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Cyclo-stationary Feature Detection using the Fractional Fourier Transform

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ABSTRACT

The Fractional Fourier Transform (FrFT) has many applications, including signal and image processing. It has been shown to provide significantly improved performance in signal detection & demodulation and image enhancement over the conventional fast Fourier transform (FFT). In this paper we apply the FrFT to cyclo-stationary feature detection (CFD), which has also been traditionally performed using the FFT. We show that the FrFT provides improved signal detection capabilities using several modulation schemes as examples, allowing detection rates of 90% or greater at lower signal-to-noise ratio (SNR) for all modulation schemes considered, depending upon the desired probability of false alarm (P_{FA}) and desired probability of detection (P_D). This method therefore has potential for more reliably detecting signals in noisy, non-stationary environments with heavy co-channel interference.

1. INTRODUCTION

The Fractional Fourier Transform (FrFT) has been applied to problems in fields such as quantum mechanics, optics, image processing, data compression, and signal processing for communications [15]. For example, in [2] it replaces the Fast Fourier Transform (FFT) to improve performance of orthogonal frequency division multiplexing (OFDM) in dispersive channels. It has been applied extensively in the field of optics [4]. The optimal linear filter to be used in the FrFT domain is derived in [11] and expanded in [16] for the case of unknown noise statistics. In [13] a least-mean square (LMS) algorithm to implement adaptive filtering in the FrFT domain is introduced. The FrFT is a very useful tool for separating a signal-of-interest (SOI) from interference in a non-stationary environment [15]. Such an environment arises when users are moving or frequency on a mobile device is drifting, as in a cellular system, or due to Doppler and time-varying co-channel interference (CCI), as in a satellite system. The improvement arises because we utilize fractional time-frequency axes not exploited by conventional methods such as the Fast Fourier Transform (FFT), which operates in the frequency domain only [1].

Cyclo-stationary feature detectors (CFDs), which are used to detect signals based on their periodic statistical properties, have conventionally used the FFT; we denote this as simply the CFD. CFDs have been studied for decades and applied also to many fields (see, for example, [8] for a comprehensive overview of past research). Past work has also assumed the environment to be stationary, such that coherent integration is possible (e.g. [6] and [14]). In this paper, we apply the FrFT, which requires estimation of the optimum rotational parameter ‘a’, i.e. the ‘t_a’ axis, to better detect the signal in a non-stationary environment. We will show that the proposed method, denoted FrFT-CFD, can provide significant improvement in detection capability. We study its performance using several modulation schemes, a range of P_{FA} and a range of signal-to-noise ratio (SNR), to be discussed later. We also consider both training mode, i.e. when some information about the signal to be detected is known, as well as blind mode.

The paper outline is as follows: Section 2 briefly reviews the FrFT. Section 3 describes conventional cyclo-stationary detection using the FFT and its theoretical performance. Section 4 discusses the proposed method of detection using the FrFT in place of the FFT. Section 5 has simulation results showing the performance of the proposed CFD-FrFT method with signals using the aforementioned modulation schemes and non-stationary interference, comparing it to the conventional CFD simulations and theoretical performance. Conclusions and remarks on future work are given in Section 6.

2. BACKGROUND: FRACTIONAL FOURIER TRANSFORM (FRFT)

In discrete time, the FrFT of an N × 1 vector **x** is

$$\mathbf{X}_a = \mathbf{F}^a \mathbf{x}, \quad (1)$$

where $\mathbf{F}^a$ is an $N \times N$ matrix with elements ([5] and [15])

$$\mathbf{F}^a[m,n] = \sum_{k=0, k \neq (N-1+(N)_2)}^{N} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n]. \quad (2)$$

and where $u_k[m]$ and $u_k[n]$ are eigenvectors of the matrix **S** defined by [5]

$$\mathbf{S} = \begin{bmatrix} C_0 & 1 & 0 & \dots & 1 \\ 1 & C_1 & 1 & \dots & 0 \\ 0 & 1 & C_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & C_{N-1} \end{bmatrix}, \quad (3)$$

and

$$C_n = 2cos(\frac{2\pi}{N}n) - 4. \quad (4)$$

The term ‘a’ is called the rotational parameter of the FrFT. Optimization of ‘a’, $0 \leq a \leq 2$, where a = 0 is time, and a = 1 is frequency, enables better estimation of a signal in non-stationary channels, and gives the FrFT its significant processing advantages over the FFT (a = 1). We will discuss how to estimate ‘a’ in Section 4.

