Capacity of Gaussian MIMO Channel for Network Intra-Chip RF Interconnect

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Abstract—Continuous increase of integration density, operating frequency, thermal stress and other complexities are pushing the conventional metal interconnects based IC technology to its material and physical limits. Radical alternative like on-chip radio frequency (RF) and wireless communication is being explored to replace the hard-wired metal interconnect. Currently for inter-chip and intra-chip RF/wireless communication various single-input- single-output (SISO) schemes like BPSK, OQPSK, MSK and GMSK are being investigated. This paper presents a multiple input-multiple output (MIMO) system for on-chip RF/wireless communication to significantly enhance system performance compared to the conventional systems. We propose to utilize a Gaussian channel environment for the MIMO system. This paper evaluates the capacity (bit/s/Hz) of the wireless Gaussian MIMO channel system and compares it with the SISO channel system currently under investigation.

Index Terms— Channel Capacity, Gaussian MIMO Channel, RF Interconnect, Signal-to-Noise-Ratio, Spectral Domain.

I. INTRODUCTION

Improvements in integrated circuit (IC) density and performance have fueled the semiconductor industry and resultant information revolution for more than 50 years. The periodic improvement in density and performance has been achieved through continuous scaling of technologies, which closely followed Moore’s law [1]. While device performance improves with the scaling of the gate length, gate dielectric thickness and junction depth, scaled chip wiring (interconnect) suffers from increased parasitic resistance due to a decrease in conductor cross-sectional area. It may also suffer from increased parasitic capacitance if metal height is not reduced with conductor spacing. As operating frequencies continue to increase, parasitic inductance of interconnect have begun influencing the circuit and system performance [2]-[3]. In fact, recent studies have shown that circuit and system performances are severely affected by the interconnect parasitics below 1-μm minimum feature sizes [4].

The initial research efforts were directed towards improving traditional metal interconnect performance in the submicron region. Repeater insertion, resistivity reduction by using copper, and reduction of dielectric constant of interlayer dielectric materials by using low-k polymers are the standard practices to minimize interconnect problems [5]-[6]. According to the International Technology Roadmap for Semiconductors (ITRS), material innovation with traditional scaling will no longer satisfy the performance requirements in the long term and radically new interconnect paradigms are needed. The continued progress of interconnect performance will require approaches that introduce materials and structures beyond the conventional metal/dielectric system, and may require information carriers other than charge. Multiple options have been envisioned to provide alternatives to the metal/dielectric system. In particular, three emerging interconnect technologies (3-dimensional integration, optical interconnect, and RF/wireless interconnect) are promising solution for future VLSI technology.

Recently, RF/wireless interconnect has drawn considerable attention due to the fact that this technology is CMOS compatible, which makes integration issue simpler. It was first proposed for inter- and intra-chip communications [7]. RF interconnects using on-chip antennas, integrated transmitters, and receivers were demonstrated in [8]. A multi-Gbits/sec board–level clock distribution network using RF interconnect has been effective in minimizing power consumption, skew, and jitter [9]. RF/wireless interconnect is also useful for intra- and inter-chip data communications. Error-correction coding (ECC) and time-diversity approaches are investigated for intra- and inter-chip RF interconnects in [3], [5] and [6] respectively. Bit-error-rate (BER) performance of intra-chip RF/wireless interconnects is derived analytically using intra-chip wireless channel characterization in [10]. Chip-multiprocessor (CMP) based design paradigm is expected to replace existing single-core solution for nanoscale integrated circuits and systems in future. CMP cores will be connected with a network-on-chip (NoC) interconnects fabric. RF/wireless interconnects have the potential to provide multiple-access communication features, which will surely be useful in a NoC interconnects system especially for cores placed far away from each other in the CMP environment. The use of multi-band RF interconnect where signal propagates at the speed of light to provide shortcuts in a many-core network-on-chip (mesh) topology is shown in [11]. In a recent work [12], an approach called wireless NoC (WiNoC) as interconnection backbone has been presented for measuring, characterizing, and modeling of interconnect systems. However, all the techniques used currently in RF/wireless interconnect is single-input single-output based system. As RF/wireless channel is employed for data communication, capacity measurement carries great significance in overall system performance. As the radio spectrum is limited, the RF/wireless interconnect channel capacity needs cannot be met without a significant increase in communication spectral efficiency by SISO system. However,
large gain in capacity of RF interconnect is possible in multiple-input multiple-output (MIMO) system. Significant improvement in spectral efficiency is possible by increasing the number of antennas both at the transmitter and receiver ends. This paper analyzes the capacity of RF interconnect for transmission over Gaussian MIMO channel and compares it with the conventional Gaussian SISO Channel.

