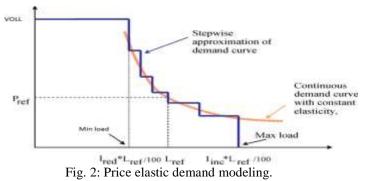
$$\frac{\delta L}{\delta P} \cdot \frac{P}{L} \tag{1}$$

where,  $\varepsilon$  is the consumer's price elasticity of the demand,  $\delta L$  is the consumer's change in load,  $\delta P$  is the price change, p is the forecasted energy price (%/MWh).

The equation indicates that: a) a price elasticity of it \$ means that a 1 percent increase in price will result in a 1 percent decrease in load, b) that zero price elasticity means that the consumers are insensitive to the price of electricity and that the load is unaffected by the price. In the latter case, the demand curve is a vertical line, as shown in Fig. 1. However, in electricity markets, the supply curve is more like a hockey stick, in which prices increase moderately for most of the supply curve except at the end, where prices increase dramatically with a steep slope. The demand responsiveness provides the greatest benefit in this region [4].

## **B.** Estimate Price Elasticity of Demand for Electricity

In general, measuring price elasticity is a complex task, and estimated elasticity coefficients usually have a wide range of uncertainty attached to them. It is common to differentiate between short-and long-run elasticity. Short-run elasticity means the loading parameter ( $\lambda$ ) will be increased at all nodes with the same value till the voltage at one node reaches the minimum voltage.



describes the price response from the system with its current infrastructure and equipment; long run elasticity takes into account the investments that can be made (e.g., in energy conservation or alternative energy supply) in response to higher prices. Table I lists examples of ranges of estimates for short-run and long-run elasticity based on several studies [10]. However, because the studies were carried out in regulated systems, they might have limited validity for restructured markets. In general, one would expect the price elasticity of demand to increase with implementation of AMI and smart grid [9].

## C. Demand-Side Bidding and Market Clearing in the Day-Ahead Market

$$P = \frac{\partial p}{\partial p} \tag{2}$$

Where the elasticity are constants, is a user input, and can easily be calculated for each hour from L and P are the Equation (2) is used to represent the demand-side bidding in the model. However, the continuous curve in Fig. 2 cannot be bid directly into the market; a stepwise approximation is necessary to calculate the market clearing as a linear (LP) Problem.

The degree of match between the continuous curve and the stepwise approximation depends on the number of steps on the demand curve, as defined for each of the consumers. Step size is constant for all the load reduction steps and also for all the load-increase steps. The corresponding prices are calculated for the load at the midpoint of each step by using the following formula [2].

$$P=Max\left(a^{\frac{1}{\varepsilon}}, L^{\frac{1}{\varepsilon}}\right)$$
(3)

Note that a maximum demand bid price is equal to the value of lost load (VOLL).

The market clears where the supply curve intersects with the demand curve, and the resulting price and load are set accordingly. The actual load in the day ahead market can therefore be higher than, lower than, or equal to the reference load. The resulting load from the clearing of the day ahead market, P ref is used as an inelastic load in the realtime market. This is illustrated in Fig. 3, where the demand curve is represented as a vertical line with a price equal to VOLL.

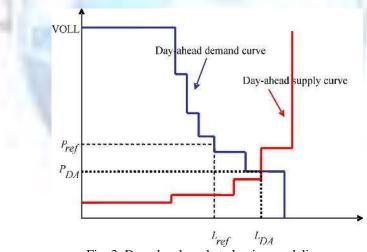


Fig. 3: Day-ahead market clearing modeling.

Note that in Fig. 4, we assume that of the generators are on forced outage, causing the real-time price, to be higher than the day-ahead price [3].

## **III. EXPERIMENTAL STUDY**

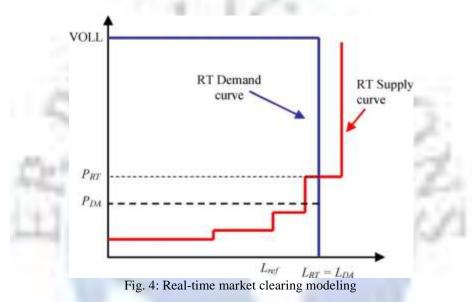
#### A . Ant Colony Optimization Algorithm

The agents (i.e. ants) are guided by the intensity of pheromone trails. The path rich in pheromone becomes the best tour with time. This concept inspired the ACO algorithm. Initially, each agent is positioned on a starting node. Agents move to feasible neighbour node following state trasition rule. During the transfer path ant modify the pheromone level by

applying the local updating rule. If the pheromone level on the chosen paths is lowered, these paths become less attractive to other agents. This property gives agents a higher probability to explore different paths and find an improved solution [5].