3. CONVENTIONAL CFD USING THE FFT

Consider a received signal sampled in discrete time, y(i), which we write as

$$y(i) = x(i) + x_I(i) + n(i), \quad (5)$$

where x(i) is modulated SOI using one of the modulation schemes to be discussed in Section 5. We upsample the SOI by a factor of SPS (samples per symbol) and filter the signal using a root-raised cosine (RRC) filter with roll-off factor α. Also, $x_I(i)$ is the interference term, and n(i) is additive white Gaussian noise (AWGN). We set the variance of the noise samples, σ^2_N, based on the desired SNR, expressed in dB, using

$$\sigma_N^2 = \frac{1}{10^{SNR/10}}. \quad (6)$$

Index i denotes the i^{th} sample, where i = 1, 2, ..., N , and N is the total number of samples per block that we process; we can therefore write the received signal in vector form as

$$\mathbf{y} = [y(1), y(2), ..., y(N)]. \quad (7)$$

Generally, in a benign, stationary environment, the larger we make N, the better our detection rate of the SOI, because we obtain a sharper peak and finer resolution when taking FFTs. However, in a non-stationary environment, we must make N small enough so that the signal is close to stationary over the N samples, else performance degrades. Hence, in a real world non-stationary channel, our detection performance suffers because of limitations in N. We further process M blocks of **y** to obtain a statistical measure of the signal detection probability. In this paper we consider one or two interferers that are approximately the same or lower power as the SOI, and are non-stationary in time or in frequency. The interferers considered will be described more specifically in Section 5.

A conventional CFD attempts to detect the signal x(i), but does not consider the effect of non-stationarity of the signals, thereby relying solely on FFT techniques and allowing coherent integration. Denoting the FFT size as N_{FFT}, we compute

$$\mathbf{y}_{CFD} = \frac{|FFT(\mathbf{y}(1:N-1)\cdot\mathbf{y}^*(2:N), N_{FFT})|}{\sqrt{N_{FFT}}}. \tag{8}$$

where $*$ denotes complex conjugate and the notation shows that the second term has been delayed from the first term by one sample. Note that conventional CFDs usually perform the operation by averaging y_{CFD} over N_C blocks of size N_{FFT} to improve detection performance, a process called coherent integration, but because of non-stationarity in real channels, this will degrade performance, so in this paper we restrict our attention to $N_C = 1$. We compute the peak of the CFD and compare it to a threshold, given by [10]

$$\gamma = 0.5\sigma_N^2\chi^{-2}(1-P_{FA}, 2N_C), \tag{9}$$

where P_{FA} denotes the desired probability of false alarm (i.e false detection), and χ^{-2} denotes the inverse of the chi-squared cumulative distribution function (CDF), given by

$$\chi^2(z,v) = \Gamma^{-1}(z, v/2, 2), \tag{10}$$

with v denoting the number of degrees of freedom in the samples in z and Γ^{-1} denoting the inverse of the Gamma function. The Gamma function is defined as [7]

$$\Gamma(t,h,c) = \int_0^t \frac{c}{h}x^{h-1}e^{-x/c}dx. \tag{11}$$

If the peak in the CFD is greater than the threshold γ, we declare a detection occurred. We count the total number of detections over the M trials and divide by M to obtain the probability of detection, P_D.