The rest of the paper is organized as follows. Section II described a traditional RF/wireless interconnect model for SISO system. Section III and Section IV calculate the capacity (bits/s/Hz) of Gaussian SISO and MIMO channel model respectively. Section V compares the capacities of SISO and MIMO channels. Finally, Section VI summarizes and concludes the paper with a brief overview of future research.

II. CONCEPT OF TRADITIONAL RF/WIRELESS INTERCONNECT OF SISO CHANNEL

Figure 1 shows a conventional SISO RF/wireless interconnect scheme, which is based on the transmission of waves through a RF-transceiver. In addition to the transmitter and the receiver, it contains capacitive couplers and an impedance-matched transmission line that are terminated by its characteristics impedance ($Z_0$) to dc-ground at the both ends. In this case, RF signals are up-linked to the shared broadcasting medium, coplanar waveguide (CPW) or micro-strip transmission line (MTL) via transmitting capacitive couplers, and then down-linked via receiving capacitive couplers [13]. Capacitive couplers work as near-field antennas. Both CPW and MTL are known to have low attenuation up to at least 200 GHz [14].

III. CAPACITY OF GAUSSIAN SISO SYSTEM

At the input of a communication system, discrete source symbols are mapped into a sequence of channel symbols. The channel symbols are then transmitted through a wireless channel that by nature is random. In addition, random noise is added to the channel symbols. In general, it is possible that two different input sequences may give rise to the same output sequence, causing different input sequences to be confusable at the output. To avoid this situation, a non-confusable subset of input sequences must be chosen so that with a high probability, there is only one input sequence causing a particular output. It is then possible to reconstruct all the input sequences at the output with negligible probability of error. A measure of how much information that can be transmitted and received with a negligible probability of error is called the channel capacity. It is desirable to design transmission schemes that exploit the channel capacity as much as possible. Representing the input
and output of a memoryless wireless channel (see Figure 2) with the random variables x and y respectively, the channel capacity is defined as in (1) [15]-[16], where \( I(x;y) \) represents the mutual information between \( x \) and \( y \). Equation (1) states that the mutual information is maximized with respect to all possible transmitter statistical distributions \( p(x) \).

\[
C = \max_{p(x)} I(x;y) \quad (1)
\]

Mutual information is a measure of the amount of information that one random variable contains about another variable. The mutual information between \( x \) and \( y \) can also be written as in (2), where \( H(y|x) \) represents the conditional entropy between the random variables \( x \) and \( y \):

\[
I(x;y) = H(y) - H(y|x) \quad (2)
\]

The entropy of a random variable can be described as a measure of the amount of information required on average to describe the random variable. It can also describe as a measure of the uncertainty of the random variable. Mutual information in (2), which can be described as the reduction in the uncertainty of one random variable due to the knowledge of the other. The ergodic (mean) capacity of a random channel with \( n_T = n_R = 1 \) and an average transmit power constraint \( P_T \) can be expressed as in (3) [16]-[17], where \( P \) is the average power of a single channel code word transmitted over the channel.

\[
C = \max_{f(x):P \leq P_T} I(x;y) \quad (3)
\]

Compared to the definition of (1), the capacity of the channel can be defined as the maximum of the mutual information between the input and the output over all statistical distributions on the input that satisfy the power constraint. By expanding \( I(x;y) \), we get the equation (4).

\[
I(x;y) = h(y) - h(y|x) = h(y) - h(x + z|x) = h(y) - h(z|x) = h(y) - h(z) \quad (4)
\]

Here \( h(z) = \frac{1}{2} \log_2 2\pi eN \). Since the output power can be expressed as in (5) and the entropy of \( y \) is bounded by \( \frac{1}{2} \log_2 2\pi e(P + N) \) [16] which applying this result to bind the mutual information.

\[
E(y)^2 = E(x + z)^2 = E(x)^2 + 2E(x)E(z) + E(z)^2 = P + N \quad (5)
\]

\[
I(x;y) = h(y) - h(z) \leq \frac{1}{2} \log_2 2\pi e(P + N) \quad (6)
\]

\[
\frac{1}{2} \log_2 2\pi eN = \frac{1}{2} \log_2 (1 + \text{SNR}) \quad (6)
\]

Therefore, the information capacity of the Gaussian channel for SISO system can be expressed as in (7).

\[
C = \frac{1}{2} \log_2 (1 + \text{SNR}) \quad (7)
\]