The number of combinations of 0-1 variables grows exponentially as the number of units grows. Over the past decades, many salient methods have been developed for solving the UC problems. The exact solution to the problem can be obtained by complete enumeration, which cannot be applied to the real power systems due to its computational burden [15].

In the experimental simulations, we use an 11-node trans- mission network configuration; this approach is based on the method. The technical specifications and the topology for the transmission lines are listed in Table II. There are eight GenCos in the system, located at various nodes in the grid (Fig. 5). All of the GenCos have the same set of generating units: one base load coal plant (CO), one combined-cycle plant (CC) to cover intermediary load, and one gas turbine (GT) peaking unit. For each GenCo, all three generating units (CO, CC, and GT) are connected to the same node. From one node to another node circuit reactance and Line capacity are given in the per unit value.



Line	From	То	Circuit	Line
No.	Node	Node	Reactance	Capacity
			(per unit)	(MW)
1	1	2	0.02	2,000
2	1	3	0.025	1,600
3	2	3	0.08	250
4	2	4	0.01	3,000
5	2	5	0.02	1,000
6	3	8	0.04	1,000
7	3	9	0.05	400
8	4	5	0.01	2,000
9	-4	6	0.02	2,000
10	4	7	0.01	3,000
11	5	7	0.015	2,000
12	6	7	0.01	2,000
13	8	10	0.025	1,600
14	8	9	0.03	1,000
15	9	10	0.04	500
16	6	11	0.02	1,500
17	7	11	0.025	1,200
18	10	11	0.04	500

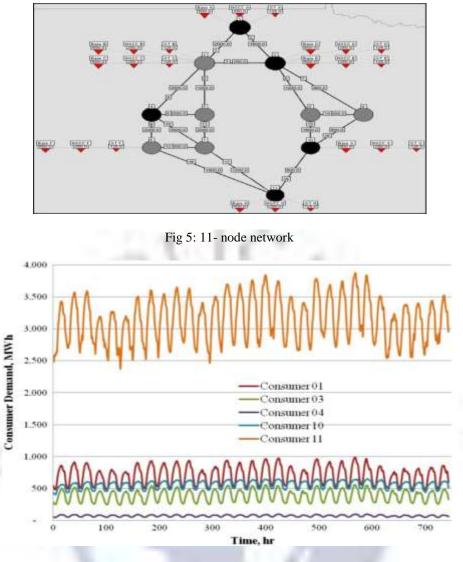


Fig. 6: Hourly consumer load in 11-node case study

We use an aggregate representation of the demand side of the market. Five aggregate consumers are included, representing total demand in the node where they are connected. The loads are connected to nodes 1, 3, 4, 10, and 11. We are simulating the month of July, which is assumed to be the peak load month of the year. The five hourly load series are shown in Fig. 6. The highest load is clearly in node 11.

## **B.** Scenarios and Price Elasticity Parameters

For the sake of simplicity, we assumed that all five consumers exhibited price elasticity. A number of scenarios were run to analyze the impact of price elasticity and the reference price of consumers.

In all of these scenarios, we assumed that the GenCos bid the Incremental production cost of their units (as listed in Table IV). In demand-side bidding, the consumers had a reference price of various price of electric coefficient. In addition, the lower and upper load decrease and increase limits were set at 90% and 105% of the base load, respectively. These scenarios are summarized in Table V. The loads served in the base case and in other scenarios for a typical day are shown in Fig. 7, which shows that consumers increase their load when prices are lower and decrease their load when prices are higher [6].

Tables VI and VII, respectively, present the reductions in peak load, total load served, and total energy cost under various scenarios. The overall peak load reduction is in the rangeof 5% to 8%. However, the peak load reduction for Consumer 10 is only in the range of 1% to 5%. The lower peak load reduction for Consumer 10 can be attributed to the LMPs at node10. The LMPs at node 10 exceed the consumers' reference price 85% of the time; at other nodes, it exceeds the reference price 91% of the time (Fig. 8). Therefore, the peak load reduction for Consumer 10 is much less than of the other consumers [7].

However, other consumers benefit from a reduction in both load and prices. Table V presents the impact of the consumers' price elasticity on GenCos and TransCos. When consumers exhibit price elasticities in the range of to, the GenCos' profits are reduced by 3.50% to 6.87% and the TransCo's congestion revenues are almost eliminated.

Here the Table III is represents the consumers 1, 3,4,10,and 11 are shows the different base cases and the total of these base cases are calculated from this five consumers. The maximum of the base case is consumer 11 and the minimum base case is consumer 4.