The theoretical, or ideal, performance of the CFD is ([3] and [9])

$$P_{FA_i} = e^{\frac{-\rho^2}{2\sigma_w{}^2}}, \tag{12}$$

and

$$P_{D_i} = Q(\frac{\sqrt{2\cdot SNR}}{\sigma_N}, \frac{\rho}{\sigma_w}), \tag{13}$$

where

$$\sigma_w^2 = \frac{\sigma_N^2}{2N+1}, \tag{14}$$

and $Q_M(A, B)$ is the generalized Marcum-Q function

$$Q_M(A,B) = \int_B^\infty x\cdot e^{-(x^2+A^2)/2}\cdot I_0(Ax)dx, \tag{15}$$

and $I_0(\cdot)$ is the modified Bessel function of the first kind, of order zero. These equations allow us to compute the theoretical P_{Di} as a function of the desired P_{FA} and SNR, which we will compare with the simulation results in Section 5. For each value of SNR, we compute σ_N, then compute σ_W from Eq. (14). Next, from P_{FA} and σ_W we compute ρ from Eq. (12). Finally, we use Eq. (13) to compute P_{Di}.

4. PROPOSED CFD METHOD USING THE FRFT

A. Training Mode

The proposed method using the FrFT modifies the above by translating the time axis to a new axis t_a given by the rotational parameter 'a' containing more of the signal energy than the original time or frequency axes. Suppose we have some information about the signal that we are trying to detect, e.g. a known training sequence, preamble, header, etc. Then we can use a modified version of the algorithm presented in [17], where we choose the axis in which the desired SOI and interference overlap as little as possible. Since we do not have a measure of the interference separate from the SOI as was used in [17], we use the received signal instead and choose the axis where the SOI and received signal overlap the most. The proposed algorithm with the required modifications is summarized as follows [17]:

(1) Initialize: $a = 0$.
(2) Compute $|\mathbf{X}_a(i)|^2 = |\mathbf{F}^a\mathbf{x}(i)|^2$.
(3) Compute $|\mathbf{Y}_a(i)|^2 = |\mathbf{F}^a\mathbf{y}(i)|^2$.
(4) Compute $\Re_{\mathbf{XY}}(a) \equiv \sum_{i=1}^{N} |\mathbf{X}_a(i)|^2|\mathbf{Y}_a(i)|^2$.
(5) Increment a and repeat Steps 2 to 4 until $a = 2$.
(6) Choose the value of a for which $\Re_{\mathbf{XY}}(a)$ in Step 4 above, is maximum. We denote this as $a_{opt,t}$, where the subscript t indicates training mode operation.

We increment 'a' in Step (5) using a nominal step size, e.g. $\Delta a = 0.1$. Recall that $\mathbf{F}^a$ was defined in Eq. (2). Then, using the above value of a_{opt}, we obtain the FrFT-CFD by computing

$$\mathbf{Y}_{a_{opt,t}} = \mathbf{F}^{a_{opt,t}}\mathbf{y}, \tag{16}$$

and compute FrFT-CFD using

$$\mathbf{y}_{FrFT-CFD,t} = |\mathbf{Y}_{a_{opt,t}}(1:N-1)\cdot\mathbf{Y}^*_{a_{opt,t}}(2:N)|. \tag{17}$$

We compare $\mathbf{y}_{FrFT-CFD,t}$ to the same threshold as with the original CFD and compute detection probabilities as before.

B. Blind Mode

Without the benefit of a training signal, we can still apply the FrFT by simply computing the energy of the composite, received signal over the range of 'a' from $0 \le a \le 2$, again using $\Delta a = 0.1$, and choosing the value of 'a' for which the energy is maximum. That is, we compute

$$a_{opt,b} = \underset{a}{arg\ max}|\mathbf{Y}_a|^2 = \sum |\mathbf{F}^a\mathbf{y}|^2, \tag{18}$$

where the summation is over all the N samples in y. Using this new value of $a_{opt,b}$ in place of $a_{opt,t}$, we compute the FrFT-CFD in blind mode

$$\mathbf{Y}_{a_{opt,b}} = \mathbf{F}^{a_{opt,b}}\mathbf{y}, \tag{19}$$

and compute FrFT-CFD using

$$\mathbf{y}_{FrFT-CFD,b} = |\mathbf{Y}_{a_{opt,b}}(1:N-1)\cdot\mathbf{Y}^*_{a_{opt,b}}(2:N)|. \tag{20}$$