This capacity is of the performance criteria for data communication and it depends on the signal-to-noise (SNR) ratio of the received signal at the receiver. Average power per bit \( P_{rb} \) is given by \( P_{rb} = P_{tb} + G_{tr} + G_r \), where \( P_{tb} \) is the transmitted power per bit in dBm, \( G_{tr} \) is the average value of the transmission gain between the transmit and receive antennas in dB, and \( G_r \) is the gain of the receiver in dB. Two sources of noise are present in the RF/wireless SISO channel: (i) thermal noise and (ii) switching noise. The thermal noise power spectral density [13] is given by (8), where \( k \) is the Boltzmann constant, \( T_0 \) is the reference temperature, \( T_{ant} \) is the antenna temperature, and \( F_r \) is the receiver noise figure. The switching noise power spectral density [18] is given by equation (9), where \( S_n \) is the LPF output of the switching noise activities, \( H_n \) is the channel transfer function, \( N \) is the total test number, \( j \) is the test index, and \( i \) represents the \( i \)-th switching noise attacker. The average bit SNR at the receiver end is expressed as \( \text{SNR} = \frac{P_{rb}}{N_0 + S_0} \) [16].

\[
N_o = kT_0 \left( \frac{T_{ant}}{T_0} + F_r \right) \quad (8)
\]

\[
S_o = \frac{1}{N} \sum_{j=1}^{N} \sum_{i=1}^{N} S_n(f, V_{ij})|H_n(f, p_{ij})|^2 \quad (9)
\]

IV. CAPACITY OF GAUSSIAN MIMO CHANNEL

Consider a single point-to-point MIMO system with arrays of \( n_T \) transmit and \( n_R \) receive antennas. The general modeling

\[
\text{Figure 3 Gaussian MIMO Channel Model between Tree and Irregular Network.}
\]
of a channel as an abstract MIMO channel allows for a unified treatment using a compact convenient vector-matrix notation. The proposed MIMO RF/wireless interconnect model is shown in Figure 3. The transmitted signals in each symbol period are represented by a \( n_T \times 1 \) column matrix \( x \), where the \( j \)-th component \( x_j \), refers to the transmitted signal from antenna \( j \). We consider a Gaussian channel so that the optimum distribution of transmitted signals are also Gaussian. Thus the elements of \( x \) are considered to be zero mean independent identically distributed (i.i.d.) Gaussian variables. The covariance matrix of the transmitted signal \((\text{XX}^H)\) is given by \( \mathbf{R}_{xx} = E(\text{XX}^H) \), where \( E(\cdot) \) denotes the expectation and the operator \( \mathbf{X}^H \) denotes the Hermitian of matrix \( \mathbf{X} \), which means the transpose and component-wise complex conjugate of \( \mathbf{X} \). The total transmitted power is constrained to \( P_T \), regardless of the number of transmit antennas \( n_T \). It can be represented as \( P = tr(\mathbf{R}_{xx}) \), where \( tr(\mathbf{A}) \) denotes the trace of matrix \( \mathbf{A} \), obtained as the sum of the diagonal elements of \( \mathbf{A} \). By using the linear model, the received vector can be represented equation (10).

\[
y = \mathbf{H}x + n
\]

where the channel Matrix \( \mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1nT} \\
. & \ddots & . \\
h_{nT1} & \cdots & h_{nTnT} \end{bmatrix} \) and \( h_{ij} = \alpha + j\beta = \sqrt{\alpha^2 + \beta^2} \cdot e^{-j\arctan(\frac{\beta}{\alpha})} \).

The received signal covariance matrix, defined as \( E(\text{yy}^H) \), by using (10), is given by \( \mathbf{R}_{yy} = \mathbf{R}_{xx} \mathbf{H}^H \), while the total received signal power can be expressed as \( tr(\mathbf{R}_{yy}) \). The capacity of a random MIMO channel for RF/wireless interconnect (see Figure 2) with power constraint \( P_T \) can be expressed as equation (11).

\[
C = \max_{p(x); tr(\mathbf{R}_{xx}) \leq P_T} I(x; y)
\]

where \( \mathbf{R}_{xx} = E(\text{xx}^H) \) is the covariance matrix of the transmit signal vector \( x \). The total transmit power is limited to \( P_T \) irrespective of the number of transmit antennas. The relationship between mutual information and entropy can be expanded as equation (12) as follows for a given MIMO channel matrix \( \mathbf{H} \).

\[
I(x; y) = h(y) - h(y|x) = h(y) - h(\mathbf{H}x + n|x) = h(y) - h(n|x) = h(y) - h(n)
\]