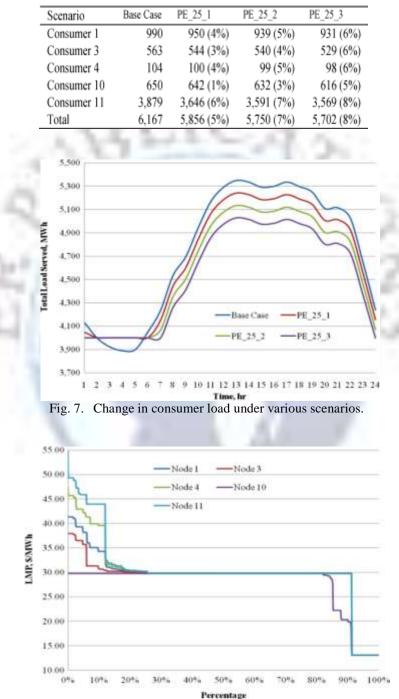
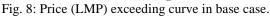


TABLE III: Peak load and its reduction in 11 node system



Scenario	Base Case	PE_25_1	PE_25_2	PE25_3
Consumer 1	15.95	15.43	15.00	14.67
Consumer 3	8.62	8.44	8.24	8.07
Consumer 4	1.76	1.68	1.62	1.58
Consumer 10	11.70	11.86	11.71	11.50
Consumer 11	72.91	68.84	66.36	64.74
Total	110.94	106.25	102.94	100.56

TABLE IV.: Total energy cost in 11-node system (MM\$)

This Table presents the impact of the consumers' price elasticity on GenCos and TransCos. When consumers exhibit price elasticities in the range of to, the GenCos' profits are reduced by 3.50% to 6.87% and the TransCo's congestion revenues are almost eliminated [13].

TABLE V: Impact of con	sumer pric	e elasticity	and refere	ence price
	Base Case	PE_30_1	PE_30_2	PE_30_3

	Base Case	PE_30_1	PE_30_2	PE_30_3
Consumers				
Total load served, GWh	3,643	3,639	3,635	3,634
Total energy cost, MM\$	110.94	110.88	109.96	109.39
Avg. energy price,	30.45	30.46	30.25	30.11
\$/MWh				
GenCos				
Total revenue, MM\$	109.26	109.98	109.40	109.90
Fuel cost, MM\$	67.71	67.26	67.08	67.04
Startup costs, MM\$	1.22	1.20	1.20	1.18
Fixed O&M, MM\$	10.30	10.30	10.30	10.30
Variable O&M, MM\$	7.14	7.08	7.05	7.05
Operating profit, MM\$	22.89	24.15	23.77	23.42
Profit increase, %	n/a	5.5	3.82	2.32
TransCo				
Line use revenue, MM\$	36.43	36.39	36.35	36.34
Congestion rev., MM\$	1.68	0.89	0.56	0.40
Congestion rev. inc., %	n/a	-47.05	-66.52	-76.47

Increase in the GenCos' profits, because even though they are generating less energy compared with the base case, the startup costs decrease; where as there is a significant reduction in the congestion charges. The table presents the profits of each GenCos, individual consumers load served and total cost respectively, when consumers have a higher reference price. When the price response is reduced because of a higher reference price, the total cost for consumers at nodes 3 and 10 increases compared with the base case. This shows that all consumers do not benefit equally, and some of them may actually face a higher cost. The level of the customers served the maximum and minimum power [14].

Because there are several consumers in the system, the results are presented here at the zonal level. There is a 2% to 4% reduction in the peak load in all zones as the consumers increase similarly; there is a 1% to 2.5% reduction in the total load. By exhibiting price elasticity, consumers were also able to reduce their total cost in the range of 2.0% to 4.0%.

### IV. RESULTS AND DISCUSSION

The range of the 11 number of the distribution loads are served from minimum to maximum from the priority of power ranges. The number of combinations of 0-11 variables grows exponentially as the number of units grows. Over the past decades, many salient methods have been developed for solving the economic dispatch problems.