The same threshold as before is used to determine detection probabilities. Note that here we are trying to detect an unknown signal, so we cannot take advantage of a training signal. However, since we are using the energy to determine the optimum rotational axis, we expect minimal degradation over training mode since this energy is present in both the training signal and the received signal. This would be sufficient for signal detection, but for signal demodulation we need the technique in [17] that employs a training signal and uses gaps in the training signal to estimate the non-stationary interference to filter it out in the fractional space. Both training and blind modes should outperform the conventional CFD which only operates in the frequency domain. All the methods will, however, suffer performance loss because we cannot coherently integrate when we operate in a real world, non-stationary environment.

5. SIMULATIONS

We present simulation examples to compare the performance of the CFD algorithm and the theoretical result to the proposed FrFT-CFD algorithm by letting $x(i)$ be a signal using all of the following modulation schemes: binary frequency shift keying (BFSK), binary phase shift keying (BPSK), quaternary phase shift keying (QPSK), minimum shift keying (MSK), and continuous phase frequency shift keying (CPFSK). We assume that the interference is composed of two signals,

$$x_I(i) = x_{I,1}(i) + x_{I,2}(i), \tag{21}$$

one taking on the form of a chirp signal,

$$x_{I,1}(i) = e^{-j1.73\pi(i/f_s)^2}, \tag{22}$$

and the other a Gaussian pulse,

$$x_{I,2}(i) = \beta e^{-\pi(i/f_s-\phi)^2}, \tag{23}$$

where f_s is the sampling rate in Hz, and β and ϕ are the amplitude and phase of the pulse, respectively, uniformly distributed in (0, 1). Letting the number of samples per symbol be SPS = 4, and taking f_S = 1 MHz, the symbol rate is therefore $R_s = f_s/SPS$ = 250 ksps. We let the block size and FFT size be $N = N_{FFT}$ = 64 samples, equivalent to 16 symbols at 4 samples per symbol, and we process M = 5,000 trials to obtain a statistical estimate of the PD . Recall that no coherent averaging is done, so the detection decisions are made using just 64 samples. We study the effect of filtering by upsampling the SOI with an RRC filter with rolloff α = 0.4, and we study the detectors' performance for $P_{FA} = 10^{-1}$, 10^{-2}, and 10^{-3}. Performance is not significantly affected by the choice of filter rolloff, but the effect of transmitter filtering does make performance worse and therefore must be included. We plot PD vs. SNR for both training mode and blind mode. The plots are shown in Figs. 1-6.

At $P_{FA} = 10^{-1}$, we see the most significant improvement in our proposed technique, which offers up to 6.5 dB improvement over the conventional algorithm in training mode, only slightly less at 6 dB in blind mode, when $P_D \geq 90\%$. This value of P_D is chosen because a smaller value would typically not be considered sufficient enough. If we can relax the requirement to $P_D \geq 80\%$ we see up to 11 dB improvement in our technique. Note that the performance of the CFD and FrFT-CFD exceeds ideal performance when P_{FA} is high. This occurs because the interference contributes non-Gaussian noise to the total noise, thereby making the theoretical expression, which assumes only AWGN, inaccurate. Interference also increases the noise floor, thereby increasing the power of the peak used in the detection finding technique, which increases the P_D; recall that the selection of the threshold in our model is unaltered by the presence of the interference. The simulation performance is further enhanced by the fact that the detection threshold is lower when P_{FA} is higher, which makes the contribution of the non-Gaussian interference more dominant over the AWGN, but again the theoretical model does not include interference. Development of a theoretical model in the FrFT domain is left for future work. We did observe that if there is no non-stationary interference, then the theoretical curves match the simulation results for both the FrFT-CFD and the CFD, as would be expected.