The differential entropy of a continuous random variable. It is assumed that the transmit vector \( x \) and the noise vector \( n \) are independent for this interconnect structure as we consider the Gaussian channel. Equation (12) is maximized when \( y \) is Gaussian since the normal distribution maximizes the entropy for a given variance [15]. The differential entropy of a real Gaussian vector \( y \in \mathbb{R}^n \) with zero mean and covariance matrix \( \mathbf{K} \) is equal to \( \frac{1}{2} \log_2((2\pi e)^n \det \mathbf{K}) \). For a complex Gaussian vector \( y \in \mathbb{C}^n \), the differential entropy is less than or equal to \( \frac{1}{2} \log_2 \det(\pi e \mathbf{K}) \) with equality if and only if \( y \) is a circularly symmetric complex Gaussian with \( E(y y^H) = \mathbf{K} \) [20]. Therefore, the optimal Gaussian distribution for the transmit vector \( x \), the covariance matrix of the received complex vector \( y \) is given by equation (13).

\[
E(\text{yy}^H) = E((\mathbf{H}x + n)(\mathbf{H}x + n)^H) = E(\mathbf{H}xx^H \mathbf{H}^H) + E(n n^H) = \mathbf{HR}_{xx} \mathbf{H}^H + \mathbf{K} = \mathbf{K}^d + \mathbf{K}^n
\]

The superscript \( d \) and \( n \) denotes the desired part and the noise part of equation (13), respectively. The maximum mutual information of a random MIMO channel is then calculated as (14).

\[
I(x; y) = h(y) - h(n)
\]

\[
= \frac{1}{2} \log_2(\det(\pi e(\mathbf{K}^d + \mathbf{K}^n))) - \frac{1}{2} \log_2(\det(\pi e \mathbf{K}^n))
\]

\[
= \frac{1}{2} \log_2(\det((\mathbf{K}^d + \mathbf{K}^n)(\mathbf{K}^n)^{-1}))
\]

\[
= \frac{1}{2} \log_2(\det(\mathbf{K}^d(\mathbf{K}^n)^{-1} + I_{n_T}))
\]

When the transmitter has no knowledge about the channel, it is optimal to use a uniform power distribution [20]. The transmit covariance matrix is then given by \( \mathbf{R}_{xx} = \frac{P_T}{n_T} I_{n_T} \). It is also common to assume uncorrelated noise in each receiver branch described by the covariance matrix \( \mathbf{K}^n = \sigma^2 I_{n_T} \). The ergodic (mean) capacity [20]-[21] for a complex Gaussian MIMO channel can be expressed as (15).

\[
C = \frac{1}{2} \log_2 \left[ \det \left( I_{n_T} + \frac{P_T}{\sigma^2 n_T} \mathbf{H} \mathbf{H}^H \right) \right]
\]

By using singular value decomposition (SVD) theorem [22], any \( n_T \times n_T \) channel matrix, \( \mathbf{H} \) can be written as \( \mathbf{H} = \mathbf{UDV}^H \), where \( \mathbf{D} \) are \( n_T \times n_T \) non-negative and diagonal matrix, \( \mathbf{U} \) and \( \mathbf{V} \) are \( n_T \times n_T \) and \( n_T \times n_T \) unitary matrices respectively. That is, \( \mathbf{UU}^H = I_{n_T} \) and \( \mathbf{VV}^H = I_{n_T} \), where \( I_{n_T} \) and \( I_{n_T} \) are \( n_T \times n_T \) and \( n_T \times n_T \) identity matrices respectively. The diagonal entries of \( \mathbf{D} \) are the non-negative square roots of the eigenvalues of matrix, \( \mathbf{HH}^H \). Therefore, the MIMO channel capacity can be written as equation (16).

\[
C = \frac{1}{2} \log_2 \left[ \det \left( I_{n_T} + \frac{P_T}{\sigma^2 n_T} \mathbf{U} \mathbf{D} \mathbf{D}^H \mathbf{U}^H \right) \right]
\]