### TABLE VI:

(A) Genco profits with higher consumer reference price (mm\$), (b) Consumers' load served with higher consumer reference price (gwh), (c) Consumers' total costs with higher consumer reference price (mm\$)

		a		
GenCo	Base Case	PE_30_1	PE_30_2	PE_30_3
GenCo A	2.93	3.08	3.03	2.99
GenCo B	3.03	3.09	3.00	2.94
GenCo C	3.01	3.08	2.99	2.93
GenCo D	2.80	3.07	3.07	3.06
GenCo E	2.75	3.02	3.03	3.01
GenCo F	3.03	3.02	2.90	2.81
GenCo G	2.27	2.76	2.86	2.91
GenCo H	3.08	3.02	2.89	2.78
		b		
Consumer	Base Case	PE_30_1	PE_30_2	PE_30_3
Consumer 1	530.10	530.06	529.50	529.24
Consumer 3	291.45	291.55	291.43	291.30
Consumer 4	57.22	57.15	57.11	57.06
Consumer 10	418.07	418.99	418.93	418.92
Consumer 11	2,346.62	2,341.71	2,338.20	2,337.76
		с		
Consumer	Base Case	BS_1.5	BS_2.0	BS_2.5
Consumer 1	30.79	30.89	30.78	30.71
Consumer 3	16.78	16.91	16.90	16.87
Consumer 4	3.36	3.35	3.33	3.32
Consumer 10	23.41	23.96	24.06	24.10
Consumer 11	138.62	137.67	136.67	136.13

TABLE VII: Impact of Consumer Price Elasticity and Reference Price

	Base	PE_55_1	PE_55_2	PE_55_3
	Case			
Consumers				
Total load served, GWh	34,660	34,673	34,736	34,592
Total energy cost,	1,902	1,902	1,906	1,899
billion Won				
Avg. energy price,	54.9	54.8	54.9	54.9
kWon/MWh				
GenCos				
Total revenue,	1,899	1,900	1,905	1,898
Billion Won				
Fuel cost, billion Won	799	779	783	773
Startup costs,	6.1	6.0	5.2	5.3
billion Won				
O&M, MM\$	368	368	368	368
Operating profit, billion	786	747	749	751
Won				
Profit increase, %	n/a	0.1	0.4	0.7
TransCo				
Line use revenue,	173	173	174	173
billion Won				
Congestion revenue,	3.7	2.1	1.5	1.1
billion Won				
Congestion rev. inc., %	n/a	-42.0	-60.2	-69.7

The minimum load served is no.5 and the maximum load served is no.11. The number of agent and maximum count are chosen same as those in the referred the value of pheromone quantity is obtained through a parameter tuning. When the pheromone quantity is more than 0.05, the cost is observed to increase. In otherwords, the solution's quality becomes worse with higher pheromone quantity.

Many practical optimization problems can be formulated as continuous optimization problems. These problems are characterized by the fact that the decision variables have continuous domains, in contrast to the discrete domains of the variables in discrete optimization. While ACO algorithms were originally introduced to solve discrete problems, their adaptation to solve continuous optimization problems enjoys an increasing attention. Early applications of ant-based algorithms to continuous optimization include algorithms such as Continuous ACO. ACO algorithms for discrete optimization problems solutions are constructed by sampling at each construction step a discrete probability distribution that is derived from the pheromone information.

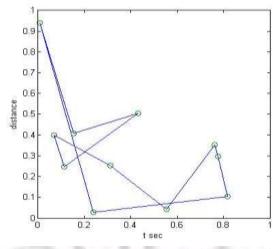
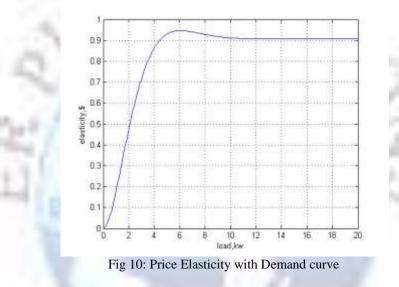


Fig 9: Output for Ant Colony Optimization technique



## **V. CONCLUSION**

This paper describes a study in which Ant Colony Optimization technique was used to demonstrate quantify the economic impact of price elasticity of demand in electricity markets when consumers are well equipped with smart grid technologies to increase their awareness of responsiveness of demand. While the impact depends on the price level at which consumers exhibit the price responsiveness, price-elastic consumers could benefit by a reduction in energy usages and prices. And also they could significantly reduce congestion charges and, potentially, reduce the market power of GenCos. The conventional ACO algorithm is known to have problems such as big memory requirement and long execution time. The proposed ACO algorithm is applied to test power systems of up to 100-unit along with 24-hour. The simulation results reveal that the proposed ACO algorithm may provide better solution for UC problems than the conventional optimization methods in a reasonable time period. The customer use the concept of elastic demand, when they are exposed and aware of the price energy and arrange their affairs in such a fashion at reduce their demands as the price of the next available offers exceeds in a certain level. The main theme of reduce energy consumption by Consumers are well equipped with smart grid technologies to increase their awareness of responsiveness of demand, and benefit by a reduction in energy usages and prices.

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