When $P_{FA} = 10^{-2}$, we are able to achieve $P_D \geq 90\%$ with $2-2.5$ dB less SNR in training mode with the FrFT-CFD vs. the CFD; this drops to about $0.5-1$ dB in blind mode, as expected. However, notice that we begin detecting signals at much lower SNRs, up to $6-7$ dB lower, but this improvement reduces as P_D increases; nonetheless, we achieve better performance at all values of P_D for all modulation schemes. There are differences in performance depending on modulation scheme, with BFSK performing the worst and CPFSK the best. This is true regardless of algorithm choice. The performance would normally be improved using coherent integration, which we cannot do here. Nonetheless, the FrFT-CFD still outperforms the conventional CFD. Note also that the CFD performance is always slightly worse than theoretical performance, as expected, by about $1-3$ dB, whereas the FrFT-CFD is able to achieve close or even better than theoretical by exploiting the FrFT domain.

When P_{FA} reduces to 10^{-3}, we begin to see a degradation in performance. Our algorithm still provides about $1-2.5$ dB improvement for $P_D \geq 90\%$ in training mode, but blind mode degrades. This is to be expected because in blind mode, as P_{FA} reduces, the threshold increases, and the interference can smear the SOI. Hence, the benefit of the algorithm is most significant when a training signal is available. If this can be obtained, we continue to see $0.5-1$ dB improvement in the FrFT-CFD over the CFD even at $P_{FA} = 10^{-5}$ for $P_D \geq 80\%$.

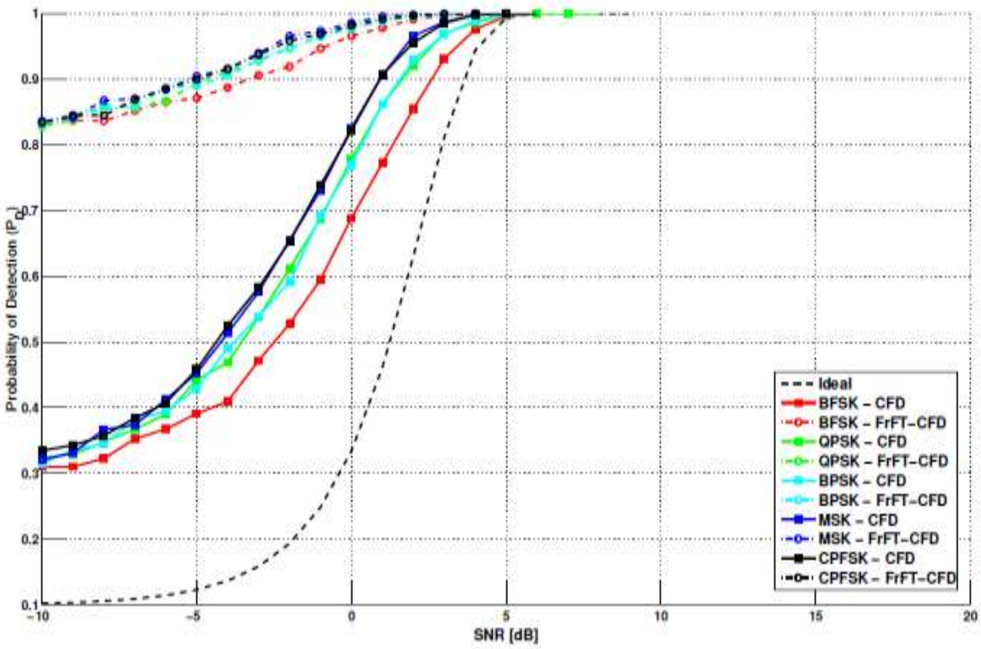

Fig. 1. **Probability of Detection (P_D) vs. SNR [dB]; Training Mode; $P_{FA} = 10^{-1}$, Filter with Rolloff, α = 0.4, M = 5000 Trials**