After diagonalizing the product matrix \( \mathbf{HH}^H \), the capacity formulas of the MIMO channel now includes unitary and diagonal matrices only. It is then easier to see that the total capacity of a MIMO channel is made up by the sum of parallel Gaussian SISO sub-channels. The number of parallel sub-channel is determined by the rank of the channel matrix which is given by \( \text{rank } (\mathbf{H}) = \min \{ n_R, n_T \} \). The transmit power from
each antenna in the equivalent MIMO channel model is \( P / n_r \) and estimate the overall channel capacity as follows in (17),
\[
C = \sum_{i=1}^{n_T} \frac{1}{2} \log_2 \left( 1 + \frac{P_{ri}}{\sigma^2} \right)
\]
where \( P_{ri} \) is the received signal power in the \( i \)-th sub-channel. It is given by \( P_{ri} = \frac{P_i}{n_r} \) where \( \sqrt{\lambda_i} \) is the singular value of channel matrix \( H \). In this channel, with the channel matrix given by equation (10), the same signal is transmitted simultaneously from \( n_r \) antennas. The received signal and the received signal power at antenna \( i \) is given by \( \sqrt{\lambda_i} n_r \) and \( P_{ri} = n_T \frac{P}{n_r} \), where \( P / n_r \) is the power transmitted from one antenna. As the power per transmit antenna is \( P / n_r \), the total received power per receive antenna is \( n_r P \). The power gain of \( n_r \) in the total received power comes due to coherent combining of the transmitted signals. The rank of channel matrix \( H \) is 1, so there is only one received signal in the equivalent channel model with the power \( P_i = n_r n_T P \). Finally, the capacity of Gaussian MIMO channel of our proposed RF/wireless interconnect is achieved as equation (18).
\[
C = \frac{1}{2} \log_2 \left( 1 + n_r n_T \frac{P}{\sigma^2} \right)
\]
\[
= \frac{1}{2} \log_2 (1 + n_r n_T \text{SNR})
\]

V. RESULTS ANALYSIS

A MATLAB simulation was carried out to simulate the data rate of Gaussian MIMO and SISO channel in network intra-chip RF/wireless interconnect system. It is evident from Error! Reference source not found. that when the number of transmitting and receiving antenna is increased, the capacity (bits/s/Hz) is significantly increased in MIMO RF/wireless interconnect system than SISO system with constant SNR and bandwidth. Table 1 shows a comparison of channel capacity between a MIMO and SISO system which revealed that how the channel capacity increased with increasing the number of transmitting and receiving antennas.

Figure 5 and Figure 6 shows the throughput (bit/s) performance in terms of frequency and SNR between MIMO and SISO channel respectively. From Figure 5, it is shown that MIMO RF/wireless channel gives higher data rate SISO RF/wireless channel at 10 dB SNR. Similar result also observe in Figure 6. At certain frequency, the data rate of \( 4 \times 4 \) MIMO is almost 4 times higher than \( 1 \times 1 \) SISO system. Therefore, it can be said that the use of multiple antennas on both the transmitter and receiver side of a communication link has greatly improve the channel capacity of RF/wireless interconnect systems by remaining constant signal-to-noise ratio as well as channel bandwidth. There are many research papers published on MIMO systems, reflecting the perception that MIMO technology will be seen as one of the most promising research in RF/wireless system applications.

<p>| Table I. A COMPARISON BETWEEN GAUSSIAN MIMO AND SISO CHANNEL |</p>
<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Number of Transmitting and Receiving antenna (( n_T \times n_r ))</th>
<th>Capacity (bits/s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 x 1</td>
<td>1.729</td>
</tr>
<tr>
<td>10</td>
<td>2 x 2</td>
<td>2.679</td>
</tr>
<tr>
<td></td>
<td>2 x 3</td>
<td>2.965</td>
</tr>
<tr>
<td></td>
<td>3 x 2</td>
<td>2.965</td>
</tr>
<tr>
<td></td>
<td>4 x 4</td>
<td>3.665</td>
</tr>
</tbody>
</table>

![Figure 4 Achievable capacity for RF/wireless Gaussian MIMO and SISO channel](image)

![Figure 5 Throughput analysis in terms of bandwidth for Gaussian MIMO and SISO channel](image)
VI. SUMMARY AND CONCLUSION

A mathematical model of the channel capacity of Gaussian MIMO channel for RF/wireless interconnect system is derived and discussed in details. Capacity and throughput is analyzed with MATLAB simulation for different MIMO antenna arrangement and compare with SISO technique. It is shown that MIMO has better performance than SISO. Therefore, we can conclude that MIMO could be a very promising approach for intra-chip network interconnect for future ultra-high frequency circuits and systems. However, implementation of multiple antenna required for MIMO scheme for chip-to-chip or core-to-core communication in a multi-chip-module or many-core system would be a daunting task. Due to strong mutual electromagnetic coupling, it would be very challenging to setup multiple transmitting and receiving antennas. A minimum spacing distance ($\lambda/2$) need to be maintained in order to reduce electromagnetic coupling effect. So far 2X2 MIMO system is experimentally developed for wireless communication. For RF interconnect application MIMO is a very new concept. To avoid electromagnetic effect in extremely dense on-chip environment the frequency of communication has to be in hundreds of GHz and THz level. Therefore, this type of MIMO RF interconnect would be suitable for THz frequency ICs.

REFERENCES