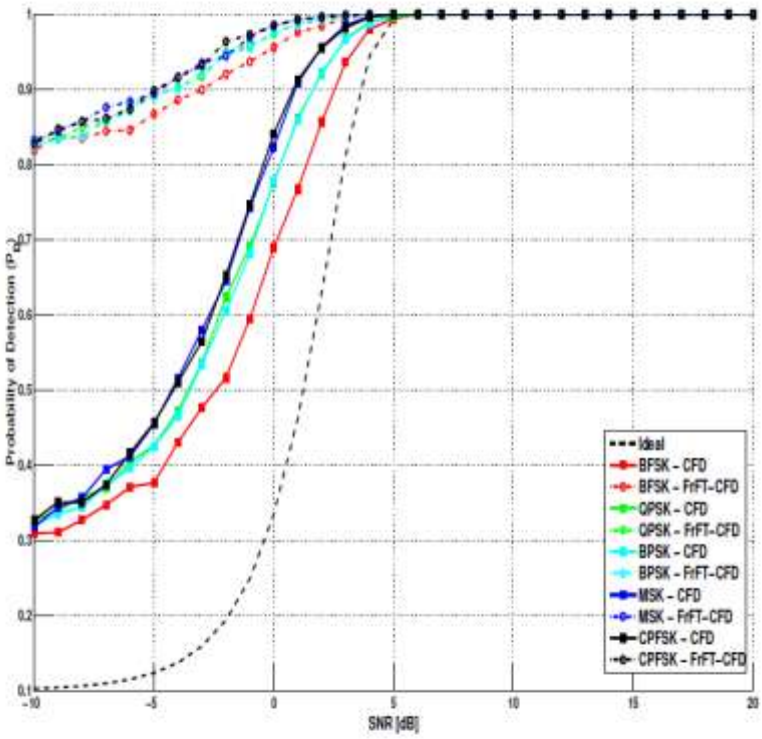

Fig. 2. **Probability of Detection (P_D) vs. SNR [dB]; Blind Mode; $P_{FA} = 10^{-1}$, Filter with Rolloff, α = 0.4, M = 5000 Trials**

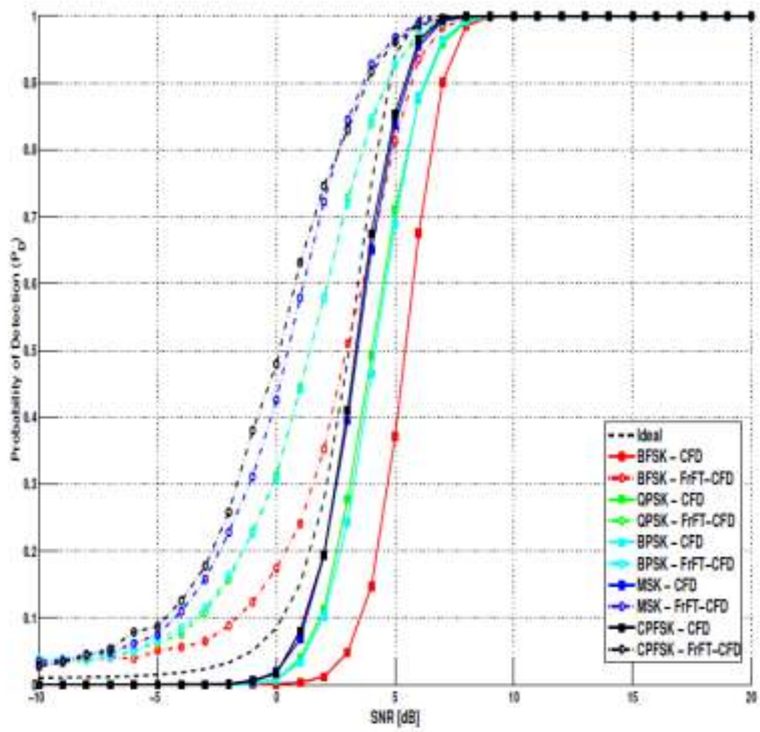

Fig. 3. **Probability of Detection (P_D) vs. SNR [dB]; Training Mode; $P_{FA} = 10^{-2}$, Filter with Rolloff, α = 0.4, M = 5000 Trials**

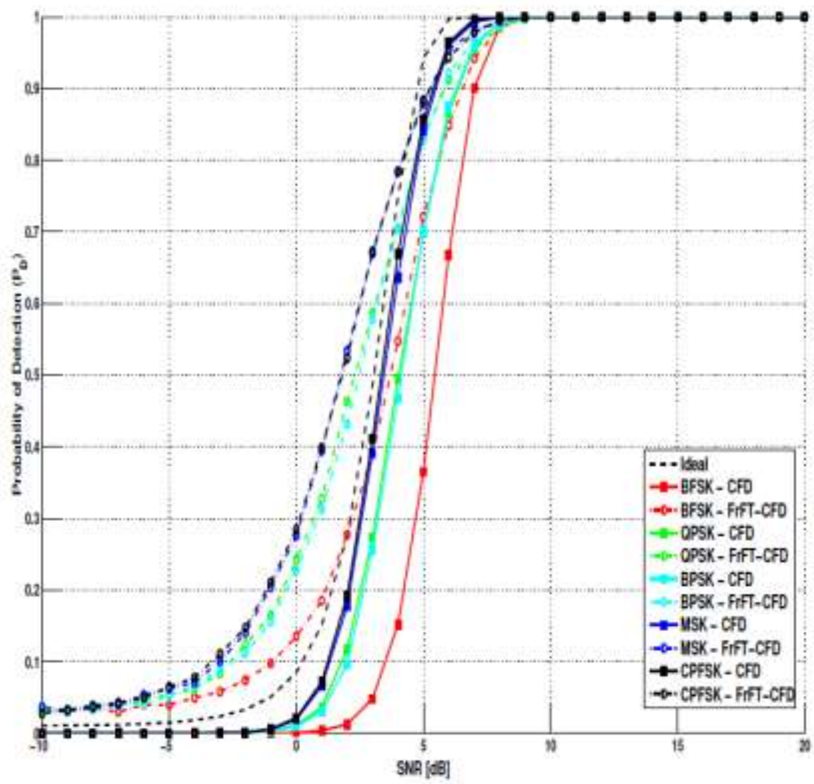

Fig. 4. **Probability of Detection (P_D) vs. SNR [dB]; Blind Mode; $P_{FA} = 10^{-2}$, Filter with Rolloff, α = 0.4, M = 5000 Trials**

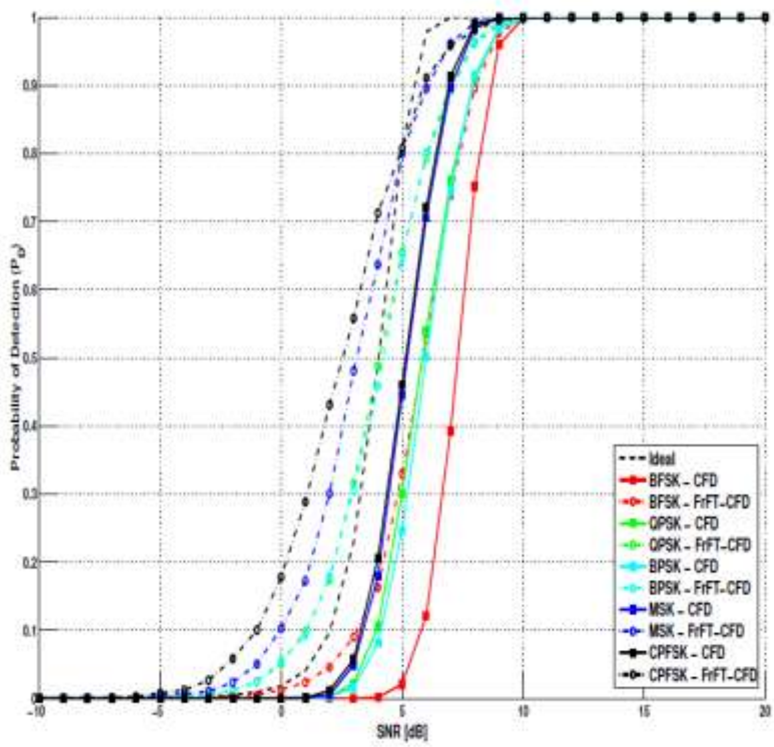

Fig. 5. **Probability of Detection (P_D) vs. SNR [dB]; Training Mode; $P_{FA} = 10^{-3}$, Filter with Rolloff, α = 0.4, M = 5000 Trials**

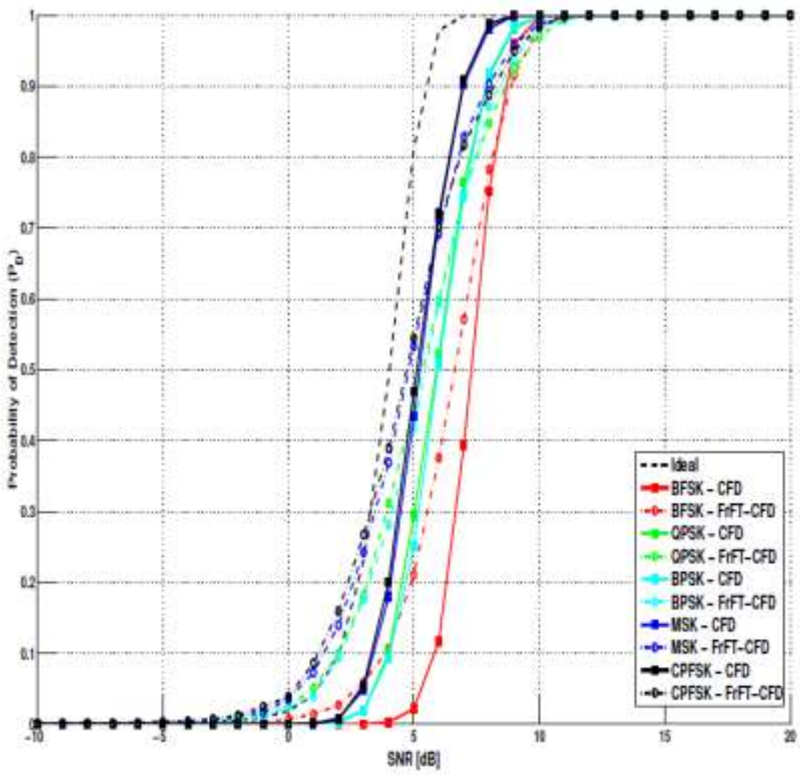

Fig. 6. Probability of Detection (P_D) vs. SNR [dB]; Blind Mode; $P_{FA} = 10^{-3}$, Filter with Rolloff, α = 0.4, M = 5000 Trials

6. CONCLUSION

In this paper, we study a new cyclo-stationary feature detector using the Fractional Fourier Transform (FrFT-CFD) in place of the traditional FFT-based CFD, simply called CFD. This requires first computing the FrFT rotational axis by searching and finding the rotational parameter 'a' for which the projection of the signal energy, i.e. the magnitude of the FrFT of the signal, is maximum. Once the best 'a' is found, we compute the FrFT-CFD by replacing the signal with its FrFT, using the parameter 'a'. This equates to multiplying the signal by a matrix, which is typically kept small due to our desire to operate in non-stationary environments. We show through simulation, that the proposed FrFT-CFD method outperforms traditional CFDs based on the FFT when non-stationary interference is present for a number of modulation schemes, and a range of false alarm probability. More significant benefit occurs when higher probability of false detections are acceptable. The algorithms can operate blindly, without any known training sequence, offering improvements over the conventional technique of $1-2.5$ dB down to $P_{FA} = 10^{-2}$; blind mode at $P_{FA} \leq 10^{-3}$ requires further study. When a training signal is used, this greatly increases to $6-11$ dB at $P_{FA} = 10^{-1}$ and $1-2.5$ dB at $P_{FA} = 10^{-3}$. The algorithm performs without the benefit of coherent integration, which is limited in non-stationary channels; however, coherent integration would improve performance if it can be applied, but this is left as a topic for future work. Future work therefore involves improving the FrFT-CFD performance when operating blindly and when $P_{FA} \leq 10^{-3}$ and could also involve modifying the theoretical detector performance to include interference.

7. ACKNOWLEDGMENTS

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